1 Base Arithmetic

1.1 Binary Numbers

We normally work with numbers in base 10. In this section we consider numbers in *base 2*, often called *binary numbers*.

In *base 10* we use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

In *base 2* we use only the digits 0 and 1.

Binary numbers are at the heart of all computing systems since, in an electrical circuit, 0 represents *no* current flowing whereas 1 represents a current flowing.

In *base 10* we use a system of place values as shown below:

1000	100	10	1		
4	2	1	5	$\rightarrow$	$4 \times 1000 + 2 \times 100 + 1 \times 10 + 5 \times 1$
3	1	0	2	$\rightarrow$	$3 \times 1000 + 1 \times 100 + 2 \times 1$

Note that, to obtain the place value for the next digit to the left, we multiply by 10. If we were to add another digit to the front (left) of the numbers above, that number would represent 10 000s.

In *base 2* we use a system of place values as shown below:

64	32	16	8	4	2	1		
1	0	0	0	0	0	0	$\rightarrow$	$1 \times 64 = 64$
1	0	0	1	0	0	1	$\rightarrow$	$1 \times 64 + 1 \times 8 + 1 \times 1 = 73$

Note that the place values begin with 1 and are multiplied by 2 as you move to the left.

Once you know how the place value system works, you can convert binary numbers to base 10, and vice versa.

Example 1

Convert the following binary numbers to base 10:

(a) 111 (b) 101 (c) 1100110

Solution

For each number, consider the place value of every digit.

(a)

4	2	1		
1	1	1	$\rightarrow$	$4 + 2 + 1 = 7$

The binary number 111 is 7 in base 10.

(b)

4	2	1		
1	0	1	→	4 + 1 = 5

The binary number 101 is 5 in base 10.

(c)

64	32	16	8	4	2	1		
1	1	0	0	1	1	0	→	64 + 32 + 4 + 2 = 102

The binary number 1100110 is 102 in base 10.

Example 2

Convert the following base 10 numbers into binary numbers:

(a) 3 (b) 11 (c) 140

Solution

We need to write these numbers in terms of the binary place value headings 1, 2, 4, 8, 16, 32, 64, 128, ..., etc.

(a)

		2	1
3 = 2 + 1	→	1	1

The base 10 number 3 is written as 11 in base 2.

(b)

		8	4	2	1
11 = 8 + 2 + 1	→	1	0	1	1

The base 10 number 11 is written as 1011 in base 2.

(c)

		128	64	32	16	8	4	2	1
140 = 128 + 8 + 4	→	1	0	0	0	1	1	0	0

The base 10 number 140 is written as 10001100 in base 2.

Exercises

1. Convert the following binary numbers to base 10:

(a) 110 (b) 1111 (c) 1001

(d) 1101 (e) 10001 (f) 11011

(g) 1111111 (h) 1110001 (i) 10101010

(j) 11001101 (k) 111000111 (l) 1100110

2. Convert the following base 10 numbers to binary numbers:

(a)	9	(b)	8	(c)	14
(d)	17	(e)	18	(f)	30
(g)	47	(h)	52	(i)	67
(j)	84	(k)	200	(l)	500

3. Convert the following base 10 numbers to binary numbers:

(a) 5 (b) 9 (c) 17 (d) 33

Describe any pattern that you notice in these binary numbers.
What will be the next base 10 number that will fit this pattern?

4. Convert the following base 10 numbers to binary numbers:

(a) 3 (b) 7 (c) 15 (d) 31

What is the next base 10 number that will continue your binary pattern?

5. A particular binary number has 3 digits.

(a) What are the *largest* and *smallest* possible binary numbers?

(b) Convert these numbers to base 10.

6. When a particular base 10 number is converted it gives a 4-digit binary number. What could the original base 10 number be?

7. A 4-digit binary number has 2 zeros and 2 ones.

(a) List all the possible binary numbers with these digits.

(b) Convert these numbers to base 10.

8. A binary number has 8 digits and is to be converted to base 10.

(a) What is the *largest* possible base 10 answer?

(b) What is the *smallest* possible base 10 answer?

9. The base 10 number 999 is to be converted to binary. How many more digits does the binary number have than the number in base 10?

10. Calculate the difference between the *base 10* number 11111 and the *binary* number 11111, giving your answer in base 10.

1.2 Adding and Subtracting Binary Numbers

It is possible to add and subtract binary numbers in a similar way to base 10 numbers. For example, $1+1+1=3$ in base 10 becomes $1+1+1=11$ in binary. In the same way, $3-1=2$ in base 10 becomes $11-1=10$ in binary. When you add and subtract binary numbers you will need to be careful when 'carrying' or 'borrowing' as these will take place more often.

Key *Addition* Results for Binary Numbers

$$1 + 0 = 1$$
$$1 + 1 = 10$$
$$1 + 1 + 1 = 11$$

Key *Subtraction* Results for Binary Numbers

$$1 - 0 = 1$$
$$10 - 1 = 1$$
$$11 - 1 = 10$$

Example 1

Calculate, using binary numbers:

(a) $111+100$ (b) $101+110$ (c) $1111+111$

Solution

(a)
$$\begin{array}{r} 1\,1\,1 \\ +\,1\,0\,0 \\ \hline 1\,0\,1\,1 \\ \hline {\scriptstyle 1} \end{array}$$

(b)
$$\begin{array}{r} 1\,0\,1 \\ +\,1\,1\,0 \\ \hline 1\,0\,1\,1 \\ \hline {\scriptstyle 1} \end{array}$$

(c)
$$\begin{array}{r} 1\,1\,1\,1 \\ +\quad 1\,1\,1 \\ \hline 1\,0\,1\,1\,0 \\ \hline {\scriptstyle 1\,1\,1} \end{array}$$

Note how important it is to 'carry' correctly.

Example 2

Calculate the binary numbers:

(a) $111-101$ (b) $110-11$ (c) $1100-101$

Solution

(a)
$$\begin{array}{r} 1\,1\,1 \\ -\,1\,0\,1 \\ \hline 1\,0 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 1\,1\,0 \\ -\quad 1\,1 \\ \hline 1\,1 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 1\,1\,0\,0 \\ -\quad 1\,0\,1 \\ \hline 1\,1\,1 \\ \hline \end{array}$$

Exercises

1. Calculate the binary numbers:

(a) $11 + 1$ (b) $11 + 11$ (c) $111 + 11$

(d) $111 + 10$ (e) $1110 + 111$ (f) $1100 + 110$

(g) $1111 + 10101$ (h) $1100 + 11001$ (i) $1011 + 1101$

(j) $1110 + 10111$ (k) $1110 + 1111$ (l) $11111 + 11101$

2. Calculate the binary numbers:

(a) $11 - 10$ (b) $110 - 10$ (c) $1111 - 110$

(d) $100 - 10$ (e) $100 - 11$ (f) $1000 - 11$

(g) $1101 - 110$ (h) $11011 - 110$ (i) $1111 - 111$

(j) $110101 - 1010$ (k) $11011 - 111$ (l) $11110 - 111$

3. Calculate the binary numbers:

(a) $11 + 11$ (b) $111 + 111$

(c) $1111 + 1111$ (d) $11111 + 11111$

Describe any patterns that you observe in your answers.

4. Calculate the binary numbers:

(a) $10 + 10$ (b) $100 + 100$

(c) $1000 + 1000$ (d) $10000 + 10000$

Describe any patterns that you observe in your answers.

5. Solve the following equations, where all numbers, including x, are binary:

(a) $x + 11 = 1101$ (b) $x - 10 = 101$

(c) $x - 1101 = 11011$ (d) $x + 1110 = 10001$

(e) $x + 111 = 11110$ (f) $x - 1001 = 11101$

6. Calculate the binary numbers:

(a) $10 - 1$ (b) $100 - 1$

(c) $1000 - 1$ (d) $10000 - 1$

Describe any patterns that you observe in your answers.

7. (a) Convert the binary numbers 11101 and 1110 to base 10.
   (b) Add together the two base 10 numbers.
   (c) Add together the two binary numbers.
   (d) Convert your answer to base 10 and compare with your answer to (b).

8. (a) Convert the binary numbers 11101 and 10111 to base 10.
   (b) Calculate the difference between the two base 10 numbers.
   (c) Convert your answer to (b) into a binary number.
   (d) Calculate the difference between the two binary numbers and compare with your answer to (c).

9. Here are 3 binary numbers:

   1110101 1011110 1010011

   Working in binary,
   (a) add together the *two smaller* numbers,
   (b) add together the *two larger* numbers,
   (c) take the smallest number away from the largest number,
   (d) add together all three numbers.

10. Calculate the binary numbers:
   (a) $111 + 101 + 100$
   (b) $11101 + 10011 + 110111$

1.3 Multiplying Binary Numbers

Long multiplication can be carried out with binary numbers and is explored in this section. Note that multiplying by numbers like 10, 100 and 1000 is very similar to working with base 10 numbers.

Example 1

Calculate the binary numbers:

(a) 1011×100 (b) 110110×1000 (c) 11011×10000

Check your answers to (a) and (c) by converting each number to base 10.

Solution

(a) $1011 \times 100 = 101100$

(b) $110110 \times 1000 = 110110000$

(c) $11011 \times 10000 = 110110000$

Checking:

(a)

8	4	2	1		
1	0	1	1	$\rightarrow$	$8 + 2 + 1 = 11$

4	2	1		
1	0	0	$\rightarrow$	4

32	16	8	4	2	1		
1	0	1	1	0	0	$\rightarrow$	$32 + 8 + 4 = 44$

and $11 \times 4 = 44$, as expected.

(c)

16	8	4	2	1		
1	1	0	1	1	$\rightarrow$	$16 + 8 + 2 + 1 = 27$

16	8	4	2	1		
1	0	0	0	0	$\rightarrow$	16

256	128	64	32	16	8	4	2	1		
1	1	0	1	1	0	0	0	0	$\rightarrow$	$256 + 128 + 32 + 16 = 432$

and $27 \times 16 = 432$, as expected.

Note: clearly it is more efficient to keep the numbers in binary when doing the calculations.

Example 2

Calculate the binary numbers:

(a) 1011×11

(b) 1110×101

(c) 11011×111

(d) 11011×1001

Solution

(a)

$$\begin{array}{rr} & 1\,0\,1\,1 \\ \times & 1\,1 \\ \hline & 1\,0\,1\,1 \\ & 1\,0\,1\,1\,0 \\ \hline & 1\,0\,0\,0\,0\,1 \\ \hline & {\scriptstyle 1\;1\;1\;1\;\;\;\;} \end{array}$$

(b)

$$\begin{array}{rr} & 1\,1\,1\,0 \\ \times & 1\,0\,1 \\ \hline & 1\,1\,1\,0 \\ & 1\,1\,1\,0\,0\,0 \\ \hline & 1\,0\,0\,0\,1\,1\,0 \\ \hline & {\scriptstyle 1\;1\;1\;\;\;\;\;\;\;\;} \end{array}$$

(c)

$$\begin{array}{r} 1\,1\,0\,1\,1 \\ \times \quad 1\,1\,1 \\ \hline 1\,1\,0\,1\,1 \\ 1\,1\,0\,1\,1\,0 \\ 1\,1\,0\,1\,1\,0\,0 \\ \hline 1\,0\,1\,1\,1\,1\,0\,1 \\ \hline {\scriptstyle 1\;1\;1\;1\;1\;1\;\;} \end{array}$$

(d)

$$\begin{array}{r} 1\,1\,0\,1\,1 \\ \times \quad 1\,0\,0\,1 \\ \hline 1\,1\,0\,1\,1 \\ 1\,1\,0\,1\,1\,0\,0\,0 \\ \hline 1\,1\,1\,1\,0\,0\,1\,1 \\ \hline {\scriptstyle 1\;1\;\;\;\;\;\;\;\;\;} \end{array}$$

Exercises

1. Calculate the binary numbers:

(a) 111×10 (b) 1100×100

(c) 101×1000 (d) 11101×1000

(e) 11000×10 (f) 10100×1000

(g) $10100 \div 10$ (h) $1100 \div 100$

Check your answers by converting to base 10 numbers.

2. Calculate the binary numbers:

(a) 111×11 (b) 1101×11

(c) 1101×101 (d) 1111×110

(e) 11011×1011 (f) 11010×1011

(g) 10101×101 (h) 10101×111

(i) 10101×110 (j) 100111×1101

3. Solve the following equations, where all numbers, including x, are binary:

(a) $\dfrac{x}{11} = 110$ (b) $\dfrac{x}{101} = 101$

(c) $\dfrac{x}{10} = 111$ (d) $\dfrac{x}{111} = 1011$

4. Multiply each of the following binary numbers by itself:

(a) 11 (b) 111 (c) 1111

What do you notice about your answers to parts (a), (b) and (c)?

What will you get if you multiply 11111 by itself?

5. Multiply each of the following binary numbers by itself:

   (a) 101 (b) 1001 (c) 10001 (d) 100001

   What would you expect to get if you multiplied 1000001 by itself?

6. Calculate the binary numbers:

   (a) $101(110 + 1101)$ (b) $1101(1111 - 110)$

   (c) $111(1000 - 101)$ (d) $1011(10001 - 1010)$

7. Here are 3 binary numbers:

   11011 11100 10011

   Working in binary,

   (a) multiply the two *larger* numbers,

   (b) multiply the two *smaller* numbers.

8. (a) Multiply the base 10 numbers 45 and 33.

   (b) Convert your answer to a binary number.

   (c) Convert 45 and 33 to binary numbers.

   (d) Multiply the binary numbers obtained in part (c) and compare this answer with your answer to part (b).

1.4 Other Bases

The ideas that we have considered can be extended to other number bases.

The table lists the digits used in some other number bases.

Base	*Digits Used*
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0 1, 2, 3, 4

The powers of the base number give the place values when you convert to base 10. For example, for base 3, the place values are the powers of 3, i.e. 1, 3, 9, 27, 81, 243, etc. This is shown in the following example, which also shows how the base 3 number 12100 is equivalent to the base 10 number 144.

Base 3	81	27	9	3	1	
	1	2	1	0	0	$\rightarrow$ $(1 \times 81) + (2 \times 27) + (1 \times 9) + (0 \times 3) + (0 \times 1) = 144$ in base 10

The following example shows a conversion from base 5 to base 10 using the powers of 5 as place values.

Base 5	625	125	25	5	1	
	4	1	0	0	1	$\rightarrow (4 \times 625) + (1 \times 125) + (0 \times 25) + (0 \times 5) + (1 \times 1) = 2626$ in base 10

Example 1

Convert each of the following numbers to base 10:

(a) 412 in base 6.

(b) 374 in base 9.

(c) 1432 in base 5.

Solution

(a)

36	6	1	
4	1	2	$\rightarrow (4 \times 36) + (1 \times 6) + (2 \times 1) = 152$ in base 10

(b)

81	9	1	
3	7	4	$\rightarrow (3 \times 81) + (7 \times 9) + (4 \times 1) = 310$ in base 10

(c)

125	25	5	1	
1	4	3	2	$\rightarrow (1 \times 125) + (4 \times 25) + (3 \times 5) + (2 \times 1) = 242$ in base 10

Example 2

Convert each of the following base 10 numbers to the base stated:

(a) 472 to base 4, (b) 179 to base 7, (c) 342 to base 3.

Solution

(a) For base 4 the place values are 256, 64, 16, 4, 1, and you need to express the number 472 as a linear combination of 256, 64, 16, 4 and 1, but with no multiplier greater than 3.

We begin by writing

$$472 = (1 \times 256) + 216$$

The next stage is to write the remaining 216 as a linear combination of 64, 16, 4 and 1.

We use the fact that

$$216 = (3 \times 64) + 24$$

and, continuing in this way,

$$24 = (1 \times 16) + 8$$
$$8 = (2 \times 4) + 0$$

Putting all these stages together,

$$472 = (1 \times 256) + (3 \times 64) + (1 \times 16) + (2 \times 4) + (0 \times 1)$$
$$= 13120 \text{ in base } 4$$

(b) For base 7 the place values are 49, 7, 1.

$$179 = (3 \times 49) + (4 \times 7) + (4 \times 1)$$
$$= 344 \text{ in base } 7$$

(b) For base 3 the place values are 243, 81, 27, 9, 3, 1.

$$342 = (1 \times 243) + (1 \times 81) + (0 \times 27) + (2 \times 9) + (0 \times 3) + (0 \times 1)$$
$$= 110200 \text{ in base } 3$$

Example 3

Carry out each of the following calculations in the base stated:

(a) $14 + 21$ base 5

(b) $16 + 32$ base 7

(c) $141 + 104$ base 5

(d) $212 + 121$ base 3

Check your answer in (a) by changing to base 10 numbers.

Solution

(a)
$$\begin{array}{r} 1\ 4 \\ +\ 2\ 1 \\ \hline 4\ 0 \\ \hline {}_{1}\ \ \ \end{array}$$

Note that $4 + 1 = 10$ in base 5.

(b)
$$\begin{array}{r} 1\ 6 \\ +\ 3\ 2 \\ \hline 5\ 1 \\ \hline {}_{1}\ \ \ \end{array}$$

Note that $6 + 2 = 11$ in base 7.

(c)
$$\begin{array}{r} 1\ 4\ 1 \\ +\ 1\ 0\ 4 \\ \hline 3\ 0\ 0 \\ \hline {\scriptstyle 1\ 1\ \ } \end{array}$$

Note that $1 + 4 = 10$ in base 5.

(d)
$$\begin{array}{r} 2\ 1\ 2 \\ +\ 1\ 2\ 1 \\ \hline 1\ 1\ 1\ 0 \\ \hline {\scriptstyle 1\ 1\ 1\ \ } \end{array}$$

Note that, in base 3,

$$\begin{aligned} 2 + 1 &= 10 \\ 1 + 2 + 1 &= 11 \\ 2 + 1 + 1 &= 11 \end{aligned}$$

Checking in (a):

(a)
$$\begin{array}{cc} 5 & 1 \\ \hdashline 1 & 4 \end{array} \rightarrow (1 \times 5) + (4 \times 1) = 9$$

$$\begin{array}{cc} 5 & 1 \\ \hdashline 2 & 1 \end{array} \rightarrow (2 \times 5) + (1 \times 1) = 11$$

$$\begin{array}{cc} 5 & 1 \\ \hdashline 4 & 0 \end{array} \rightarrow (4 \times 5) + (0 \times 1) = 20$$

and $9 + 11 = 20$, as expected.

Example 4

Carry out each of the following multiplications in the base stated:

(a) 141×23 in base 5

(b) 122×12 in base 3

(c) 512×24 in base 6

Check your answer to (b) by converting to base 10 numbers.

Solution

(a)
$$\begin{array}{r} 1\ 4\ 1 \\ \times\quad 2\ 3 \\ \hline 1\ 0\ 2\ 3 \\ 3\ 3\ 2\ 0 \\ \hline 4\ 3\ 4\ 3 \\ \hline \end{array}$$

Note that, in base 5,

$$\begin{aligned} 3 \times 4 &= 22 \\ 2 \times 4 &= 13 \end{aligned}$$

(b)

$$
\begin{array}{r}
1\ 2\ 2 \\
\times \quad 1\ 2 \\
\hline
1\ 0\ 2\ 1 \\
1\ 2\ 2\ 0 \\
\hline
1\ 0\ 0\ 1\ 1 \\
\hline
{\scriptstyle 1\ \ 1\ \ 1\ \ \ \ \ \ }
\end{array}
$$

Note that, in base 3,

$2 \times 2 = 11$

(c)

$$
\begin{array}{r}
5\ 1\ 2 \\
\times \quad 2\ 4 \\
\hline
3\ 2\ 5\ 2 \\
1\ 4\ 2\ 4\ 0 \\
\hline
2\ 1\ 5\ 3\ 2 \\
\hline
{\scriptstyle 1\ \ \ \ 1\ \ \ \ \ \ }
\end{array}
$$

Note that, in base 6,

$2 \times 4 = 12$

$4 \times 5 = 32$

$2 \times 5 = 14$

Checking in (b):

(b)

$$
\begin{array}{ccc}
9 & 3 & 1 \\
\hdashline
1 & 2 & 2
\end{array}
\rightarrow (1 \times 9) + (2 \times 3) + (2 \times 1) = 17
$$

$$
\begin{array}{cc}
3 & 1 \\
\hdashline
1 & 2
\end{array}
\rightarrow (1 \times 3) + (2 \times 1) = 5
$$

$$
\begin{array}{ccccc}
81 & 27 & 9 & 3 & 1 \\
\hdashline
1 & 0 & 0 & 1 & 1
\end{array}
\rightarrow (1 \times 81) + (0 \times 27) + (0 \times 9) + (3 \times 1) + (1 \times 1) = 85
$$

and $17 \times 5 = 85$, as expected.

Exercises

1. Convert the following numbers from the base stated to base 10:

(a)	412	base 5	(b)	333	base 4
(c)	728	base 9	(d)	1210	base 3
(e)	1471	base 8	(f)	612	base 7
(g)	351	base 6	(h)	111	base 3

2. Convert the following numbers from base 10 to the base stated:

(a) 24 to base 3 (b) 16 to base 4
(c) 321 to base 5 (d) 113 to base 6
(e) 314 to base 7 (f) 84 to base 9
(g) 142 to base 3 (h) 617 to base 5

3. Carry out the following additions in the base stated:

(a) $3+2$ in base 4 (b) $5+8$ in base 9
(c) $4+6$ in base 8 (d) $2+2$ in base 3
(e) $6+7$ in base 9 (f) $3+4$ in base 6

4. In what number bases could each of the following numbers be written:

(a) 123 (b) 112 (c) 184

5. Carry out each of the following calculations in the base stated:

(a) $13+23$ in base 4 (b) $120+314$ in base 5
(c) $222+102$ in base 3 (d) $310+132$ in base 4
(e) $624+136$ in base 7 (f) $211+142$ in base 5
(g) $333+323$ in base 4 (h) $141+424$ in base 5

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

6. Carry out each of the following multiplications in the base stated:

(a) 3×2 in base 4 (b) 4×3 in base 5
(c) 4×2 in base 6 (d) 3×5 in base 6
(e) 2×2 in base 3 (f) 8×8 in base 9

7. Carry out each of the following multiplications in the base stated:

(a) 121×11 in base 3 (b) 133×12 in base 4
(c) 13×24 in base 5 (d) 142×14 in base 5
(e) 161×24 in base 7 (f) 472×32 in base 8
(g) 414×22 in base 5 (h) 2101×21 in base 3

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

8. In which base was each of the following calculations carried out?

(a) $4 + 2 = 11$ (b) $7 + 5 = 13$

(c) $8 \times 2 = 17$ (d) $4 \times 5 = 32$

(e) $11 - 3 = 5$ (f) $22 - 4 = 13$

9. (a) Change 147 in base 8 into a base 3 number.

(b) Change 321 in base 4 into a base 7 number.

(c) Change 172 in base 9 into a base 4 number.

(d) Change 324 in base 5 into a base 6 number.

10. In which base was each of the following calculations carried out?

(a) $171 \times 12 = 2272$ (b) $122 \times 21 = 11102$

(c) $24 \times 32 = 1252$ (d) $333 \times 33 = 23144$

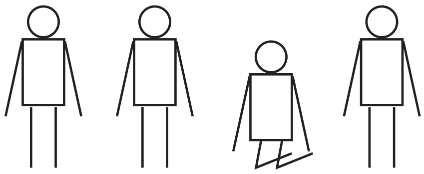
UNIT 1 *Base Arithmetic* Activities

Activities

ACTIVITY 1.1 *Binary Digits*

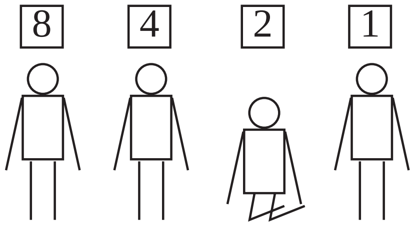
This activity is based on the idea of using children to represent the digit 1 by *standing*, and 0 by *squatting*.

e.g.



This represents 1101.

It may also be useful to have cards available that can be held up to indicate place value.



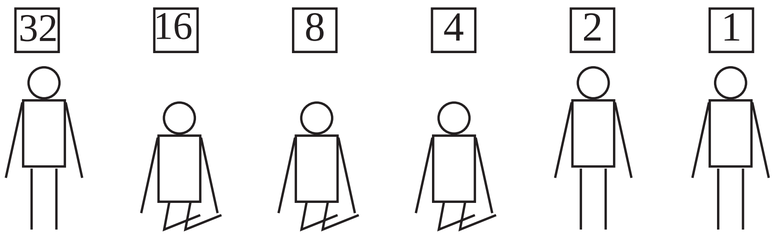
The activity can be used to introduce binary numbers and explain how the place value is used.

A team competition could then be carried out, with each team required to express a base 10 number in binary form.

For example,

"*Express 35 in base 2*."

should give:



ACTIVITY 1.2 *ASCII Codes*

'ASCII' stands for 'American Standard Code for Information Interchange'.

An ASCII code is a code assigned to every keyboard character and is used extensively in computing. Each character has a code that is normally expressed as a 7-digit binary number.

For example, 'A' has an ASCII code of 1000001 in binary, 65 in base 10 (and 41 in hexadecimal). The table below gives the ASCII codes as binary numbers for all the capital letters.

ASCII System

A	1000001
B	1000010
C	1000011
D	1000100
E	1000101
F	1000110
G	1000111
H	1001000
I	1001001
J	1001010
K	1001011
L	1001100
M	1001101
N	1001110
O	1001111
P	1010000
Q	1010001
R	1010010
S	1010011
T	1010100
U	1010101
V	1010110
W	1010111
X	1011000
Y	1011001
Z	1011010

1. Use the ASCII code to decode the following messages:

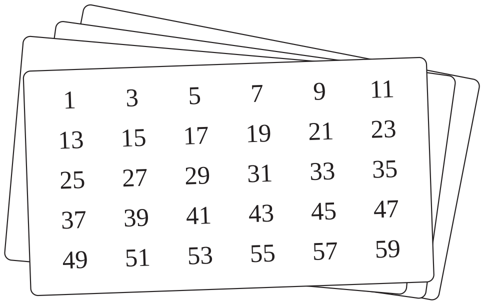
   (a) 1001000 1000101 1001100 1001100 1001111

   (b) 1001001 1000011 1000001 1001110 1010111
   1001111 1010010 1001011 1001001 1010100
   1001111 1010101 1010100

2. Now code and decode a short message to a friend.

You can find a full list of ASCII codes at:
www.mindspring.com/~jc1/serial/Resources/ASCII.html

ACTIVITY 1.3 *Choose a Number*



You have probably seen this game in Christmas crackers. You ask someone to choose a whole number between 1 and 60. You then show them each card and ask them to say whether or not their chosen number is on the card. Miraculously you can tell them the number!

Construction

Writing down the page, list all the whole numbers, 1 to 60, and express each one as a base 2 number, as shown below.

The 1s in the first *column correspond to numbers on the* first *card.*

$1 = 2^0 = 1$

$2 = 2^1 = 10$

$3 = 2^1 + 2^0 = 11$

$4 = 2^2 = 100$

$5 = 2^2 + 2^0 = 101$

$6 = 2^2 + 2^1 = 110$

. . . = . . .

. . . = . . .

$60 = 2^5 + 2^4 + 2^3 + 2^2 = 111100$

The 1s in the second *column correspond to numbers on the* second *card.*

- You will need 6 cards, each with a possible 6×5 array of numbers.
- Each card corresponds to a power of 2. The numbers on the card are those for which there is a 1 entry in the corresponding column, as shown opposite.
- So the first card has on it 1, 3, 5, 7, . . ., 59 and the second card has on it 2, 3, 6, 7, 10, 11, . . ., 59. The third card has 4, 5, 6, etc.
- *Complete all six cards.*

How it works

Ask someone to choose a number, and show them each card in turn. If the number is on the card, note the *power* of 2 on the card (this will be the smallest number).

For example, the number 51 will occur on the four cards corresponding to

$2^5 (= 32),\ 2^4 (= 16),\ 2^1 (= 2),\ 2^0 (= 1).$

Add up these numbers to give the chosen number.

Why it works

Taking 51 as an example, we see that

$51 = 32 + 16 + 2 + 1 = 2^5 + 2^4 + 2^1 + 2^0$

and 51 will occur on the 1st, 2nd, 5th and 6th cards. Adding the powers of 2 uniquely gives the number.

Extension

Design cards to cope with the numbers 1 to 120.

How many cards would be needed to cope with numbers 1 to 1000 ? How many numbers would be on each card?

ACTIVITY 1.4 *Hexadecimal Numbers*

'Hexadecimal' numbers are numbers in base 16. In order to deal with hexadecimal numbers we require extra digits and so use the letters A to F.

Base 10	Hexadecimal
10	A
11	B
12	C
13	D
14	E
15	F

So the hexadecimal number $4AC$ is:

$$4 \times 16^2 + 10 \times 16 + 12 = 1196$$

Hexadecimal numbers are used in computing because each hexadecimal digit is a byte of 4 bits.

1. Convert the following base 10 numbers to hexadecimal:

   (a) 5 (b) 15 (c) 17

   (d) 47 (e) 69 (f) 149

2. Convert the following hexadecimal numbers to base 10:

   (a) AA (b) $2AF$ (c) $BF3$

   (d) $4AF2$ (e) $36B9$ (f) $ABFD$

3. Calculat, giving your answers in base 10:

   (a) $A + B$ (b) $E + 11$ (c) $F \times 2A$

   (d) $F - 9$ (e) $D2 \times F2$ (f) $26 \times 1C4$

ACTIVITY 1.5 *Roman Numerals*

The Romans developed a system of numbers based on the letters listed below:

I	1
V	5
X	10
L	50
C	100
D	500
M	1000

The values of the consecutive letters are *added* unless a letter with a lower value appears in front of a letter with a higher value. In this case, the lower value letter is *subtracted* from the higher value.

For example,

VII = 5 + 1 + 1 = 7

XXV = 10 + 10 + 5 = 25

IV = 5 − 1 = 4

IM = 1000 − 1 = 999

1. Convert the following Roman numerals to base 10:

   (a) III (b) VIII (c) IX (d) CD

   (e) DI (f) DVI (g) LIX (h) XXD

2. Convert the following base 10 numbers to Roman numerals:

   (a) 14 (b) 6 (c) 27 (d) 56

   (e) 84 (f) 109 (g) 1010 (h) 1499

Extension

You can perform calculations using Roman Numerals. Calculate:

(a) VIII + XXV (b) XIX − IV (c) LIX × V (d) CDLXXX ÷ XXIV

Design your own questions and use them with a friend.

ACTIVITIES 1.1 - 1.3 *Notes and Solutions*

Notes and solutions given only where appropriate.

1.2 1. (a) HELLO (b) I CAN WORK IT OUT

1.3 The puzzle uses base 2 arithmetic, but knowledge of this concept is not necessary. You can also make the working easier by just using the numbers 1 to 30, say, for which you will need 5 cards.

For 1 to 60, we have

1	=					1	31	=		1	1	1	1	1
2	=				1	0	32	=	1	0	0	0	0	0
3	=				1	1	33	=	1	0	0	0	0	1
4	=			1	0	0	34	=	1	0	0	0	1	0
5	=			1	0	1	35	=	1	0	0	0	1	1
6	=			1	1	0	36	=	1	0	0	1	0	0
7	=			1	1	1	37	=	1	0	0	1	0	1
8	=		1	0	0	0	38	=	1	0	0	1	1	0
9	=		1	0	0	1	39	=	1	0	0	1	1	1
10	=		1	0	1	0	40	=	1	0	1	0	0	0
11	=		1	0	1	1	41	=	1	0	1	0	0	1
12	=		1	1	0	0	42	=	1	0	1	0	1	0
13	=		1	1	0	1	43	=	1	0	1	0	1	1
14	=		1	1	1	0	44	=	1	0	1	1	0	0
15	=		1	1	1	1	45	=	1	0	1	1	0	1
16	=	1	0	0	0	0	46	=	1	0	1	1	1	0
17	=	1	0	0	0	1	47	=	1	0	1	1	1	1
18	=	1	0	0	1	0	48	=	1	1	0	0	0	0
19	=	1	0	0	1	1	49	=	1	1	0	0	0	1
20	=	1	0	1	0	0	50	=	1	1	0	0	1	0
21	=	1	0	1	0	1	51	=	1	1	0	0	1	1
22	=	1	0	1	1	0	52	=	1	1	0	1	0	0
23	=	1	0	1	1	1	53	=	1	1	0	1	0	1
24	=	1	1	0	0	0	54	=	1	1	0	1	1	0
25	=	1	1	0	0	1	55	=	1	1	0	1	1	1
26	=	1	1	0	1	0	56	=	1	1	1	0	0	0
27	=	1	1	0	1	1	57	=	1	1	1	0	0	1
28	=	1	1	1	0	0	58	=	1	1	1	0	1	0
29	=	1	1	1	0	1	59	=	1	1	1	0	1	1
30	=	1	1	1	1	0	60	=	1	1	1	1	0	0

The first card is given in Activity 1.3; the others are shown below:

2	3	6	7	10	11
14	15	18	19	22	23
26	27	30	31	34	35
38	39	42	43	46	47
50	51	54	55	58	59

4	5	6	7	12	13
14	15	20	21	22	23
28	29	30	31	36	37
38	39	44	45	46	47
52	53	54	55	60	

8	9	10	11	12	13
14	15	24	25	26	27
28	29	30	31	40	41
42	43	44	45	46	47
56	57	58	59	60	

16	17	18	19	20	21
22	23	24	25	26	27
28	29	30	31	48	49
50	51	52	53	54	55
56	57	58	59	60	

32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55
56	57	58	59	60	

To make it more difficult to understand how or why the puzzle works, it would be better to mix up all the numbers on each card (but remember that the key number is always the smallest).

ACTIVITIES 1.4 - 1.5 *Notes and Solutions*

1.4	1.	(a)	5	(b)	F	(c)	11		
		(d)	2F	(e)	45	(f)	95		
	2.	(a)	170	(b)	687	(c)	3059		
		(d)	19186	(e)	14009	(f)	44029		
	3.	(a)	21	(b)	31	(c)	630		
		(d)	6	(e)	50820	(f)	17176		
1.5	1.	(a)	3	(b)	8	(c)	9	(d)	400
		(e)	501	(f)	506	(g)	59	(h)	480
	2.	(a)	XIV	(b)	VI	(c)	XXVII	(d)	LVI
		(e)	LXXXIV	(f)	CIX	(g)	MX	(h)	MID

Extension

(a)	X X X I I I	(b)	X V	(c)	C C V C	(d)	X X

UNIT 1 *Base Arithmetic* Extra Exercises 1.1

1. Convert each of the following binary numbers to base 10:

(a)	11	(b)	1011	(c)	11101
(d)	100011	(e)	101101	(f)	1001001
(g)	110010	(h)	111101	(i)	110111

2. Write each of the following base 10 numbers as binary numbers:

(a)	12	(b)	38	(c)	15
(d)	61	(e)	102	(f)	90
(g)	82	(h)	44	(i)	56

3. (a) What is the *largest* possible binary number with 5 digits?
   (b) What is the *smallest* possible binary number with 5 digits?
   (c) Convert your answers to (a) and (b) to base 10.

UNIT 1 *Base Arithmetic* Extra Exercises 1.2

1. Calculate the following, in binary arithmetic:

(a) $1+1$ (b) $1+101$ (c) $11+101$

(d) $111+101$ (e) $110+101$ (f) $111+111$

(g) $11011+1101$ (h) $1110+1011$ (i) $11011+11101$

2. Calculate the following, in binary arithmetic:

(a) $11-1$ (b) $10-1$ (c) $111-100$

(d) $1011-110$ (e) $1111-101$ (f) $1000-11$

(g) $10000-1110$ (h) $11010-1101$ (i) $110111-1101$

3. Solve these equations, where all the numbers are binary numbers.

(a) $x-111=1010$ (b) $x-101=1101$

(c) $x+11=110$ (d) $x+111=1101$

(e) $x-1011=1101$ (f) $x+10111=11100$

UNIT 1 *Base Arithmetic* Extra Exercises 1.3

1. Calculate the following, in binary arithmetic:

   (a) 1101×10 (b) 10110×100

   (c) 11101×1000 (d) $1010100 \div 100$

2. Carry out these multiplications, in binary arithmetic:

   (a) 101×11 (b) 1111×101

   (c) 1011×110 (d) 1101×111

   (e) 10001×111 (f) 10011×110

   (g) 1101×110 (h) 10111×111

3. Multiply each of the following binary numbers by itself:

   (a) 1 (b) 10 (c) 110

   (d) 1101 (e) 1011 (f) 10011

UNIT 1 *Base Arithmetic* Extra Exercises 1.4

1. Convert the following numbers from the base stated to base 10:

(a)	122 base 3		(b)	312 base 4	
(c)	142 base 5		(d)	1125 base 6	
(e)	178 base 9		(f)	243 base 5	
(g)	615 base 7		(h)	342 base 6	

2. Convert these base 10 numbers to the base stated:

(a)	47 to base 3	(b)	17 to base 4
(c)	108 to base 5	(d)	99 to base 6
(e)	142 to base 7	(f)	362 to base 8
(g)	142 to base 9	(h)	97 to base 4

3. Carry out the following calculations in the base stated:

(a)	$142 + 233$ base 5	(b)	$463 + 354$ base 8
(c)	$121 + 122$ base 3	(d)	$683 + 478$ base 9
(e)	$412 - 332$ base 5	(f)	12×32 base 4
(g)	36×25 base 7	(h)	64×16 base 8

Extra Exercises 1.1 Answers

1.	(a)	3	(b)	11	(c)	29
	(d)	35	(e)	45	(f)	73
	(g)	50	(h)	61	(i)	55
2.	(a)	1100	(b)	100110	(c)	1111
	(d)	111101	(e)	1100110	(f)	1011010
	(g)	1010010	(h)	101100	(i)	111000
3.	(a)	11111	(b)	10000	(c)	31 and 16

Extra Exercises 1.2 Answers

1.	(a)	10	(b)	110	(c)	1000
	(d)	1100	(e)	1011	(f)	1110
	(g)	101000	(h)	11001	(i)	111000
2.	(a)	10	(b)	1	(c)	11
	(d)	101	(e)	1010	(f)	101
	(g)	10	(h)	1101	(i)	101010
3.	(a)	10001	(b)	10010	(c)	11
	(d)	110	(e)	11000	(f)	101

Extra Exercises 1.3 Answers

1.	(a)	11010	(b)	1011000	(c)	11101000	(d)	10101
2.	(a)	1111	(b)	1001011	(c)	1000010	(d)	1011011
	(e)	1110111	(f)	1110010	(g)	1001110	(h)	10100001
3.	(a)	1	(b)	100	(c)	100100		
	(d)	10101001	(e)	1111001	(f)	101101001		

Extra Exercises 1.4 Answers

1.	(a)	17	(b)	54	(c)	47	(d)	269
	(e)	152	(f)	73	(g)	306	(h)	134
2.	(a)	1202	(b)	101	(c)	413	(d)	243
	(e)	262	(f)	552	(g)	167	(h)	1201
3.	(a)	430	(b)	1037	(c)	1020	(d)	1272
	(e)	30	(f)	1110	(g)	1332	(h)	1330

UNIT 1 *Base Arithmetic* Lesson Plans **St**

These are based on 45/50 minute lessons.

Lesson No.	*Suggested Plan*	*References*
1.	**Binary Numbers**	
	Introduction to concept	
	Binary to base 10	OS 1.1
	Exercises - interactively	PB 1.1, Q1
	Base 10 to binary	OS 1.2
	Exercises - interactively	PB 1.1, Q2
	Set homework	PB 1.1, Q3, Q4 and Q5
2.	**Binary Addition**	
	Discuss homework	
	Activity	Activity 1.1
	Introduction	OS 1.3
	Exercises - interactively	PB 1.2, Q1
	Activity	Activity 1.2
	Set homework	Complete Activity 1.2 or PB 1.2, Q3 and Q4
3.	**Binary Subtraction**	
	Discuss homework	
	Introduction	OS 1.4
	Exercises - interactively	PB 1.2, Q2
	Mental Test	M 1.1
	Review answers	
	Set homework	PB 1.2, Q6 and Q7
4.	**Multiplying Binary Numbers**	
	Discuss homework	
	Introduction	OS 1.5
	Exercises	PB 1.3, Q2
	Review answers	
	Activity	Activity 1.3 or Activity 1.5
	Set homework	Complete Activity 1.3 or Activity 1.5 or PB 1.3, Q1
5.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 1.1
6.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 1 *Base Arithmetic* A

Lesson No.	*Suggested Plan*	*References*
1.	**Binary Numbers**	
	Introduction to concept	
	Binary to base 10	OS 1.1
	Exercises - interactively	PB 1.1, Q1
	Base 10 to binary	OS 1.2
	Exercises - interactively	PB 1.1, Q2
	Activity	Activity 1.1
	Exercises	PB 1.1, Q3
	Review answers	
	Set homework	PB 1.1, Q5, Q7 and Q8
2.	**Binary Arithmetic**	
	Discuss homework	
	Introduction to binary addition	OS 1.3
	Exercises - interactively	PB 1.2, Q1
	Introduction to binary subtraction	OS 1.4
	Exercises - interactively	PB1.2, Q2
	Activity	Activity 1.2
	Set homework	PB 1.2, Q4, Q5 and Q6
3.	**Multiplying Binary Numbers**	
	Discuss homework	
	Introduction	OS 1.5
	Exercises - interactively	PB 1.3, Q1
	Exercises	PB 1.3, Q3
	Review answers	
	Mental Test	M 1.2
	Review answers	
	Set homework	PB 1.3, Q2 or Activity 1.5
4.	**Other Bases**	
	Discuss homework	
	Introduction	OS 1.6 and OS 1.7
	Exercises	PB 1.4, Q1
	Review answers	
	Activity	Activity 1.3
	Set homework	PB 1.4, Q5 and Q6

UNIT 1 *Base Arithmetic* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
5.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 1.2
6.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 1 *Base Arithmetic* E

Lesson No.	*Suggested Plan*	*References*
1.	**Binary Numbers**	
	Introduction to concept	
	Binary to base 10	OS 1.1
	Exercises - interactively	PB 1.1, Q1
	Base 10 to binary	OS 1.2
	Exercises - interactively	PB 1.1, Q2
	Activity	Activity 1.1
	Exercises	PB 1.1, Q3
	Review answers	
	Set homework	PB 1.1, Q5, Q7 and Q8
2.	**Binary Arithmetic**	
	Discuss homework	
	Introduction to binary addition	OS 1.3
	Exercises - interactively	PB 1.2, Q1
	Introduction to binary subtraction	OS 1.4
	Exercises - interactively	PB1.2, Q2
	Activity	Activity 1.2
	Set homework	PB 1.2, Q4, Q5 and Q6
3.	**Multiplying Binary Numbers**	
	Discuss homework	
	Introduction	OS 1.5
	Exercises - interactively	PB 1.3, Q1
	Exercises	PB 1.3, Q3
	Review answers	
	Mental Test	M 1.3
	Review answers	
	Set homework	PB 1.3, Q4, Q5 and Q6
4.	**Other Bases**	
	Discuss homework	
	Introduction	OS 1.6 and OS 1.7
	Exercises	PB 1.4, Q1
	Review answers	
	Exercises	PB 1.4, Q4
	Review answers	
	Activity	Activity 1.4
	Set homework	PB 1.4, Q7 and Q8 or Activity 1.3

UNIT 1 *Base Arithmetic* Lesson Plans E

Lesson No.	*Suggested Plan*	*References*
5.	**Revision Test** Discuss homework Revision Test	 RT 1.3
6.	**Recap** Give back marked tests Go over test questions interactively Revise topics	

UNIT 1 Base Arithmetic Mental Tests

M 1.1 Standard Route *(no calculator)*

1. In binary, calculate $1+1$. (10)
2. Convert the base 10 number 4 to a binary number. (100)
3. Convert the binary number 11 to base 10. (3)
4. In binary, calculate 11×10. (110)
5. In binary, calculate $11+1$. (100)
6. Convert the base 10 number 5 to binary. (101)
7. Convert the binary number 1000 to base 10. (8)
8. In binary, calculate $10-1$. (1)
9. In binary, calculate $10+10$. (100)
10. Convert 110 from binary to base 10. (6)

M 1.2 Academic Route *(no calculator)*

1. In binary, calculate $11+1$. (100)
2. In binary, calculate $10-1$. (1)
3. Convert the binary number 1000 to base 10. (8)
4. Convert the binary number 101 to base 10. (5)
5. Convert the binary number 111 to base 10. (7)
6. Convert the base 10 number 10 to binary. (1010)
7. In binary, calculate 101×10. (1010)
8. Convert 1111 from binary to base 10. (15)
9. Convert the base 10 number 33 to binary. (100001)
10. Convert the base 10 number 22 to binary. (10110)

UNIT 1 Base Arithmetic Mental Tests

M 1.3 Express Route *(no calculator)*

1. In binary, calculate $111+1$. (1000)
2. In binary, calculate $1010 \div 10$. (101)
3. Convert the base 10 number 33 to binary. (100001)
4. In binary, calculate $100-1$. (11)
5. Convert the binary number 1011 to base 10. (11)
6. Convert the binary number 10001 to base 10. (17)
7. Convert the base 10 number 66 to binary. (1000010)
8. Convert 100 from base 10 to binary. (1100100)
9. Convert the binary number 11011 to base 10. (27)
10. In binary, calculate 11×11. (1001)

Practice Book *UNIT 1 Base Arithmetic* Answers

1.1 Binary Numbers

1.	(a)	6	(b)	15	(c)	9	(d)	13
	(e)	17	(f)	27	(g)	127	(h)	113
	(i)	170	(j)	205	(k)	455	(l)	102
2.	(a)	1001	(b)	1000	(c)	1110	(d)	10001
	(e)	10010	(f)	11110	(g)	101111	(h)	110100
	(i)	1000011	(j)	1010100	(k)	11001000	(l)	111110100
3.	(a)	101	(b)	1001	(c)	10001	(d)	100001

The binary equivalents always begin and end with 1 and the number of zeros in the middle increases by 1 each time.

The next base 10 number that fits this pattern is 65 (corresponding to the binary number 1000001).

4. (a) 11 (b) 111 (c) 1111 (d) 11111

The next base 10 number that fits this pattern is 63 (corresponding to the binary number 111111).

5. (a) Largest = 111, smallest = 100 (b) 7 and 4

6. Any whole number from 8 to 15

7. (a) 1100, 1010 and 1001 (b) 12, 10 and 9

8. (a) Largest = 255 (binary 11111111) (b) smallest = 128 (binary 10000000)

9. 999 converts to binary 1111100111 with 10 digits, so the binary equivalent has 7 more digits than the 3 digits in base 10.

10. The binary number 11111 is 31 in base 10, so the difference is 11111 – 31 = 11080.

1.2 Adding and Sutracting Binary Numbers

1.	(a)	100	(b)	110	(c)	1010	(d)	1001
	(e)	10101	(f)	10010	(g)	100100	(h)	100101
	(i)	11000	(j)	100101	(k)	11101	(l)	111100
2.	(a)	1	(b)	100	(c)	1001	(d)	10
	(e)	1	(f)	101	(g)	111	(h)	10101
	(i)	1000	(j)	101011	(k)	10100	(l)	10111
3.	(a)	110	(b)	1110	(c)	11110	(d)	111110

The answers always end in a single zero and all the other digits are ones. Also, the number of ones increases by 1 each time.

1.1 Answers

Alternatively, the answer is the binary number in the question with an extra zero on the right hand end. This is because adding a number to itself is the same as doubling, which in binary means multiplying by 10, so you just add a zero onto the right hand end of the number you are adding to itself.

4. (a) 100 (b) 1000 (c) 10000 (d) 100000

The answer always begins with 1 and the remaining digits are all zero. Also, the number of zeros increases by 1 each time.

Alternatively, the answer is the binary number in the question with an extra zero on the right hand end. Again this is because adding a number to itself is the same as doubling, which in binary means multiplying by 10, so you just add a zero onto the right hand end of the number you are adding to itself.

5. (a) 1010 (b) 111 (c) 101000 (d) 11
(e) 10111 (f) 100110

6. (a) 1 (b) 11 (c) 111 (d) 1111

The answers only involve the digit one. Also, the number of ones increases by 1 each time. The number of ones is equal to the number of zeros in the original calculation.

7. (a) $11101 \rightarrow 29$ in base 10, $1110 \rightarrow 14$ in base 10 (b) $29 + 14 = 43$
(c) $11101 + 1110 = 101011$ (d) $101011 \rightarrow 43$ in base 10

The answers to (b) and (d) are the same.

8. (a) $11101 \rightarrow 29$ in base 10, $10111 \rightarrow 23$ in base 10 (b) $29 - 23 = 6$
(c) $6 \rightarrow 110$ in binary (d) $11101 - 10111 = 110$

The answers to (c) and (d) are the same.

9. (a) 10110001 (b) 11010011 (c) 100010 (d) 100100110

10. (a) 10000 (b) 1100111

1.3 Multiplying Binary Numbers

1. (a) 1110 (check $7 \times 2 = 14$) (b) 110000 (check $12 \times 4 = 48$)
(c) 101000 (check $5 \times 8 = 40$) (d) 11101000 (check $29 \times 8 = 232$)
(e) 110000 (check $24 \times 2 = 48$) (f) 10100000 (check $20 \times 8 = 160$)
(g) 1010 (check $20 \div 2 = 10$) (h) 11 (check $12 \div 4 = 3$)

2. (a) 10101 (b) 100111 (c) 1000001 (d) 1011010
(e) 100101001 (f) 100011110 (g) 1101001 (h) 10010011
(i) 1111110 (j) 111111011

1.3 Answers

3. (a) 10010 (b) 11001 (c) 1110 (d) 1001101

4. (a) 1001 (b) 110001 (c) 11100001

   The number of ones at the start increases by 1 each time, as does the number of zeros that follow. The answer always ends with a single 1. Also, the number of ones in the original number being squared is one more than the number of ones at the start of the answer, and is the same as the number of zeros that follow.

   $11111 \times 11111 = 1111000001$

5. (a) 11001 (b) 1010001 (c) 100100001 (d) 10001000001

   $1000001 \times 1000001 = 1000010000001$

6. (a) 1011111 (b) 1110101 (c) 10101 (d) 1001101

7. (a) 1011110100 (b) 1000000001

8. (a) 1485 (b) 10111001101 (c) $45 \rightarrow 101101$ in binary, $33 \rightarrow 100001$ in binary

   (d) $101101 \times 100001 = 10111001101$, the same answer as part (b).

1.4 Other Bases

1. (a) 107 (b) 63 (c) 593 (d) 48
   (e) 825 (f) 303 (g) 139 (h) 13

2. (a) 220 (b) 100 (c) 2241 (d) 305
   (e) 626 (f) 103 (g) 12021 (h) 4432

3. (a) 11 (b) 14 (c) 12 (d) 11
   (e) 14 (f) 11

4. (a) Bases 4, 5, 6, 7, 8, 9 and 10 (b) Bases 3, 4, 5, 6, 7, 8, 9, and 10
   (c) Bases 9 and 10

5. (a) 102 check

   $13 \rightarrow (1 \times 4) + (3 \times 1) = 7$

   $23 \rightarrow (2 \times 4) + (3 \times 1) = 11$

   $102 \rightarrow (1 \times 16) + (0 \times 4) + (2 \times 1) = 18$

   and $7 + 11 = 18$

   (b) 434

1.4 Answers

(c) 1101 check $222 \rightarrow (2 \times 9) + (2 \times 3) + (2 \times 1) = 26$

$102 \rightarrow (1 \times 9) + (0 \times 3) + (2 \times 1) = 11$

$1101 \rightarrow (1 \times 27) + (1 \times 9) + (0 \times 3) + (1 \times 1) = 37$

and $26 + 11 = 37$

(d) 1212

(e) 1063 check $624 \rightarrow (6 \times 49) + (2 \times 7) + (4 \times 1) = 312$

$136 \rightarrow (1 \times 49) + (3 \times 7) + (6 \times 1) = 76$

$1063 \rightarrow (1 \times 343) + (0 \times 49) + (6 \times 7) + (3 \times 1) = 388$

and $312 + 76 = 388$

(f) 403 (g) 1322 (h) 1120

6. (a) 12 (b) 22 (c) 12 (d) 23
   (e) 11 (f) 71

7. (a) 2101 check $121 \rightarrow (1 \times 9) + (2 \times 3) + (1 \times 1) = 16$

$11 \rightarrow (1 \times 3) + (1 \times 1) = 4$

$2101 \rightarrow (2 \times 27) + (1 \times 9) + (0 \times 3) + (1 \times 1) = 64$

and $16 \times 4 = 64$

(b) 2322

(c) 422 check $13 \rightarrow (1 \times 5) + (3 \times 1) = 8$

$24 \rightarrow (2 \times 5) + (4 \times 1) = 14$

$422 \rightarrow (4 \times 25) + (2 \times 5) + (2 \times 1) = 112$

and $8 \times 14 = 112$

(d) 3143

(e) 4554 check $161 \rightarrow (1 \times 49) + (6 \times 7) + (1 \times 1) = 92$

$24 \rightarrow (2 \times 7) + (4 \times 1) = 18$

$4554 \rightarrow (4 \times 343) + (5 \times 49) + (5 \times 7) + (4 \times 1) = 1656$

and $92 \times 18 = 1656$

(f) 17744 (g) 20213 (h) 121121

1.4 Answers

8.	(a)	Base 5	(b)	Base 9	(c)	Base 9	(d)	Base 6
	(e)	Base 7	(f)	Base 5				
9.	(a)	Base 8	(b)	Base 3	(c)	Base 6	(d)	Base 5
10.	(a)	10211	(b)	111	(c)	2102	(d)	225

UNIT 1 *Base Arithmetic* Revision Test 1.1 (Standard)

1. Convert each of the following binary numbers to base 10:

(a)	11	(b)	1011	(c)	1101
(d)	1110	(e)	100001	(f)	10101

(11 marks)

2. Convert each of the following numbers from base 10 to binary:

(a)	16	(b)	8	(c)	9
(d)	7	(e)	25	(f)	39

(10 marks)

3. In binary, calculate:

(a)	110×10	(b)	$11 + 1$	(c)	$11 - 10$
(d)	$111 + 1$	(e)	$101 + 11$	(f)	$111 + 11$

(9 marks)

UNIT 1 *Base Arithmetic* Revision Test 1.2 (Academic)

1. Convert each of the following binary numbers to base 10:

   (a) 10000 (b) 10011 (c) 11001

   (d) 110011 (e) 10001 (f) 10101

   (11 marks)

2. Convert each of the following numbers from base 10 to binary:

   (a) 32 (b) 19 (c) 25

   (d) 47 (e) 65

   (9 marks)

3. In binary, calculate:

   (a) 10101×10 (b) $110 + 1$ (c) $111 + 1011$

   (d) $111 + 111$ (e) 11×101 (f) 1101×11

   (10 marks)

UNIT 1 *Base Arithmetic* Revision Test 1.3 (Express)

1. Convert each of the following binary numbers to base 10:

   (a) 110011 (b) 110110 (c) 1011011

   (6 marks)

2. Convert each of the following numbers from base 10 to binary:

   (a) 28 (b) 47 (c) 162

   (6 marks)

3. In binary, calculate:

   (a) $111 + 111$ (b) 1101×11

   (c) $1100 - 101$ (d) 110×111

   (8 marks)

4. Convert the base 5 number 124 to base 10.

   (2 marks)

5. Convert the base 10 number 242 to base 8.

   (2 marks)

6. Calculate 2×8 in base 9.

   (2 marks)

7. Calculate $142 + 243$ in base 6.

   (2 marks)

8. Convert the base 9 number 147 to base 10.

   (2 marks)

Revision Test 1.1 (Standard) Answers

1.	(a)	$2+1=3$	B1	
	(b)	$8+2+1=11$	M1 A1	
	(c)	$8+4+1=13$	M1 A1	
	(d)	$8+4+2=14$	M1 A1	
	(e)	$32+1=33$	M1 A1	
	(f)	$16+4+1=21$	M1 A1	*(11 marks)*
2.	(a)	10000	B1	
	(b)	1000	B1	
	(c)	1001	B2	
	(d)	111	B2	
	(e)	11001	B2	
	(f)	100111	B2	*(10 marks)*
3.	(a)	$110\times 10=1100$	B1	
	(b)	$11+1=100$	B1	
	(c)	$11-10=1$	B1	
	(d)	$111+1=1000$	B2	
	(e)	$101+11=1000$	B2	
	(f)	$111+11=1010$	B2	*(9 marks)*

(TOTAL MARKS 30)

Revision Test 1.2 (Academic) Answers

1.	(a)	16	B1	
	(b)	$1 + 2 + 16 = 19$	M1 A1	
	(c)	$1 + 8 + 16 = 25$	M1 A1	
	(d)	$1 + 2 + 16 + 32 = 51$	M1 A1	
	(e)	$1 + 16 = 17$	M1 A1	
	(f)	$1 + 4 + 16 = 21$	M1 A1	*(11 marks)*
2.	(a)	100000	B1	
	(b)	10011	B2	
	(c)	11001	B2	
	(d)	101111	B2	
	(e)	1000001	B2	*(9 marks)*
3.	(a)	101010	B1	
	(b)	111	B1	
	(c)	10010	B2	
	(d)	1110	B2	
	(e)	1111	B2	
	(f)	100111	B2	*(10 marks)*

(TOTAL MARKS 30)

Revision Test 1.3 (Express) Answers

1.	(a)	$1 + 2 + 16 + 32 = 51$	M1 A1	
	(b)	$2 + 4 + 16 + 32 = 54$	M1 A1	
	(c)	$1 + 2 + 8 + 16 + 64 = 91$	M1 A1	*(6 marks)*
2.	(a)	11100	B2	
	(b)	101111	B2	
	(c)	10100010	B2	*(6 marks)*
3.	(a)	1110	B2	
	(b)	100111	B2	
	(c)	111	B2	
	(d)	101010	B2	*(8 marks)*
4.		$25 + 10 + 4 = 39$	M1 A1	*(2 marks)*
5.		$3 \times 64 + 6 \times 8 + 2 = 362$	M1 A1	*(2 marks)*
6.		17	B2	*(2 marks)*
7.		425	B2	*(2 marks)*
8.		$81 + 4 \times 9 + 7 = 124$	M1 A1	*(2 marks)*

(TOTAL MARKS 30)

UNIT 1 *Base Arithmetic* Teaching Notes

The representation of numbers in binary form has come into general usage only comparatively recently, due to its applications in *logic theory* in the 19th century and in *coding* in the 20th century.

Binary is, of course, the simplest way of representing numbers as it uses just 0 and 1, but it could be argued that it is not very efficient. In fact, a variety of bases and not the usual base 10 were used in early times. For example, the Babylonians essentially used base 60, and this has survived as the measurement used for both time and angles. It is not entirely clear why they used base 60, but it is conjectured that it was because 60 is easily divisible by many small integers.

The use of binary came to prominence with the advent, during the last century (1950 onwards), of electronic (rather than mechanical) number machines, where a particular device is either 'on' or 'off ' (i.e., '1' or '0'). Applications have grown with the development of both hardware and software, although other bases (particularly base 16) have also found application.

Additionally, binary is the building block of both logic theory and Boolean Algebra, and, most recently, the enormous growth in coding is in part dependent on binary type analysis, since again it involves the concept of using just '1' and '0', i.e., 'on' or 'off'. So, clearly, binary is a topic with key modern applications.

Binary has been included here both for its applicability and also for the fact that it underpins basic number theory. It provides a way of revising and reinforcing basic number work, and, above all, it should be an enjoyable and stimulating topic.

Routes		**Standard**	**Academic**	**Express**
1.1	Binary Numbers	✓	✓	✓
1.2	Adding and Subtracting Binary Numbers	✓	✓	✓
1.3	Multiplying Binary Numbers	(✓)	✓	✓
1.4	Other Bases	✕	✓	✓

Language	**Standard**	**Academic**	**Express**
Base 2	✓	✓	✓
Binary numbers	✓	✓	✓

UNIT 1 *Base Arithmetic* Teaching Notes

Misconceptions

- it must be understood that in base 2, the only possible digits are 0 and 1 (0, 1, 2 in base 3, etc.), e.g. $1+1=10$, and not 2.
- the digits must be read in the correct order, e.g. 1 1 0 1 represents

$$1\times 8+1\times 4+0\times 2+1\times 1 = 13$$

and not

$$1\times 1+1\times 2+0\times 4+1\times 8 \quad (=11)$$

- pupils should know that there is no need, when adding, subtracting or multiplying numbers in binary, to first transform them to base 10 and then back to binary.

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9A	1.4	8, 10	15