1 Logic

This unit introduces ideas of *logic*, a topic which is the foundation of all mathematics. We will be looking at logic puzzles and introducing some work on sets.

1.1 Logic Puzzles

Here we introduce logic puzzles to help you think mathematically.

Example

Rana, Toni and Millie are sisters. You need to deduce which sister is 9 years old, which one is 12 and which one is 14. You have two clues:

Clue 1 : *Toni's age is not in the 4-times table.*

Clue 2 : *Millie's age can be divided exactly by the number of days in a week.*

Solution

You can present this information in a logic table, shown opposite.

A *cross* in any box means that the statement is *not true*.

A *tick* in any box means that the statement *is true.*

	9 *yrs*	12 *yrs*	14 *yrs*
Rana			
Toni			
Millie			

Clue 1 : *Toni's age is not in the 4-times table.*

This tells you that Toni's age is not 12.

Put a cross in Toni's row and column 12.

	9 *yrs*	12 *yrs*	14 *yrs*
Rana			
Toni		×	
Millie			

Clue 2 : *Millie's age can be divided exactly by the number of days in a week.*

This tells you that Millie's age is 14.

Put 2 crosses and a tick in Millie's row.

	9 *yrs*	12 *yrs*	14 *yrs*
Rana			
Toni		×	
Millie	×	×	✓

Looking at column '12 yrs', you can see that Rana must be 12.

Fill in the ticks and crosses in Rana's row.

	9 *yrs*	12 *yrs*	14 *yrs*
Rana	✗	✓	✗
Toni		✗	
Millie	✗	✗	✓

Looking at column '9 yrs', you can see that Toni must be 9.

Toni's row can now be completed.

	9 *yrs*	12 *yrs*	14 *yrs*
Rana	✗	✓	✗
Toni	✓	✗	✗
Millie	✗	✗	✓

Answer : Toni is 9 years old.

Rana is 12 years old.

Millie is 14 years old.

Exercises

1. Jane, Bill and Kelly each have one pet. They all own different types of pet.

   Clue 1: Kelly's pet does not have a beak.

   Clue 2: Bill's pet lives in a bowl.

   Use this logic table to find out which pet each person owns.

	Goldfish	*Dog*	*Budgie*
Jane			
Bill			
Kelly			

2. Karen, John and Jenny each play one sport: badminton, tennis or football. Use these clues to decide who plays which sport.

   Clue 1: John hits a ball with a racket.

   Clue 2: Karen kicks a ball.

	Badminton	*Tennis*	*Football*
Karen			
John			
Jenny			

3. Three children are asked to name their favourite subject out of Maths, PE and Art. They each give a different answer. Decide which child names which subject.

Clue 1: Daniel likes working with numbers.

Clue 2: Sarah does not like to draw or paint.

	Maths	*PE*	*Art*
Daniel			
Sarah			
Jane			

4. The three children in a family are aged 8, 12 and 16. Use these clues to find the age of each child.

Clue 1: Alan is older than Charlie.

Clue 2: John is younger than Charlie.

	8 *yrs*	12 *yrs*	16 *yrs*
John			
Alan			
Charlie			

5. A waiter brings these meals to the table in a restaurant.

Chips, steak and salad

Baked potato, cheese and beans

Chips, mushroom pizza and salad

Use the clues to decide who eats which meal.

- *Chris does not eat salad.*
- *Adam is a vegetarian.*

6. Amanda, Jo, Alex and Zarah each have different coloured cars. One car is red, one blue, one white and the other is black.

Decide which person has which coloured car.

- *Amanda's car is not red or white.*
- *Jo's car is not blue or white.*
- *Alex's car is not black or blue.*
- *Zarah's car is red.*

	Red	*Blue*	*White*	*Black*
Amanda				
Jo				
Alex				
Zarah				

7. Bill, John, Fred and Jim are married to one of Mrs Brown, Mrs Green, Mrs Black and Mrs White.

Use these clues and the table to decide who is married to who.

	Bill	*John*	*Fred*	*Jim*
Mrs Brown				
Mrs Green				
Mrs Black				
Mrs White				

Clues

- *Mrs Brown's husband's first name does not begin with J.*
- *Mrs Black's husband has a first name which does have the same letter twice.*
- *The first name of Mrs White's husband has 3 letters*

8. In a race the four fastest runners were Alice, Leah, Nadida and Anna.

Decide who finished in 1st, 2nd, 3rd and 4th places.

- *Alice finished before Anna.*
- *Leah finished before Nadida.*
- *Nadida finished before Alice.*

9. There are 4 children in a family. They are 6, 8, 11 and 14 years old. Use these clues and the table to find out the age of each child.

Clues

- *Dipak is 3 years older than Ali.*
- *Mohammed is older than Dipak.*

	6 years	*8 years*	*11 years*	*14 years*
Ali				
Mohammed				
Dipak				
Nesima				

10. Here is a completed logic table.

	Football	*Tennis*	*Hockey*	*Rugby*
Ben	✓	✗	✗	✗
Tom	✗	✗	✗	✓
Helen	✗	✗	✓	✗
Abbie	✗	✓	✗	✗

(a) Write a set of clues that will give this answer.

(b) Try your clues out on a friend.

1.2 Two Way Tables

Here we extend the ideas of the first section and present data in two way tables, from which we can either complete the tables or deduce information.

Example

Emma collected information about the cats and dogs that children in her class have. She filled in the table below, but missed out one number.

	Has a dog	*Does not have a dog*
Has a cat	8	4
Does not have a cat	12	

(a) Explain how to find the missing number if there are 30 children in Emma's class.

(b) How many children own at least one of these pets?

(c) Do more children own cats rather than dogs?

(d) Could it be true that some of the children do not have any pets?

Solution

(a) As there are 30 children in the class, each one has one entry in the complete table.
As there are already

$$8 + 4 + 12 = 24$$

entries, the missing number is

$$30 - 24 = 6$$

	Has a dog	*Does not have a dog*
Has a cat	8	4
Does not have a cat	12	?

(b) All the children, except those in the bottom right hand square, own at least one cat or dog.

	Has a dog	*Does not have a dog*
Has a cat	8	4
Does not have a cat	12	6

Hence,

number of children owning at least one cat or dog is

$$30 - 6 = 24$$

(c) The total number of children owning a dog is given in the first column,

i.e. $8 + 12 = 20$

The total number of children owning a cat is given in the first row,

i.e. $8 + 4 = 12$

	Has a dog	*Does not have a dog*
Has a cat	8	4
Does not have a cat	12	6

So the answer to the question is NO, since there are more dog owners than cat owners.

(d) There are 6 children that do not own either a cat or a dog, but they might own a hamster or rabbit, etc., so we cannot deduce that some children have no pets.

Exercises

1. People leaving a football match were asked if they supported Manchester United or Newcastle. They were also asked if they were happy. The table below gives the results.

	Manchester United	*Newcastle*
Happy	40	8
Not happy	2	20

(a) How many Manchester United supporters were happy?

(b) How many Manchester United supporters were asked the questions?

(c) How many Newcastle supporters were not happy?

(d) How many people were asked the questions?

(e) Which team do you think won the football match? What are your reasons for your answer?

2. The children in a class conducted a survey to find out how many children had videos at home and how many had computers at home. Their results are given in the table.

	Video	*No Video*
Computer	8	2
No Computer	20	3

(a) How many children did *not* have a video at home?

(b) How many children had a computer at home?

(c) How many children did *not* have a computer or a video at home?

(d) How many children were in the class?

3. The children in a school are to have extra swimming lessons if they cannot swim. The table gives information about the children in Years 7, 8 and 9.

	Can swim	*Cannot swim*
Year 7	120	60
Year 8	168	11
Year 9	172	3

(a) How many children need swimming lessons?

(b) How many children are there in Year 8?

(c) How many of the Year 7 children *cannot* swim?

(d) How many children in Years 7 and 8 *can* swim?

(e) How many children are there altogether in Years 7, 8 and 9?

4. 40 children are members of a cycling club. Details of their bikes are given below. Each child has one bike.

	Mountain Bike	*Racing Bike*	*BMX Bike*
15-speed	2	0	0
12-speed	8		0
10-speed	1	8	0
1-speed	0	0	15

(a) How many children have 12-speed racing bikes?

(b) How many children have mountain bikes?

(c) Which type of bike is most popular?

5. The headteacher of a school with 484 pupils collected information about how many of the pupils wear glasses.

	Always wear glasses	*Sometimes wear glasses*	*Never wear glasses*
Boys	40		161
Girls	36	55	144

(a) Explain how to find the number of boys who *sometimes* wear glasses.

(b) How many of the pupils wear glasses some of the time?

(c) How many of the pupils *never* wear glasses?

(d) Are there more boys or girls in the school?

6. During one month, exactly half of the 180 babies born in a hospital were boys, and 40 of the babies weighed 4 kg or more. There were 26 baby boys who weighed 4 kg or more.

	Less than 4 kg	*4 kg or more*
Boys		
Girls		

(a) Copy and complete the table above.

(b) How many baby girls weighed less than 4 kg when they were born?

7. In a school survey pupils chose the TV programme they liked best from a list. Some of the results are given in the table.

	Blue Peter	*Grange Hill*	*Newsround*
Year 7	8		1
Year 8	12	5	

The same number of pupils took part from Year 7 and Year 8. Six pupils chose Newsround. Copy and complete the table and state which programme was the most popular.

8. 18 people who took part in a survey had blue eyes and 22 people had other coloured eyes. In the same survey, 16 people had blond hair and 24 did not have blond hair.

(a) How many people took part in the survey?

(b) Explain why it is impossible to complete the table below.

	Blue eyes	*Not blue eyes*
Blond hair		
Not blond hair		

(c) Complete the table if $\frac{3}{4}$ of the people with blond hair had blue eyes.

(d) How many people did *not* have blond hair and did *not* have blue eyes?

9. In a car showroom there are 8 blue cars, one of which is a hatchback.
If 6 of the 20 cars in the showroom are hatchbacks, find how many cars are not hatchbacks and are not blue.

10. In a class of 32 pupils, there were 8 girls who played hockey and 5 boys who did not. Find how many boys played hockey if there were 15 girls in the class.

1.3 Sets and Venn Diagrams

We use the idea of *sets* to classify numbers and objects and we use *Venn diagrams* to illustrate these sets.

Example

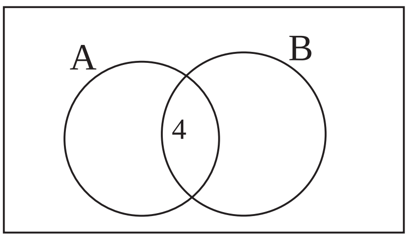
The sets A and B consist of numbers taken from the numbers 0, 1, 2, 3, . . . , 9 so that

Set A = { 4, 7, 9 }

Set B = { 1, 2, 3, 4, 5 }

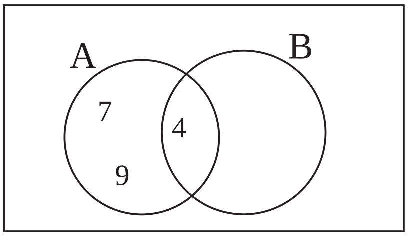
Illustrate these sets in a Venn diagram.

Solution

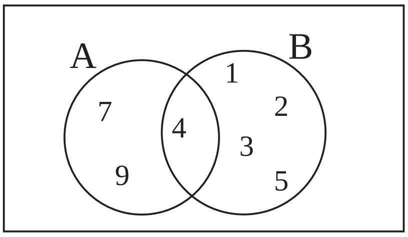


The framework for a Venn diagram is shown opposite, with the sets A and B indicated by the circles.

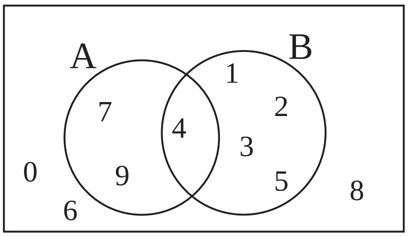
Since 4 is in both sets, it must be placed in the *intersection* of the two sets.



To complete set A, you put 7 and 9 in the part that does not intersect with B.



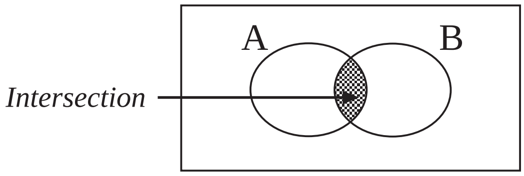
Similarly for B, you put 1, 2, 3 and 5 in the part that does not intersect with A.



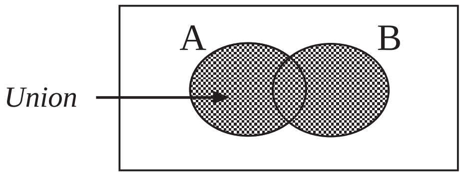
Finally, since the numbers 0, 6 and 8 have not been used in A or B, they are placed outside both A and B.

Note

The *intersection* of two sets consists of any numbers (or objects) that are in both A and B.



The *union* of two sets consists of any numbers (or objects) that are in A or in B or in both.



In the example above,

the intersection of A and B = { 4 }

the union of A and B = { 1, 2, 3, 4, 5, 7, 9 }

Note that, although the number 4 occurs in both A and B, it is *not* repeated when writing down the numbers in the union.
The complement of a set consists of any numbers (or objects) that are not in that set. In the example above,

the complement of A = { 0, 1, 2, 3, 5, 6, 8 }

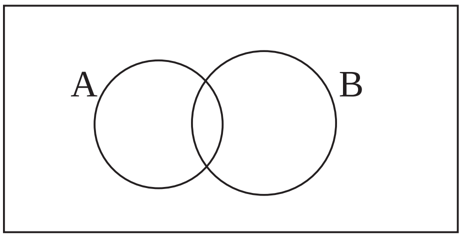
the complement of B = { 0, 6, 7, 8, 9 }

Exercises

1. Set A = { 1, 4, 5, 7, 8 }

   Set B = { 2, 6, 8, 10 }

   (a) Copy and complete the Venn diagram. Include all the whole numbers from 1 to 10.



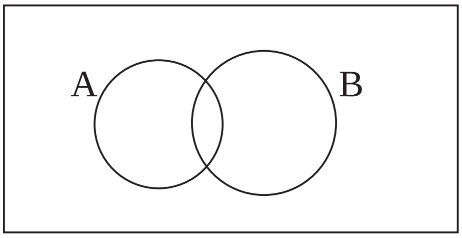
   (b) What is the intersection of A and B?

2. The whole numbers 1 to 10 are organised into 2 sets, set A and set B.

   Set A contains all the odd numbers.

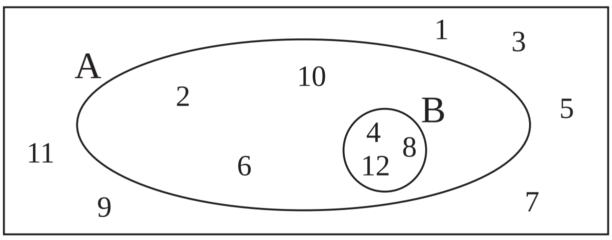
   Set B contains all the numbers greater than 4.

   (a) Copy and complete this diagram.



   (b) What is the union of A and B?

3. The whole numbers 1 to 12 are included in the Venn diagram.



   (a) List set A.

   (b) List set B.

   (c) Describe both sets in words.

   (d) What is the complement of A?

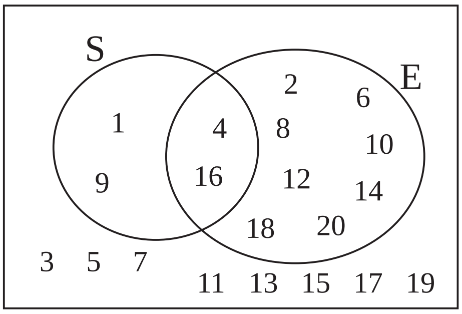
4. (a) Draw a Venn diagram to illustrate the sets P and Q. Include all the whole numbers from 1 to 15 in your diagram.

P = { 3, 5, 7, 9 }

Q = { 1, 3, 5, 7, 9, 11, 13, 15 }

(b) What is the intersection of P and Q?

5. The whole numbers 1 to 20 are organised into sets as shown in the Venn diagram below.



(a) List set E.

(b) List set S.

(c) Describe each set in words.

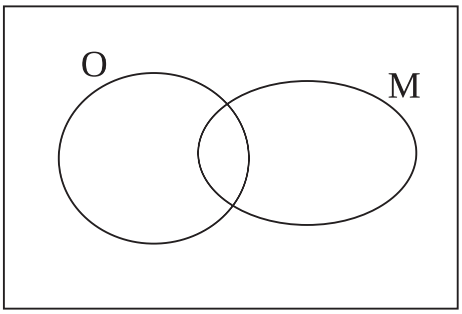
(d) What is the union of E and S?

6. The whole numbers 1 to 20 are organised into two sets,

O : Odd numbers

M: Multiples of 5

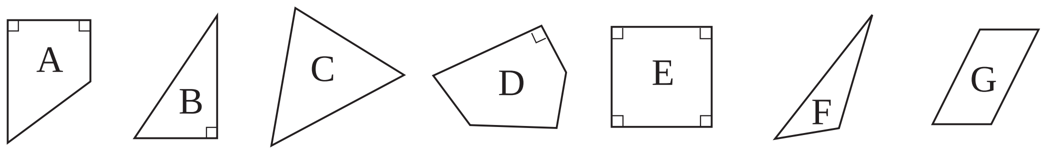
Copy and complete the Venn diagram, placing each number in the correct place.

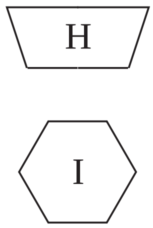


7. The shapes shown below are to be sorted into 2 sets, R and Q.

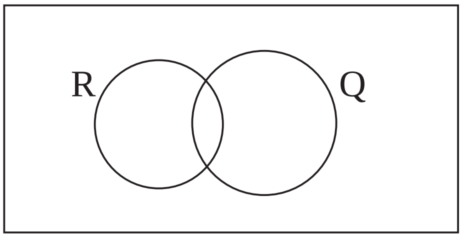
R contains shapes with a right angle.

Q contains shapes with four sides.





(a) Sort the shapes using the Venn diagram below.



(b) Which shapes are in both sets?

(c) Which shapes are in R but not in Q?

(d) Which shapes are not in R or Q?

8. Set P contains the letters needed to spell 'JENNY'.

Set Q contains the letters needed to spell 'JEN'.

Set R contains the letters needed to spell 'TED'.

(a) Draw a Venn diagram for the two sets, P and R.

(b) Draw a Venn diagram for the two sets, P and Q.

(c) What is the union of P and R?

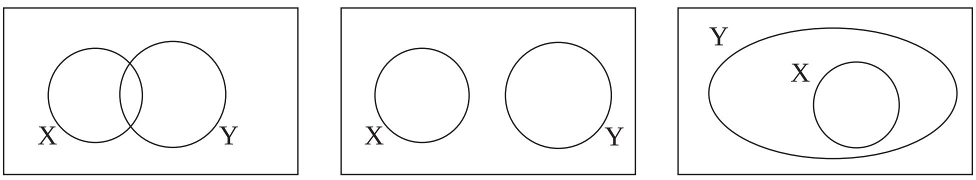
(d) What is the intersection of P and R?

9. Set S contains silver coins in circulation in the UK.

Set R contains circular coins in circulation in the UK.

Draw a Venn diagram to illustrate these two sets. You should include *all* UK coins in the Venn diagram.

10. Which of these Venn diagrams would be best for the sets described below?



A B C

(a) X is the set of all squares.
Y is the set of all rectangles.

(b) X is the set of all triangles.
Y is the set of all squares.

(c) X is the set of all quadrilaterals (4-sided shapes).
Y is the set of all triangles.

(d) X is the set of all shapes containing a right angle.
Y is the set of all triangles.

1.4 Set Notation

We use ξ to denote the *universal* set, that is, the set from which we are picking the members of A, B, ...

$A \cap B$, the intersection of A and B, is the set of members in set A *and* in set B.

$A \cup B$, the union of A and B, is the set of members in set A or in set B or in both.

A' , the complement of A, is the set of members in ξ but not in A.

$A \subset B$ means that A is a subset of B, i.e. every element in A is also in B.

$\varnothing$ is the empty set, i.e. the set with no numbers (or objects).

Example 1

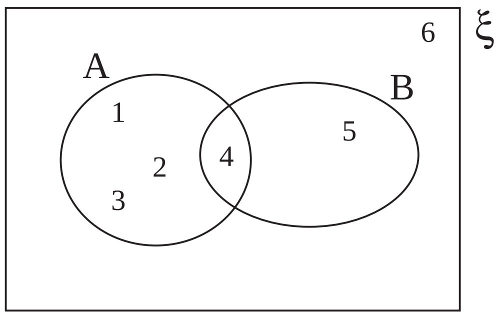
If $\xi = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 2, 3, 4\}$, $B = \{4, 5\}$

find (a) $A \cap B$, (b) $A \cup B$ (c) A' (d) B'

Is $B \subset A$?

Solution

First put the numbers in a Venn diagram.



(a) $A \cap B = \{4\}$

(b) $A \cup B = \{1, 2, 3, 4, 5\}$

(c) $A' = \{5, 6\}$

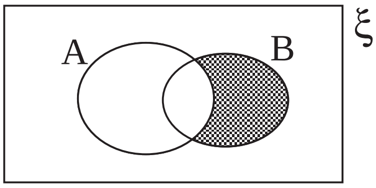
(d) $B' = \{1, 2, 3, 6\}$

No, B is not a subset of A since the number 5 is in B but not in A.

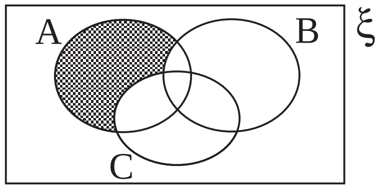
Example 2

Use set notation to describe the shaded regions of these diagrams.

(a)



(b)



Solution

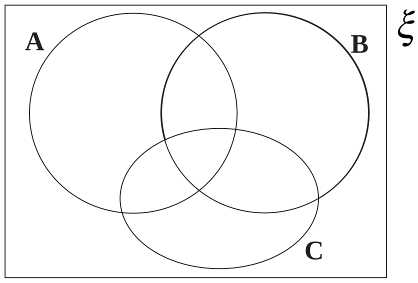
(a) This is the intersection of B with A', i.e. $B \cap A'$.

(b) This is the intersection of A with the complement of the union of B and C, i.e. $A \cap (B \cup C)'$.

Example 3

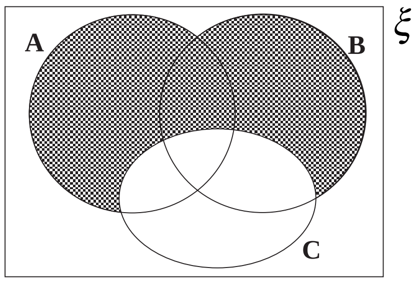
On this diagram, shade the region that represents

$$(A \cup B) \cap C'$$



Solution

You want the union of A and B which is not in C.



Exercises

1. If $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

   $A = \{2, 4, 6, 8\}$

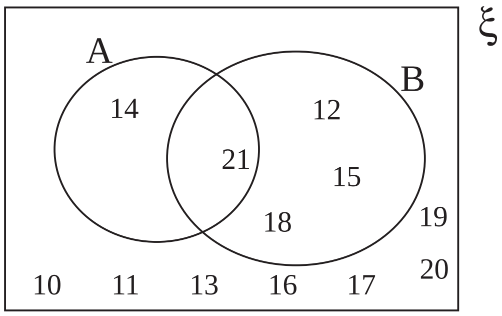
   and $B = \{3, 6, 9\}$

   find:

   (a) $A \cap B$ (b) $A \cup B$ (c) A'

   (d) B' (e) $A' \cap B'$ (f) $A' \cup B'$

2. The Venn diagram illustrates sets A, B and ξ.



Find:

(a) $A \cap B$ (b) $(A \cap B)'$ (c) $A \cup B$

(d) A' (e) B' (f) $A' \cap B'$

(g) $A' \cap B$

3. If $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{1, 3, 6, 10\}$

$B = \{1, 5, 10\}$

and $C = \{3, 6, 9, 12\}$,
find:

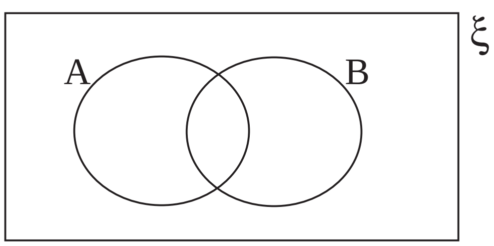
(a) $A \cap B$ (b) $A \cap C$ (c) $B \cap C$

(d) $A \cup B$ (e) $A \cup C$ (f) C'

(g) $A \cap C'$ (h) B' (i) $B' \cup C'$

(j) $A \cap B \cap C$ (k) $A \cup B \cup C$

4. Make a separate copy of this diagram for each part of the question.

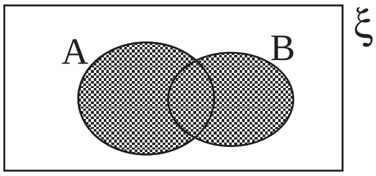


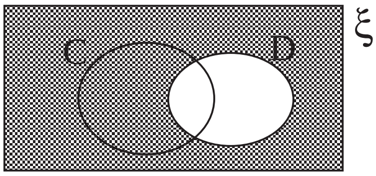
Shade the region on the diagram that represents:

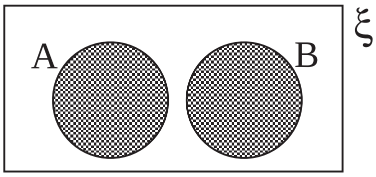
(a) $A \cap B$ (b) A' (c) $A \cup B'$

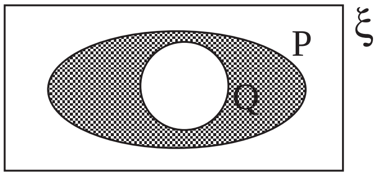
(d) $A' \cap B'$ (e) $A \cap B'$ (f) $(A \cup B)'$

5. Use set notation to describe the region shaded in each of these diagrams.

(a)


(b)


(c)


(d)


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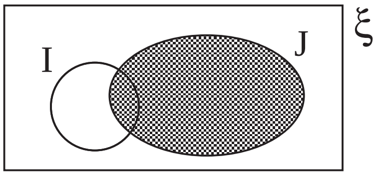

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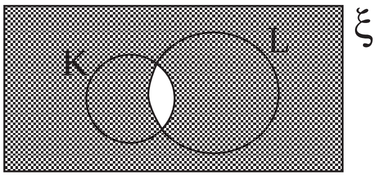
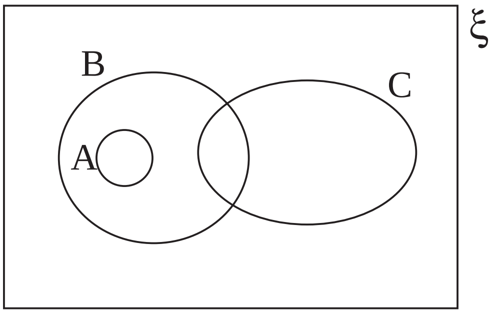


6. The diagram illustrates 3 sets, A, B and C.

Say whether each of these statements is *true* or *false*.



(a) $A \subset B$

(b) $B \subset C$

(c) $A \cap B = A$

(d) $A \cap B = C$

(e) $A \cap C = \varnothing$

(f) $B \cap C = \varnothing$

(g) $B \cup A = B$

7. If $\xi = \{a, b, c, d, e, f, g, h\}$

$A = \{a, c, e\}$

$B = \{b, d, g, h\}$

and $C = \{a, c, e, f\}$,

say whether each of these statements is *true* or *false*. Write correct statements to replace those that are false.

(a) $B \cap C = \varnothing$

(b) $C \subset A$

(c) $B \cup C = \xi$

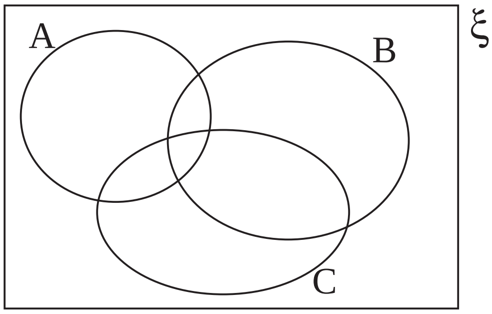
(d) $A \cap C = \{a, c, e, f\}$

(e) $(A \cap C)' = \{b, d, f, g, h\}$

(f) $B \subset \xi$

(g) $A \cap B' = C$

8. For each part of the question, use a copy of the diagram.



Shade the region of the diagram that represents:

(a) $A \cap B \cap C$

(b) $(A \cup B) \cap C$

(c) $(A \cap B) \cup C$

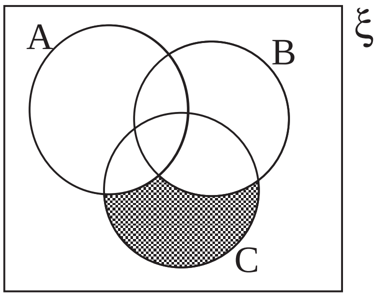
(d) $A' \cap (B \cup C)$

(e) $A' \cap B \cap C$

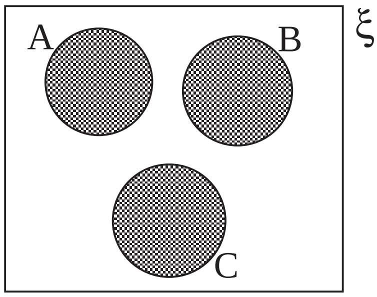
(f) $A' \cap B' \cap C'$

9. Use set notation to describe the regions shaded in each of these diagrams.

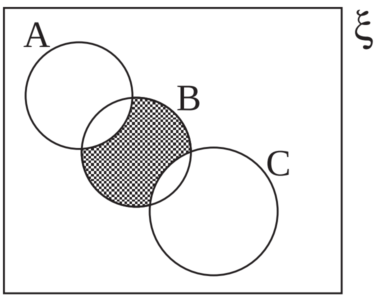
(a)



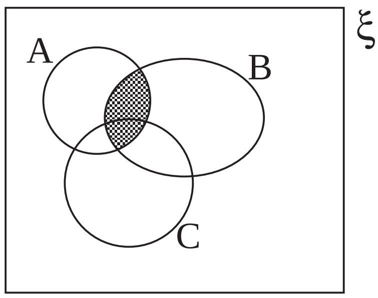
(b)



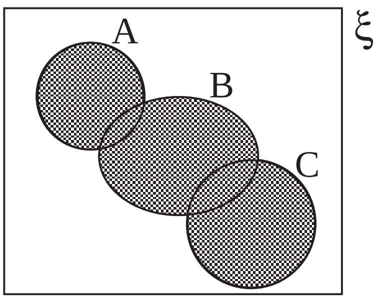
(c)



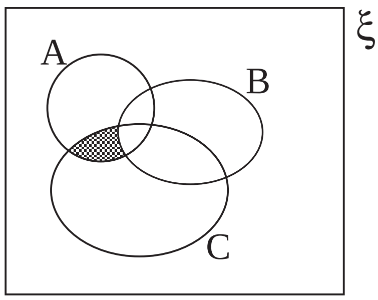
(d)



(e)



(f)



10. If $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 3, 5, 7\}$

$B = \{2, 4, 6, 8\}$

$C = \{1, 3, 5, 7, 9\}$,

draw a Venn diagram to represent these sets.

Then find

(a) $A \cap B$ (b) C'

(c) $A \cap C$ (d) $B \cap C$

(e) $A \cup B$ (f) $(A \cup B)'$

(g) $(A \cup B) \cap C$ (h) $(A \cup B) \cap C'$

1.5 Logic Problems and Venn Diagrams

Venn diagrams can be very helpful in solving logic problems.

Example

In a class there are

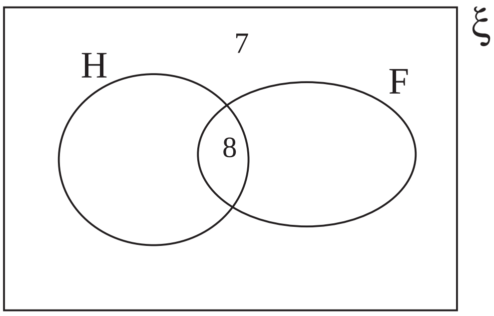
- 8 students who play football and hockey
- 7 students who do not play football or hockey
- 13 students who play hockey
- 19 students who play football.

How many students are there in the class?

Solution

You can use a Venn diagram to show the information.

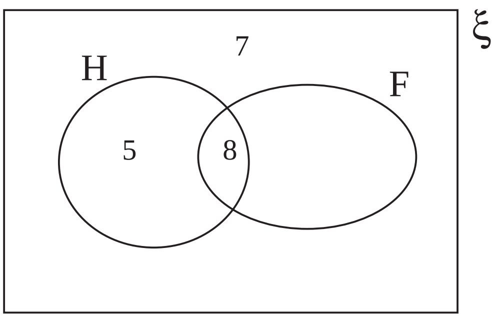
The first two sets of students can be put directly on to the diagram.



If there are 13 students who play hockey, and we already know that 8 play hockey and football, then there must be

$$13 - 8 = 5$$

who play just hockey.



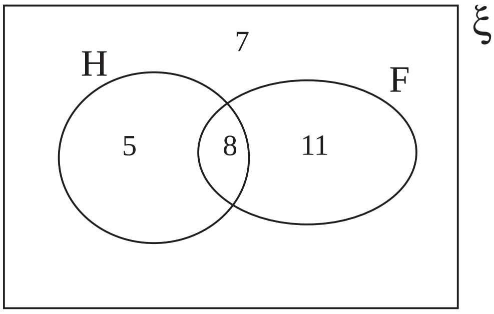
Similarly for football,

$$19 - 8 = 11$$

play just football.

So the total number of students in the class is

$$7 + 5 + 8 + 11 = 31$$

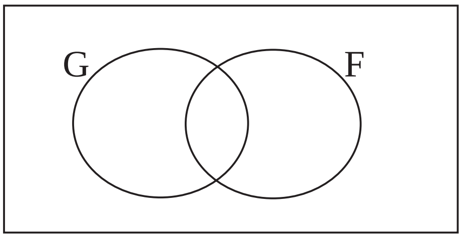


Exercises

1. In a family of six, everybody plays football or hockey. 4 members of the family play both sports and 1 member of the family plays only hockey. How many play only football?

2. John's mum buys 5 portions of chips. All the portions have salt or vinegar on them. Some have salt and vinegar. There are 2 portions with salt and vinegar and one portion with only vinegar. How many portions have only salt on them?

3. This diagram represents a class of children. G is the set of girls and F is the set of children who like football. Make 4 copies of this diagram.



On separate diagrams, shade the part that represents:

(a) girls who like football, (b) girls who dislike football,

(c) boys who like football, (d) boys who do not like football.

4. In a class of 32 pupils, 20 say that they like pancakes and 14 say that they like maple syrup. There are 6 pupils who do not like either. How many of them like both pancakes and maple syrup?

5. On a garage forecourt there are 6 new cars, 12 red cars and no others.

   (a) What is the maximum possible number of cars on the forecourt?

   (b) What is the smallest possible number of cars on the forecourt?

   (c) If 2 of the new cars are red, how many cars are on the forecourt?

6. There are 20 people in a room. Of these, 15 are holding newspapers and 8 are wearing glasses. Everyone wears glasses or holds a newspaper. How many people are wearing glasses *and* holding a newspaper?

7. A pencil case contains 20 pens that are red or blue. Of these, 8 are blue and 6 do not work. How many of the blue pens do not work if there are 8 red pens that do work?

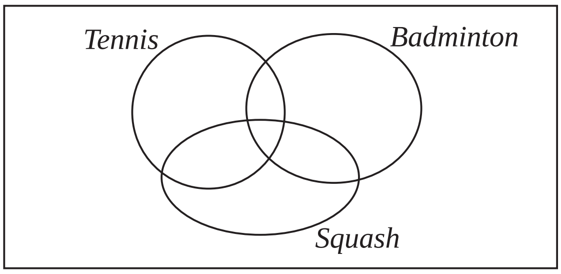
8. In a school canteen there are 45 children. There are 16 who have finished eating. The others are eating either fish or chips, or both fish and chips. There are 26 eating chips and 17 eating fish.

   (a) How many are eating fish and chips?

   (b) How many are eating fish without chips?

   (c) How many are eating only chips?

9. Youth club members can choose to play tennis, badminton or squash. The diagram below represents the possible combinations.



   Make 3 copies of the diagram.

   On separate diagrams shade the parts that represent:

   (a) those who play all three sports,

   (b) those who play tennis and badminton, but not squash,

   (c) those who play only tennis.

10. All the members of a group of 30 teenagers belong to at least one club. There are 3 clubs, chess, drama and art.

    6 of the teenagers belong to only the art club.

    5 of the teenagers belong to all 3 clubs.

    2 of the teenagers belong to the chess and art clubs but not to the drama club.

    15 of the teenagers belong to the art club.

    2 of the teenagers belong only to the chess club.

    3 of the teenagers belong only to the drama club.

(a) How many of the group belong to the chess club and the drama club, but not the art club?

(b) How many of the group belong to each club?

11. In a class of 32 pupils:

5 pupils live in New Town, travel to school by bus and eat school dinners,

3 pupils live in New Town, travel to school by bus but do not eat school dinners.

9 pupils do not live in New Town, do not travel to school by bus and do not eat school dinners.

11 pupils live in New Town and have school dinners.

16 pupils live in New Town.

9 pupils travel by bus and eat school dinners.

13 pupils travel by bus.

How many pupils eat school dinners?

2 Arithmetic: Place Value

This unit concentrates on place value, both for whole numbers and for decimals, and revises the important techniques of rounding numbers to a given accuracy.

2.1 Place Value and Rounding

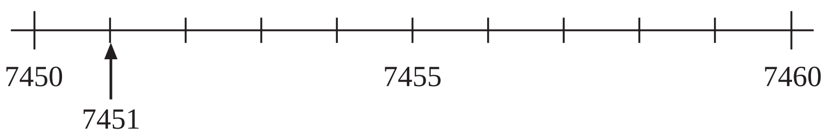
First note how we write and speak numbers.

Hundreds	*Tens* *of Millions*	*Units*	*Hundreds*	*Tens* *of Thousands*	*Units*	*Hundreds*	*Tens*	*Units*
5	7	2	6	4	1	8	3	9

We say

"Five hundred and seventy two million, six hundred and forty one thousand, eight hundred and thirty nine."

7451 is 7450 *to the nearest 10*, since it is nearer to 7450 than to 7460 (see the number line below).



7451 is 7500 *to the nearest 100*, since it is nearer to 7500 than to 7400.

7451 is 7000 *to the nearest 1000*, since it is nearer to 7000 than to 8000.

Note

The convention is that '5' rounds up to the nearest 10,
e.g. 35 to the nearest 10 is 40.

Example 1

Write 2716 to the nearest

(a) 10 (b) 100 (c) 1000

Solution

(a) 2720 (b) 2700 (c) 3000

Example 2

What is the value of the '6' in each of these numbers?

(a) 167 (b) 2006 (c) 6423

Solution

(a) '6' means 6 tens = 60

(b) '6' means 6 units = 6

(c) '6' means 6 thousands = 6000

Exercises

1. Write each of these numbers to the nearest 10.

(a) 89	(b) 45	(c) 72
(d) 12	(e) 9	(f) 2
(g) 4713	(h) 5629	(i) 4755

2. Write each of these numbers to the nearest 100.

(a) 376	(b) 1417	(c) 24 699
(d) 101	(e) 149	(f) 251

3. Write each of these numbers to the nearest 1000.

(a) 1001	(b) 2500	(c) 3999
(d) 132 400	(e) 56 471	(f) 555 511

4. A milkman delivered 109 865 bottles of milk in one year.
Write the number of bottles to:
(a) the nearest 100,
(b) the nearest 1000,
(c) the nearest 10,
(d) the nearest 10 000.

5. A school has 1256 pupils. Write this number to:
(a) the nearest 10,
(b) the nearest 100,
(c) the nearest 1000.

6. Explain what the '9' represents in each of these numbers.

(a)	19	(b)	91	(c)	190
(d)	1971	(e)	19 800	(f)	2190
(g)	9 100 001	(h)	9 001 111	(i)	900 371 423

7. Write these numbers in words.

(a)	32	(b)	14	(c)	86
(d)	124	(e)	328	(f)	1463
(g)	3 000 000	(h)	4 713 000	(i)	3 991 001

8. Write each of the following in figures.

(a) Twenty four

(b) Eighty six

(c) Nineteen

(d) One hundred and twenty

(e) Three hundred and four

(f) One thousand and twenty six

(g) Three million, four hundred thousand

(h) One thousand and five

9. For each statement below, explain whether it is likely to be *true* or *false*.

(a) The average shoe size in your class is size 6.

(b) The average length of a car is 10 metres.

(c) The average height of an adult is 176 cm.

(d) The number of matches in a matchbox is 50.

(e) The average height of children in a class of Year 7 pupils is 138 cm.

(f) John counted 12 people on his bus this morning.

(g) Sarah saw 120 people get onto her bus on her way to school.

10. Place the numbers below in order, with the smallest first.

(a) 147, 222, 316, 47, 32, 1004.

(b) 1472, 3416, 621, 3813, 1471, 15 721.

(c) 6000, 60 000, 3000, 30 000, 4 000 000.

11. (a) What is the largest possible number you can make using each of these digits once only: 4, 6, 3, 2 and 8 ?

    (b) What is the smallest number you can make using all the digits in (a)?

    (c) What do you notice about the order of the digits in your answers to (a) and (b)?

    (d) How do your answers change if you can use 0 as well?

12. You are given the number 1735. You are allowed to swap the positions of any two digits.

    For example, 1 7 3 5 gives 1 3 7 5

    or, 1 7 3 5 gives 5 7 3 1

    (a) Explain how to make the *largest* possible number using one swap.

    (b) Explain how to make the *smallest* possible number with one swap.

13. Ramesh says that there are 120 pupils in his year at school. If he has rounded the number of pupils in his year to the nearest 10, how many pupils could there be in his school year? (Write *all* the possible answers.)

14. A newspaper report states that 42 000 people watched a football match at Wembley. The actual number has been rounded to the nearest 1000.

    (a) What is the *largest* possible number of people that watched the match?

    (b) What is the *smallest* possible number of people that watched the match?

15. The table gives the results and attendances for some football matches. Answer these questions using the table.

    (a) Which match had the largest attendance?

    (b) Find the total attendance at all the matches to the nearest 1000.

    (c) How many more people watched Newcastle than watched Wimbledon, to the nearest 100?

BLACKBURN 28,212	1	ARSENAL	4
DERBY 29,126	4	BOLTON	0
LIVERPOOL 43,007	2	CRYSTAL PALACE	1
NEWCASTLE 36,534	2	BARNSLEY	1
SHEFFIELD WED 28,036	1	WEST HAM	1
TOTTENHAM 33,463	1	COVENTRY	1
WIMBLEDON 15,131	0	EVERTON	0

2.2 Decimals and Place Value

Note that the number

1 . 7 4 3

means

1 unit   7 tenths   4 hundredths   3 thousandths

Example 1

Write these numbers in order, smallest first.

0.5, 0.95, 0.905, 0.59, 0.509, 0.6, 0.9

Solution

0.5, 0.509, 0.59, 0.6, 0.9, 0.905, 0.95

Example 2

Write 8.4751 correct to
(a) 3 decimal places,
(b) 2 decimal places,
(c) 1 decimal place.

Solution

(a) 8.475, since 8.4751 is nearer to 8.475 than to 8.476.

(b) 8.48, since 8.4751 is nearer to 8.48 than to 8.47.

(c) 8.5, since 8.4751 is nearer to 8.5 than to 8.4.

Exercises

1. What is the value of the '5' in each of these numbers?

   (a) 0.45   (b) 0.54   (c) 5.74
   (d) 3.415   (e) 4.258   (f) 3.502

2. Write the numbers in order, smallest first.

   0.85, 0.9, 0.8, 0.58, 0.6, 0.5, 0.87

3. Write each of these numbers correct to 1 decimal place.

   (a) 1.47   (b) 3.68   (c) 0.45
   (d) 3.751   (e) 4.08   (f) 5.005

4. Write each of these numbers correct to 2 decimal places.

   (a) 3.444 (b) 8.555 (c) 0.321

   (d) 4.7612 (e) 0.3002 (f) 4.1050

5. Sally is given a number correct to 3 decimal places. She writes it to 2 decimal places as 4.71.

   Write down a list of the numbers she could have been given.

6. Write these numbers in figures.

   (a) Four and six tenths

   (b) Five and four hundredths

   (c) Sixteen, three tenths and four hundredths

   (d) One hundred and five hundredths

   (e) One thousand and twenty six and five thousandths

7. Write these numbers in words.

   (a) 5.7 (b) 5.006 (c) 3.02

8. What is the difference between four tenths and forty hundredths? Explain your answer.

9. Write these numbers in order, largest first.

   0.7, 0.2991, 1.05, 1.508, 0.58, 2.4

10. You are given the digits 3, 4, 0, 7 and a decimal point. Using each number only once, what is

    (a) the *largest* number you can make,

    (b) the *smallest* number you can make?

3 Graphs

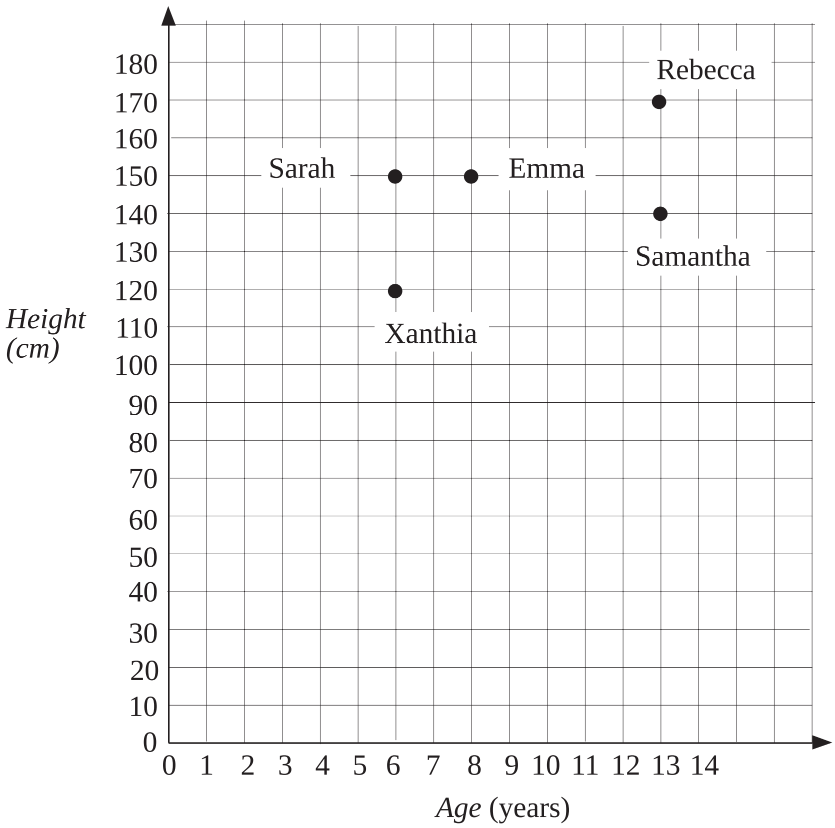
This is a key building block in in mathematics that is used both to illustrate data and algebraic formulae.

3.1 Scatter graphs

This is a particularly useful way of illustrating "paired" data - that is, information which has *two* related values e.g. height and weight; price and sales; latitude and longtitude; rain and sunshine.

Example

The scatter graph below shows height and age. Information for five girls has been plotted on the graph.



(a) Who is the tallest and how tall is she?

(b) Who is the youngest and how old is she?

(c) How much taller is Rebecca than Emma?

(d) How much younger is Sarah than Samantha?

(e) Is it true that older people are taller?

Solution

(a) Rebecca is the tallest, and her height is 170 cm.

(b) Sarah and Xanthia are the youngest - they are both 6 years old.

(c) Emma is 150 cm in height, so Rebecca is

$$170 - 150 = 20 \text{ cm}$$

taller than Emma.

(d) Samantha is 13 years old, so Sarah is

$$13 - 6 = 7 \text{ years}$$

younger than Samantha.

(e) There is some evidence from the graph to deduce that older people are usually taller, but it is not true in general for girls less than 14 years old.

Exercises

1. Ten children took part in a sponsored walk. The scatter plot below shows how much money they raised and how far they walked.

   (a) (i) How far did Karen walk?

   (ii) How much money did she raise?

   (b) Two children walked 15 miles. What are their names?

   (c) Who walked 20 miles and raised £40?

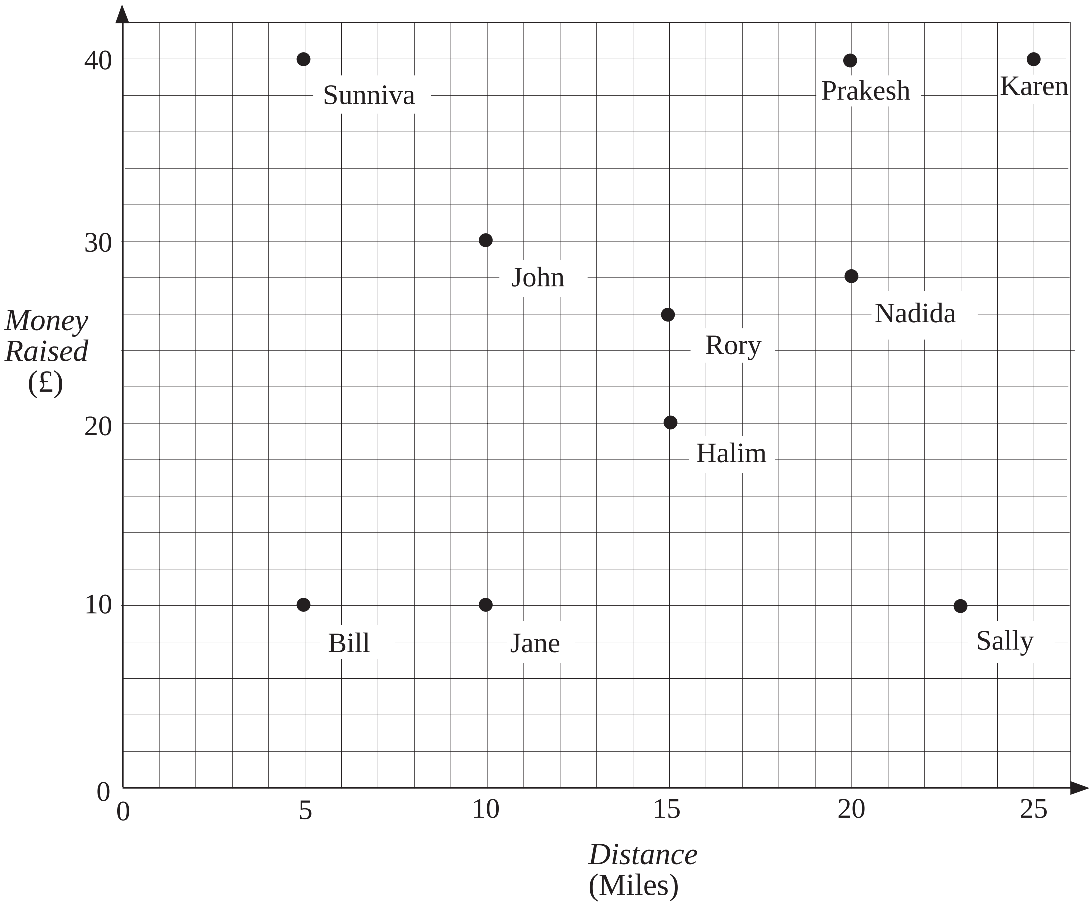
   (d) Explain how to work out how much Bill was sponsored for each mile.

   (e) How much was Sunniva sponsored for each mile?

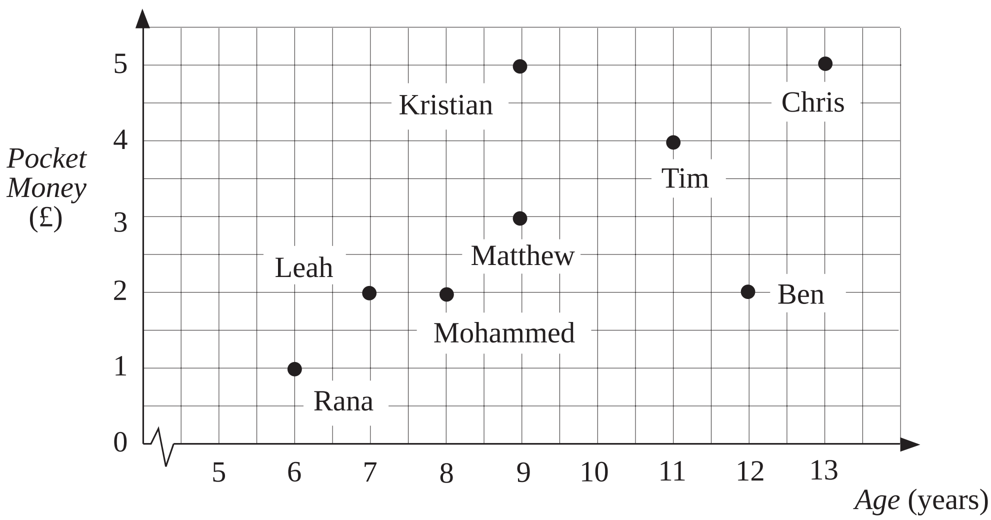
   (f) How much money did Rory raise?

   (g) How far did Sally walk?

   (h) Generally, was more money raised by walking further?



2. The scatter plot below shows the ages of the children who live in one street and how much pocket money they get each week.



(a) (i) How old is Tim?

(ii) How much pocket money does he get?

(b) Which children are older than Tim?

(c) Who gets more pocket money than Tim?

(d) Who is 9 and gets £ 3 pocket money?

(e) Who gets the same amount of pocket money as Kristian?

(f) Who is the same age as Kristian?

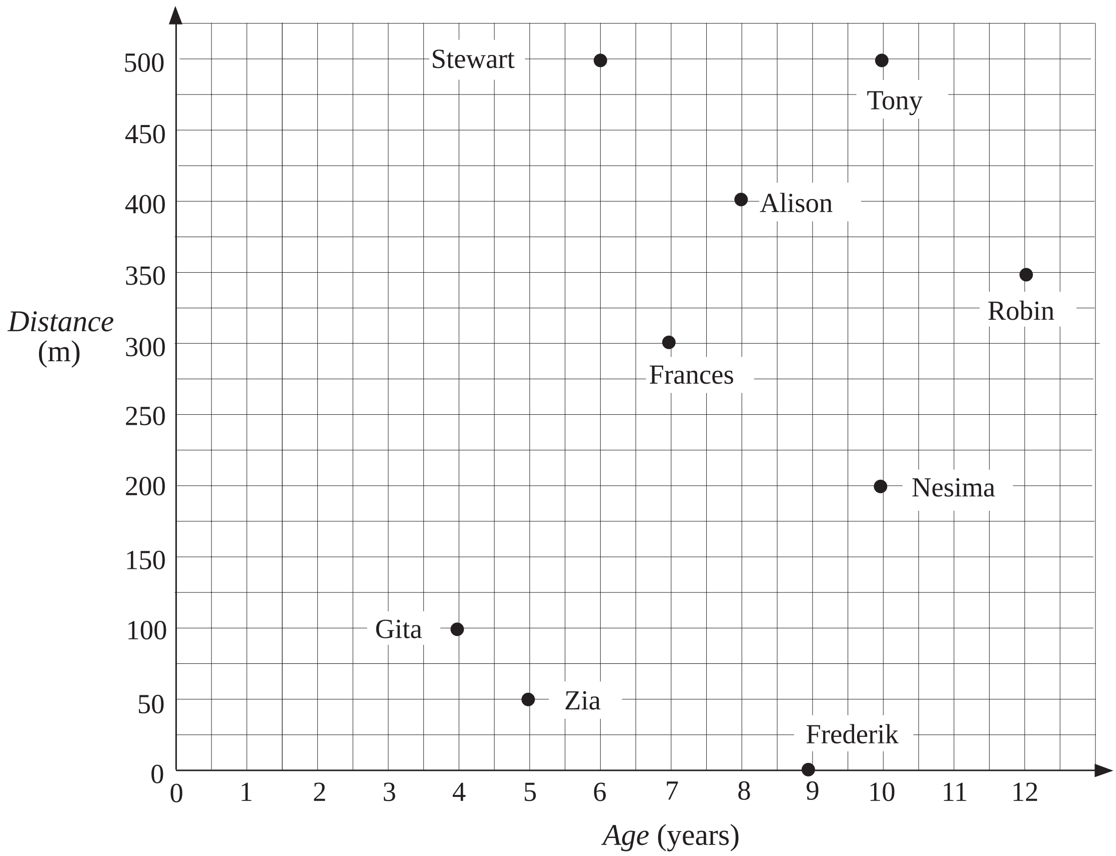
(g) Who gets twice as much pocket money as Ben?

(h) Who gets half the pocket money Ben gets?

(i) Ben is trying to pursuade his parents to give him more pocket money. How would Ben use this graph to support his claim?

What would be a reasonable amount for Ben to have?

3. The scatter plot shows the ages of some children and the greatest distance they can swim.



(a) Who is the youngest person that can swim 500 m?

(b) Who cannot swim at all?

(c) Who is 8 and can swim 400 m?

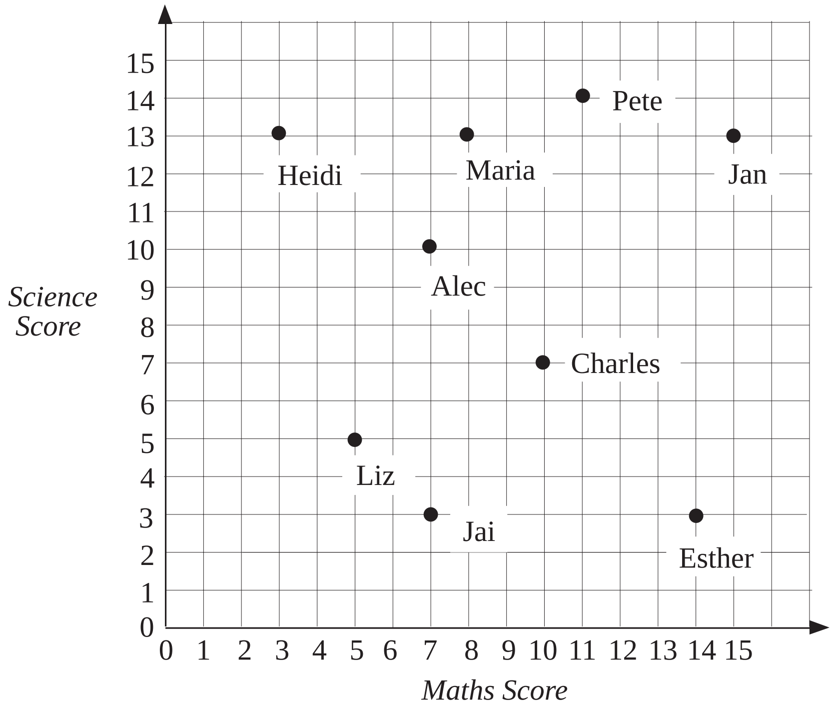
(d) Who is 12 and can swim 350 m?

(e) Who can swim the furthest?

(f) Who is the youngest child?

(g) Who can swim further than Robin?

4. The maths and science teachers at a school gave the Year 7 pupils two tests. The results for 9 pupils are shown on the scatter plot.



(a) Who had the highest score in science?

(b) Who had the highest score in maths?

(c) Who is good at science and poor at maths?

(d) Who is good at maths and poor at science?

The two test scores are added together.

(e) Who has the highest total?

(f) Who has the lowest total?

The results for six other pupils are given in the table below.

Copy the set of axes used above. Draw a scatter plot for these pupils.

Name	*Maths Score*	*Science Score*
Rola Reesh	10	8
Karen Eccles	5	14
Jenny Sharp	15	13
Zia Uddin	7	4
Adrian Smith	8	6
Wendy Maull	5	9

5. In a gymnastics competition the performance of each competitor is given a mark out of 10 by two different judges. The results for a competition are given below.

Competitor	*Judge 1*	*Judge 2*
Jane	4	5
Nishi	6	3
Julie	8	9
Andrea	2	1
Veronica	5	4
Chiori	3	2

Plot the scores in a scatter plot.

The two scores are added together to give a total.

Use your scatter plot to find:

(a) Who has the highest total?

(b) Who is second in the competition?

6. In a sponsored swim, the children raised the money listed in the table below.

Name	*Distance (m)*	*Money Raised*
Mark	300	£12
Kingsley	500	£20
Caroline	200	£4
Kevin	400	£10
Pushpa	300	£8
Alex	250	£15
Jai	300	£20
Zahra	50	£10
Lynda	450	£18

(a) Draw a scatter plot for the distance and the money raised.

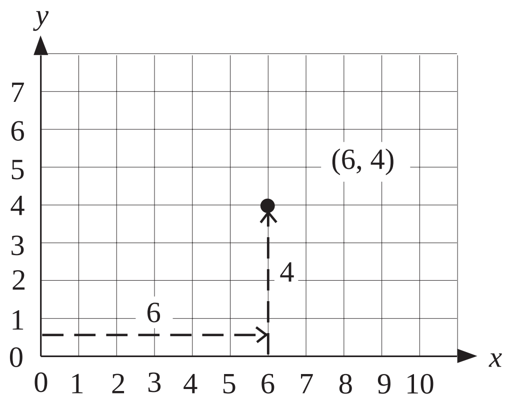
(b) Copy and complete this sentence using one of these words

"more" "less" "no"

The children who swam the furthest raised ☐ money.

3.2 Plotting Points

We will now see how to plot points on a graph.



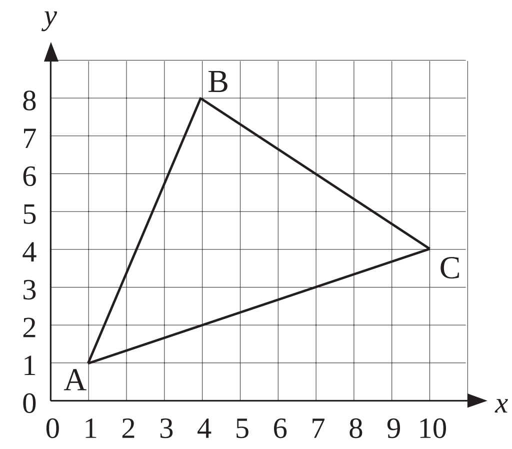
The x number comes first then the y number:

$$(x , y)$$

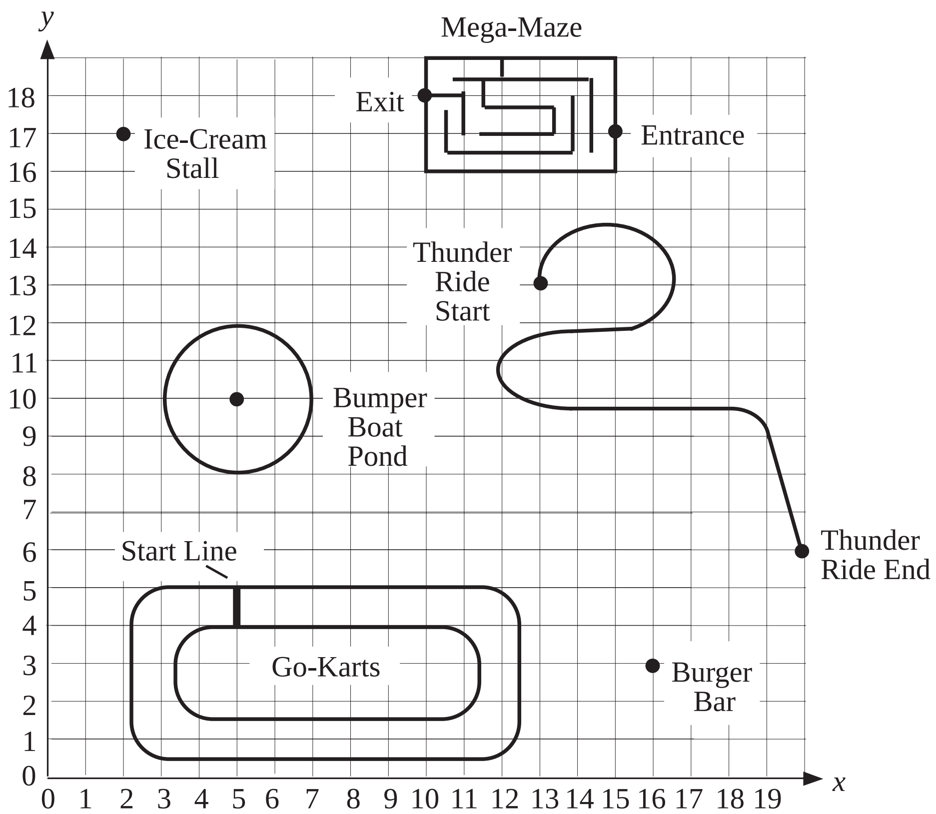
These number are called coordinates.

Exercises

1. Write down the coordinates of the three corners of this triangle.



2. The diagram shows a map of a theme park drawn on a set of axes.



Write down the coordinates of:

(a) the Burger Bar

(b) the start of the Thunder Ride

(c) the end of the Thunder Ride

(d) the centre of the Bumper Boat Pond

(e) the Ice Cream Stall

(f) the Mega-Maze entrance

(g) the Mega-Maze exit

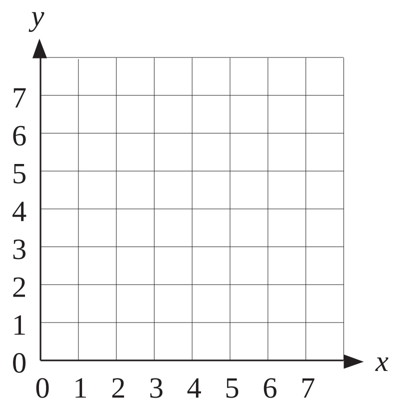
(h) both ends of the Go-Karts start line.

3. Draw a grid like this one.

(a) Join the points with coordinates (0, 3), (5, 6) and (5, 0) to draw a triangle.

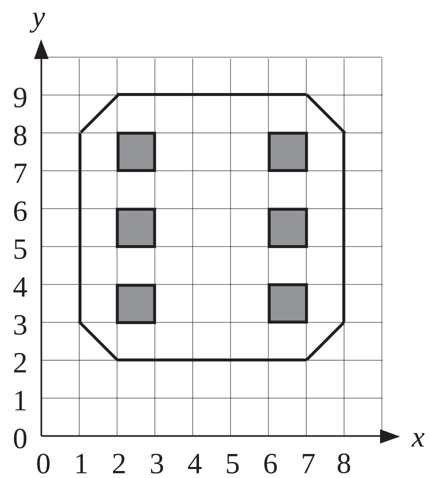
(b) On the same diagram join the points with coordinates (2, 0) (2, 6) and (7, 3) to draw a second triangle.

(c) Describe the shape you have now drawn.

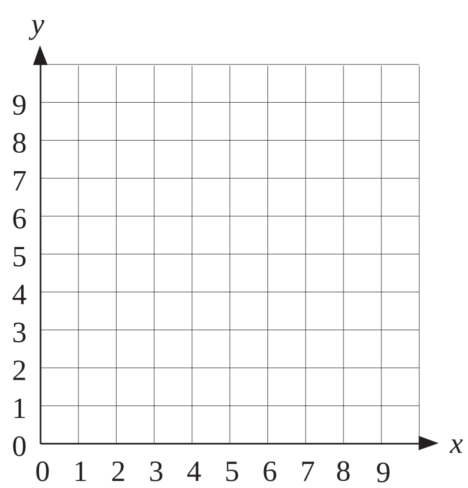


4. The diagram shows the face of a dice showing a 6.

Write a set of instructions that would give the face of the dice that shows a 1.



5. Draw a grid like this.



Join these points in order.

Use the same grid for all four parts.

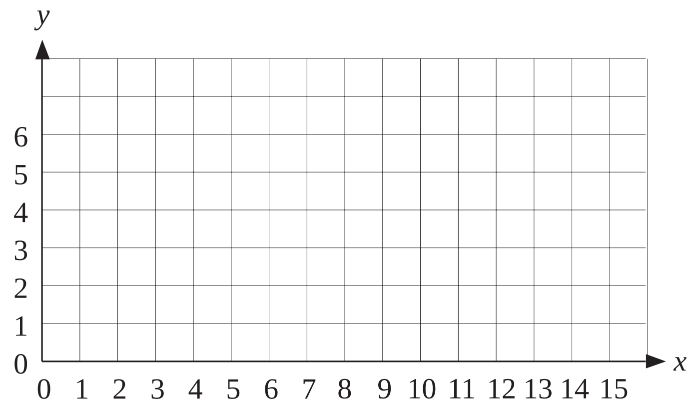
(a) (4, 6), (5, 7), (6, 6), (4, 6).

(b) (5, 8), (4, 8), (4, 7), (5, 8), (6, 8), (6, 7), (5, 8).

(c) (4, 5), (5, 4), (6, 5), (5, 3), (4, 5).

(d) (5, 2), (3, 4), (3, 5), (2, 5), (2, 8), (3, 8), (3, 9), (7, 9), (7, 8), (8, 8), (8, 5), (7, 5), (7, 4), (5, 2).

6. For this question you will need a grid like the one below.



Join each set of points in order to discover a message.

(a) (14, 4), (14, 5), (15, 5), (15, 4), (14, 4)

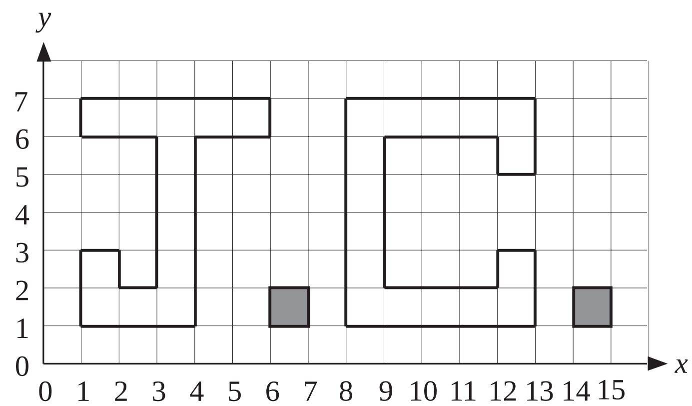
(b) (5, 1), (5, 6), (8, 6), (8, 5), (6, 5), (6, 4), (8, 4), (8, 3), (6, 3), (6, 2), (8, 2), (8, 1), (5, 1)

(c) (9, 6), (10, 6), (10, 2), (12, 2), (12, 1), (9, 1), (9, 6)

(d) (14, 1), (14, 3), (16, 3), (16, 6), (13, 6), (13, 1), (14, 1)

(e) (4, 6), (3, 6), (3, 4), (2, 4), (2, 6), (1, 6), (1, 1), (2, 1), (2, 3), (3, 3), (3, 1), (4, 1), (4, 6)

7. The picture shows a set of initials.

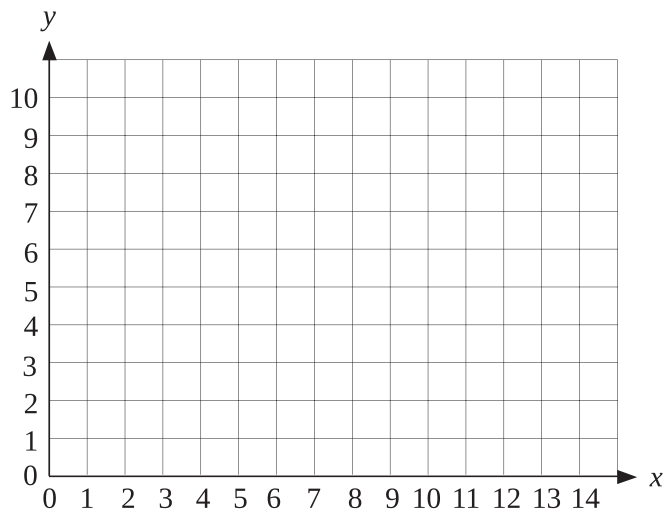


(a) Write out a set of instructions to draw these initials.

(b) Draw your initials on a grid in a similar way.

Write out a set of instructions. Give the instructions to a friend and see if they can use them to draw your initials.

8. Answer this question on a copy of this grid.



(a) Shade in the square with corners at the points with coordinates (5, 3), (5, 4), (6, 4), (6, 3).

(b) Join these points in order: (2, 4), (4, 8), (6, 9).

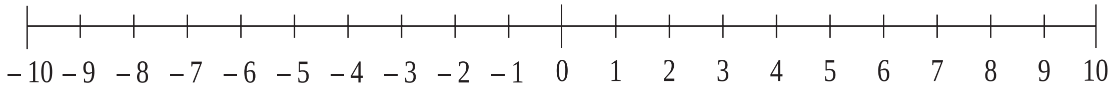
(c) Join these points in order: (6, 4), (7, 5), (8, 5), (9, 4), (9, 3), (8, 2), (7, 2) and (6, 3).

(d) Complete the picture to show a pair of glasses and write down the coordinates of the extra points that you use.

9. Draw a picture on a grid. Write a set of instructions using the coordinates so that a friend can draw the picture.

3.3 Negative Numbers

We extend our number system now to include negative numbers. It is useful to use a number line to illustrate this concept.



You can see for example

$$-2 < 4$$

$$-6 < -3$$

$$5 > -4$$

$$-1 > -7$$

You can check all these by looking at their positions on the number line.

Example

Make each statement below true by using the symbols < or > .

(a) – 5 ☐ 4

(b) 3 ☐ 7

(c) – 6 ☐ – 9

(d) 2 ☐ – 2

Solution

(a) – 5 < 4 (b) 3 < 7 (c) – 6 > – 9 (d) 2 > – 2

Exercises

1. What temperature is:

   (a) 3°C warmer than – 1°C

   (b) 6°C colder than – 3°C

   (c) 5°C warmer than – 5°C

   (d) 8°C warmer than – 7°C

   (e) 5°C colder than – 2°C

   (f) 3°C colder than 1°C

   (g) 6°C colder than 2°C

   (h) 8°C warmer than – 12°C

   (i) 10°C colder than – 2°C

   (j) 20°C warmer than – 12°C?

2. What number is;

   (a) 3 more than – 2 (b) 6 less than 1

   (c) 5 more than – 7 (d) 6 more than – 10

   (e) 5 less than – 4 (f) 16 less than 3

   (g) 5 more than – 20 (h) 6 more than 5

   (i) 12 less than 10 (j) 20 more than – 8?

3. Write each set of numbers in order with the smallest first.

   (a) 6, – 7, 8, – 2, – 5, – 10, 3

   (b) 3, – 2, 8, 0, – 1, 1, – 3

   (c) 5, – 7, – 20, 100, – 50, – 90, 60

4. Put either a $<$ or $>$ sign between each pair of numbers to give a true statement.

   (a) 4   2   (b) – 6   – 2

   (c) – 3   4   (d) 2   – 4

   (e) – 6   – 7   (f) – 6   – 5

   (g) 0   1   (h) – 1   0

5. Is each statement below true or false?

   (a) $6 > 7$ (b) $4 > 3$

   (c) $8 > -1$ (d) $5 > -6$

   (e) $-6 < -7$ (f) $-1 > 0$

   (g) $-3 < 2$ (h) $-7 < 6$

   (i) $-4 > -3$ (j) $-5 < -2$

6. Write down any integer that could go in the boxes below.

   (a) $5 < \square < 7$

   (b) $-5 < \square < -3$

   (c) $-3 > \square > -7$

   (d) $-6 < \square < 0$

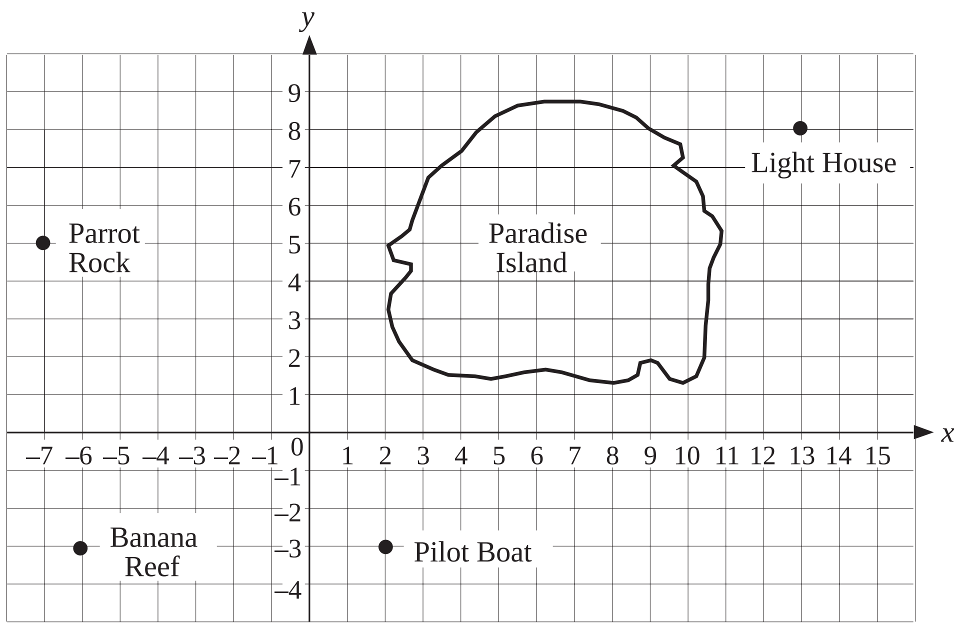
   (e) $-1 < \square < 2$

3.4 Coordinates

With the introduction of negative numbers, we can bring in coordinate axes with positive and negative numbers.

Example

A map of Paradise Island is drawn on the grid below.



What are the coordinates of;

(a) the Lighthouse

(b) the Pilot Boat

(c) Parrot Rock

(d) Banana Reef?

Solution

(a) $x = 13$, $y = 8$ which is written as (13, 8)

(b) (2, – 3)

(c) (– 7, 5)

(d) (– 6, – 3)

Exercises

1. The map below shows a small island.

   (a) What are the coordinates of;

      (i) the cafe

      (ii) the beach shop

      (iii) the hotel

      (iv) the campsite

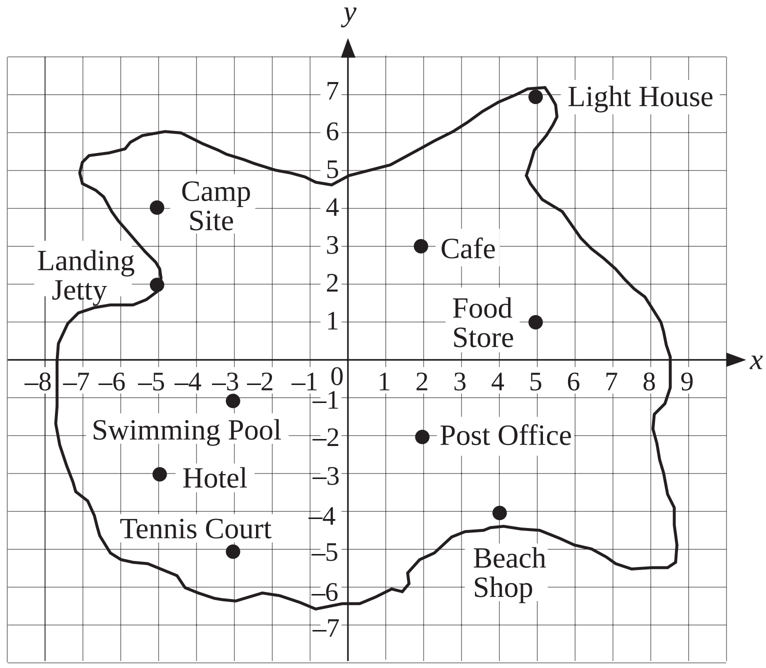
      (v) the swimming pool?

(b) Nisha moves from the place with coordinates (– 3, – 5) to the place with coordinates (– 5, 4).

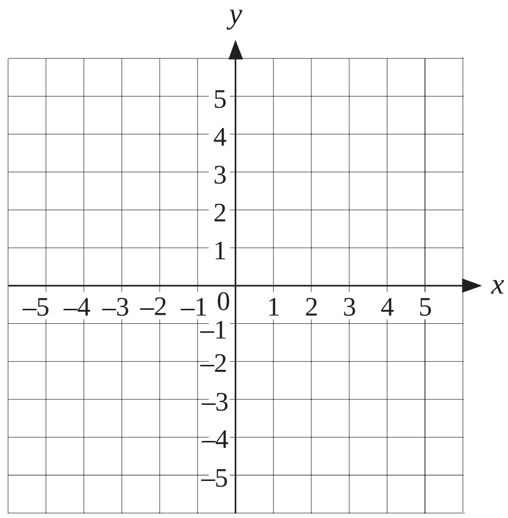
Where did she start?

Where did she finish?

(c) Explain why Jacob cannot walk in a straight line from the place with coordinates (5, 7) to the place with coordinates (– 5, 4).



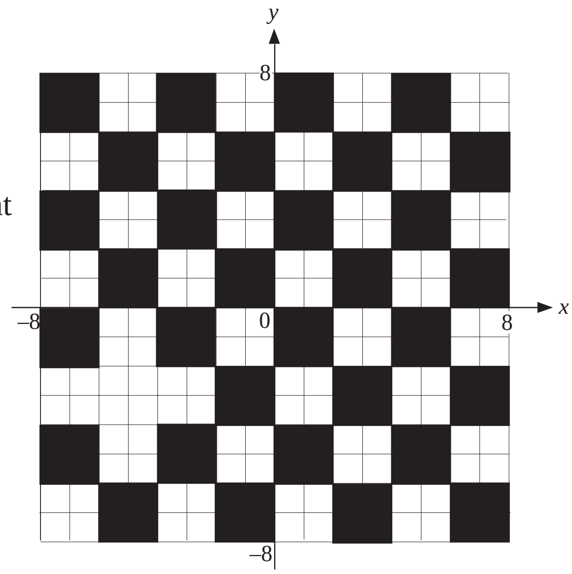
2. (a) Draw a set of axes like those in the diagram.



(b) Mark the points with coordinates (4, 0), (– 4, 0), (0, 4), (0, – 4), (1, 2), (1, – 2), (3, 3), (3, – 3), (2, 1), (2, – 1), (– 1, 2), (– 1, – 2), (– 3, 3), (– 3, – 3), (– 2, 1), (– 2, – 1).

(c) Join the points to form an 8 pointed star.

3. The picture shows a giant chess board that has been drawn on the surface of a playground.



(a) What are the coordinates of the corners of the square that is the wrong colour?

(b) John always looks in the direction of the y-axis. When he starts at the point with the coordinates (0, 7), he has one foot on a black square and one foot on a white square.

Is his left foot on the black square or the white square?

Describe where his feet are if he starts at;

(i) (2, 5)

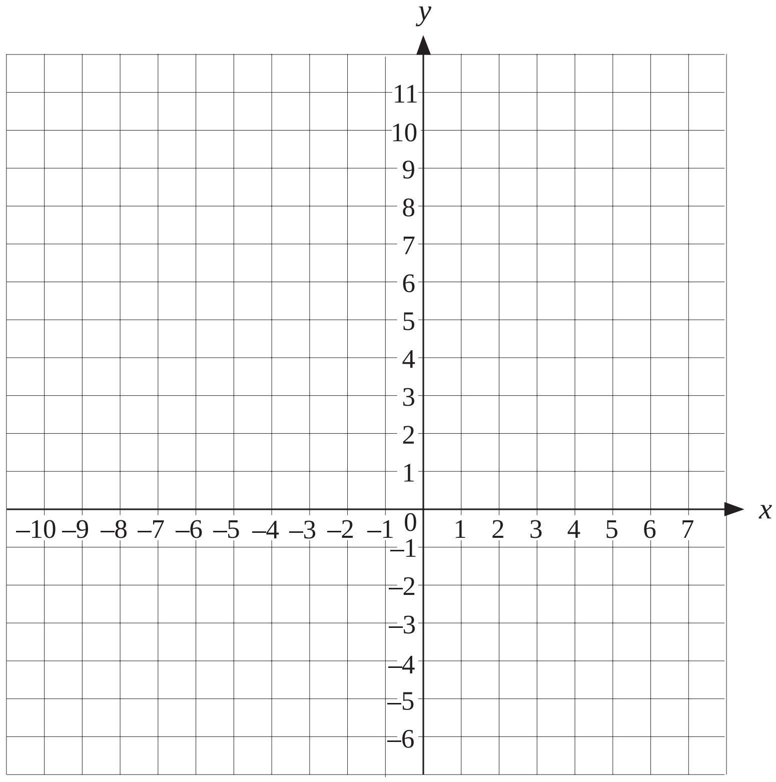
(ii) (– 5, 3)

(iii) (6, – 3)

(iv) (– 8, – 1)

(v) (– 2, 3)

4. Draw the grid shown in the diagram.



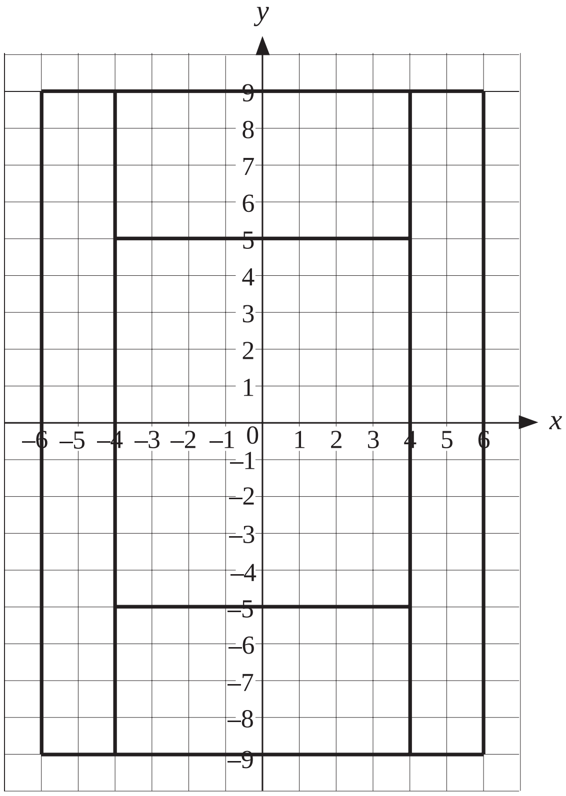
On the grid plot the points below joining each point to the next point.

(0, 0), (2, – 2), (0, – 5), (– 2, – 5), (0, – 2), (– 2, 0), (– 4, – 2), (– 8, – 5), (– 10, – 5), (– 6, – 2), (– 4, 0), (– 2, 6), (– 5, 5), (– 7, 3), (– 7, 4), (– 5, 6), (– 2, 7), (– 1, 7), (– 1, 8), (– 2, 9), (– 2, 10), (– 1, 11), (1, 11), (2, 10), (2, 9), (1, 8), (1, 7), (2, 7), (4, 5), (6, 5), (6, 4), (4, 4), (2, 6), (0, 0).

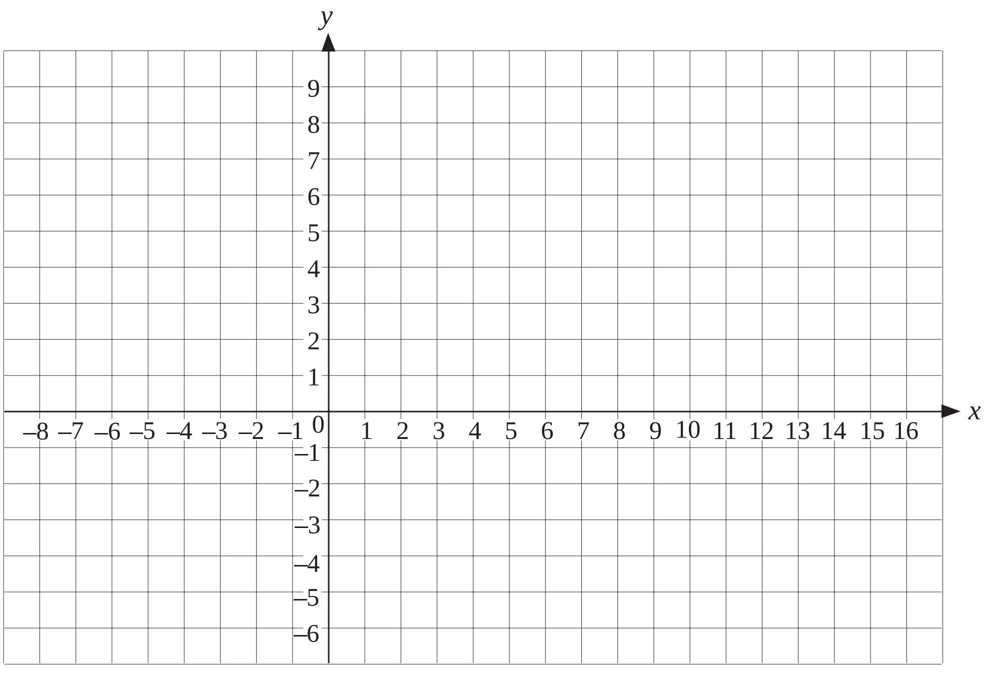
5. The diagram shows a tennis court drawn on a set of axes. The position of the ball is directly above the coordinates given.

The ball is served at (1, 9) and hit at (– 3, – 7). It travels back over the net and is hit again at (– 3, 8). The ball bounces next at (6, – 9).

Draw the path of the tennis ball on a copy of this diagram.



6. (a) Draw a copy of this grid.



(b) On this grid draw the rectangles with corners at the following points with coordinates;

(i) (– 6, 6), (– 5, 6), (– 5, 4), (– 6, 4)

(ii) (– 2, 1), (– 3, 1), (– 3, 3), (– 2, 3)

(iii) (3, 1), (3, 3), (4, 3), (4, 1)

(iv) (10, 1), (10, 3), (9, 3), (9, 1)

(v) (12, 4), (13, 4), (13, 6), (12, 6)

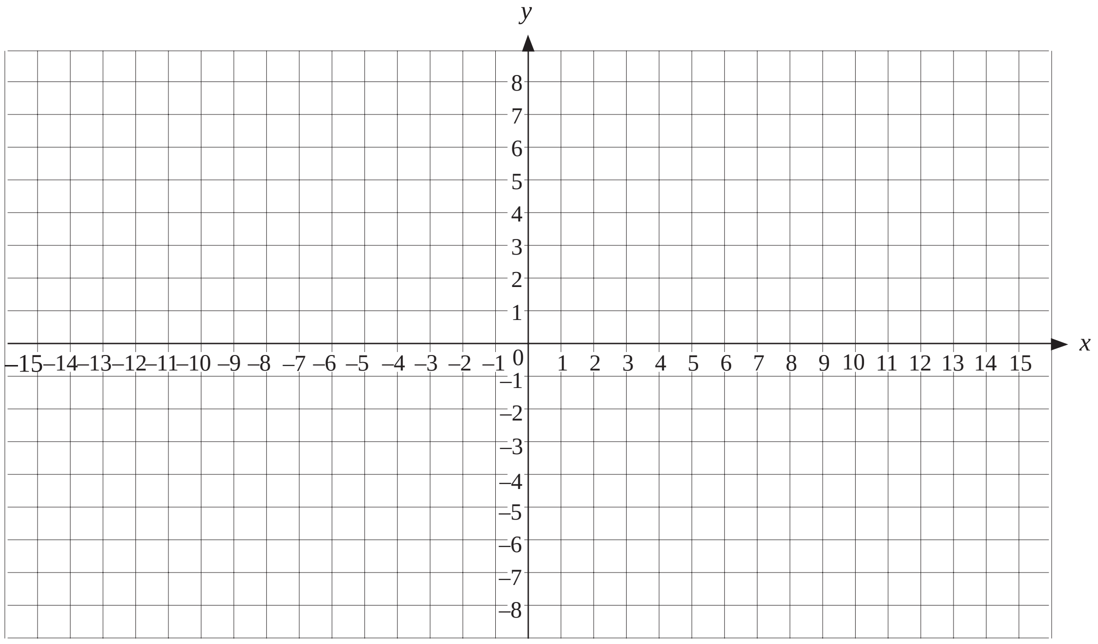
(c) Join the points with coordinates

(1, – 5), (1, – 1), (2, 0), (5, 0), (6, – 1), (6, – 5)

(d) Join the points with coordinates

(– 7, – 5), (– 7, 5), (– 8, 7), (– 8, 9), (– 7, 9), (– 7, 8), (– 6, 8), (– 6, 9), (– 5, 9), (– 5, 8), (– 4, 8), (– 4, 9), (– 3, 9), (– 3, 7), (– 4, 5), (– 3, 5), (– 3, 4), (– 2, 4), (– 2, 5), (– 1, 5), (– 1, 4), (0, 4), (0, 5), (1, 5), (1, 4), (2, 4), (2, 5), (3, 5), (3, 4), (4, 4), (4, 5), (5, 5), (5, 4), (6, 4), (6, 5), (7, 5), (7, 4), (8, 4), (8, 5), (9, 5), (9, 4), (10, 4), (10, 5), (11, 5), (10, 7), (10, 9), (11, 9), (11, 8), (12, 8), (12, 9), (13, 9), (13, 8), (14, 8), (14, 9), (15, 9), (15, 7), (14, 5), (14, – 5), (– 7, – 5).

7. (a) Draw a picture of your own on a copy of the axes below.



(b) Write a set of instructions and give them to a friend, so they can draw your picture.

3.5 Plotting Polygons

Here we look at polygons plotted on coordinate axes, but first, we must recap the names of polygons.

Names of Polygons

Number of Sides	*Name*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Note

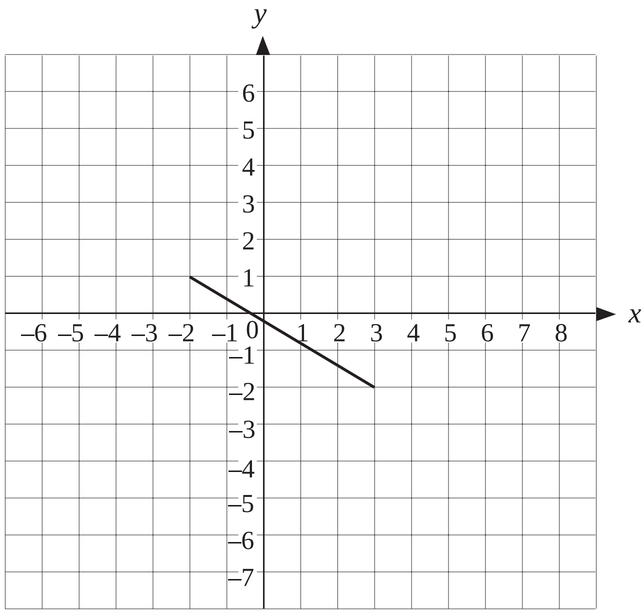
In a *regular* polygon:

(a) All the sides are the same length.

(b) All the angles are the same size.

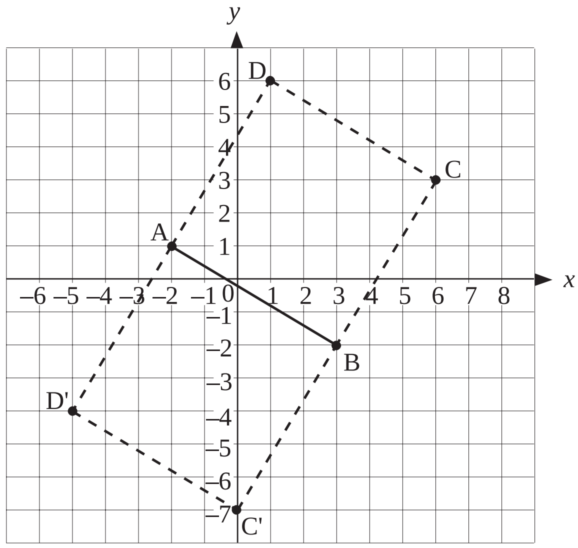
Example

The line is one side of a square. What are the possible coordinates of the corners of the square?



Solution

You can construct a square in two ways using the given lines.



These are shown opposite.

To go from A to B, *x* increases by 5 units, *y* by 3 units.
So, to go to C, you increase *x* by 3 and *y* by 5 etc.
This gives

C (6, 3) , D (1, 6)

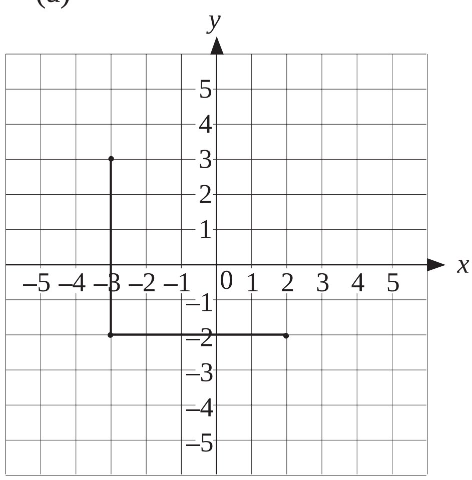
or, alternatively

C' (0, – 7) , D' (– 5, – 4)

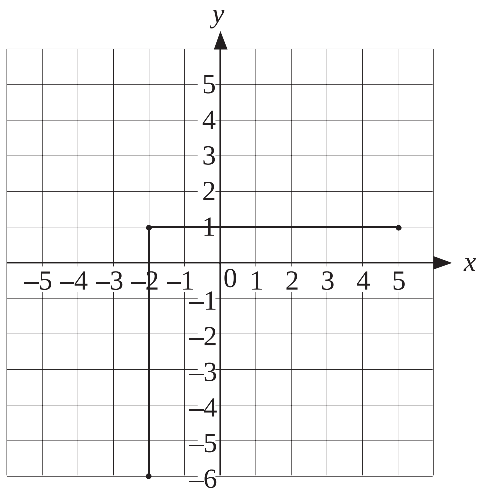
Exercises

1. Write down the coordinates of the missing corner of each square.

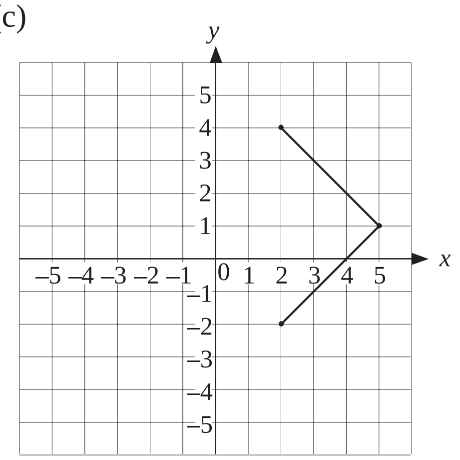
(a)



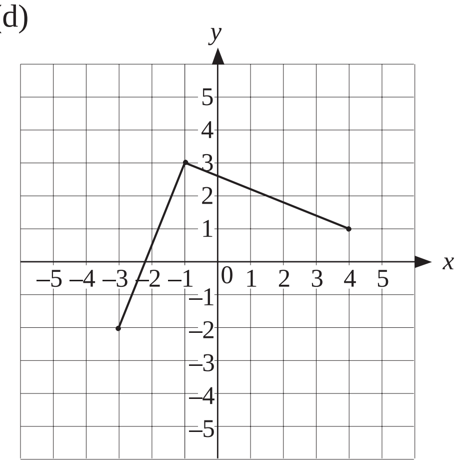
(b)



(c)



(d)



2. In each case the coordinates of 3 corners of a square are given. Find the coordinates of the other corner.

   (a) (2, – 2), (2, 3) and (– 3, 3)

   (b) (2, 3), (3, 4) and (1, 4)

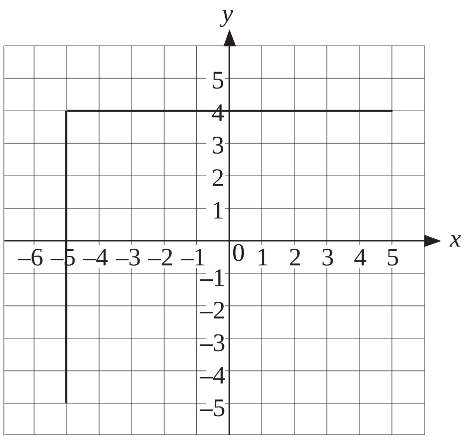
   (c) (2, 2), (4, 4) and (4, 0)

   (d) (– 6, 2), (– 5, – 5) and (1, 3)

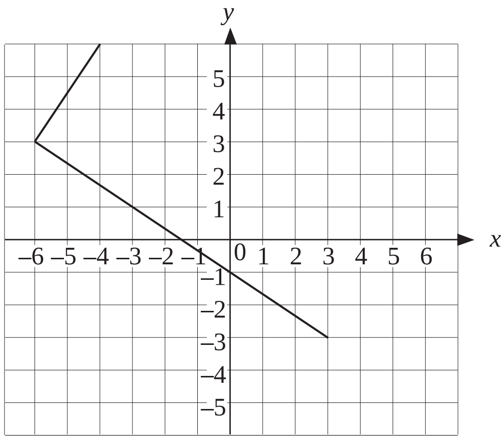
   (e) (– 5, – 2), (– 2, – 1) and (– 1, – 4)

3. Write down the coordinates of the missing corner of the rectangles.

(a)



(b)



4. The coordinates of 3 corners of a rectangle are given below. Find the coordinates of the other corner of each rectangle.

   (a) (– 4, 2), (– 4, 1) and (6, 1)

   (b) (0, 2), (– 2, 0) and (4, – 6)

   (c) (– 4, 5), (– 2, – 1) and (1, 0)

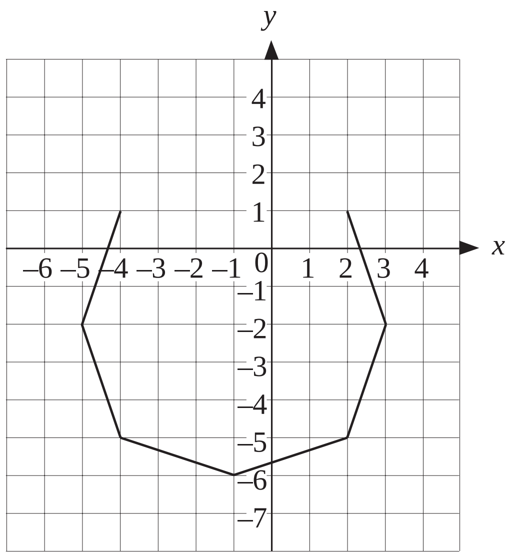
   (d) (– 5, 1), (– 2, 5) and (6, – 1)

5. (a) The coordinates of 2 corners of a square are (– 4, 4) and (1, – 1). Explain why it is possible to draw three different squares using these two points.

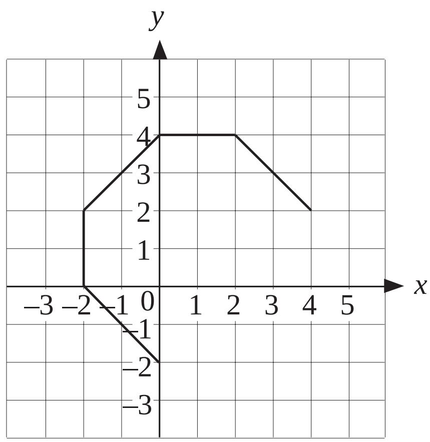
(b) Draw the three different squares.

(c) If the coordinates of the corners had been (– 5, 1) and (1, 3) would it still be possibe to draw 3 squares? Draw the possible squares.

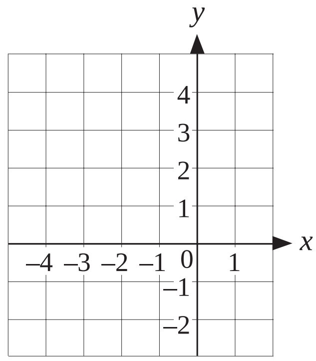
6. The sides of an octagon are all the same length. The diagram shows part of the octagon. Find the coordinates of the missing corner.



7. The angles between the sides of an octagon are all the same. The sides are not all the same length. Find the coordinates of the missing corners of the octagon.



8. (a) Join the points with coordinates (– 2, – 1), (– 3, – 1), (– 4, 1) and (– 2, 2) on this grid.



(b) This shape is half of a pentagon that has one line of symmetry. Complete the pentagon.

Write down the coordinates of the extra corners.

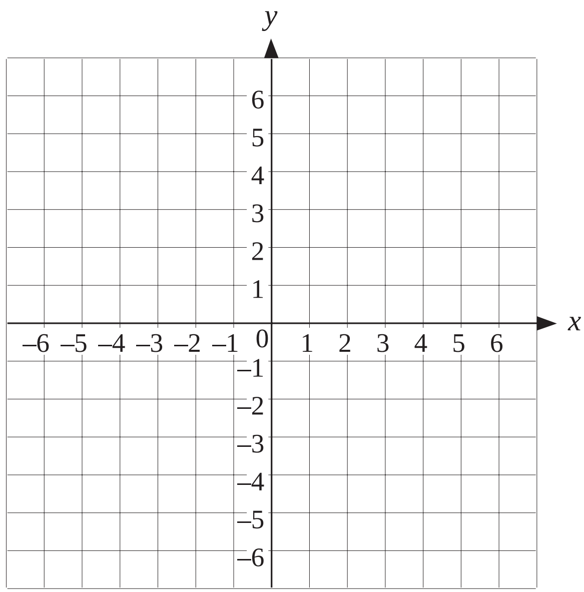
9. Half of a heptagon with one line of symmetry can be drawn by joining the points with coordinates:

(2, 4), (– 2, 1), (– 2, – 1), (0, – 3) and (2, – 3).

Join the coordinates. You have drawn one half of the heptagon. Complete the heptagon. Write down the coordinates.

10. (a) Mark the points with these coordinates on a grid like this one.

(0, 5), (4, 3), (– 6, 0), (– 5, – 3) and (2, – 5).



(b) Add extra points and draw a nonagon with the y-axis as a line of symmetry.

(c) Write down the coordinates of the extra points.

3.6 Conversion Graphs

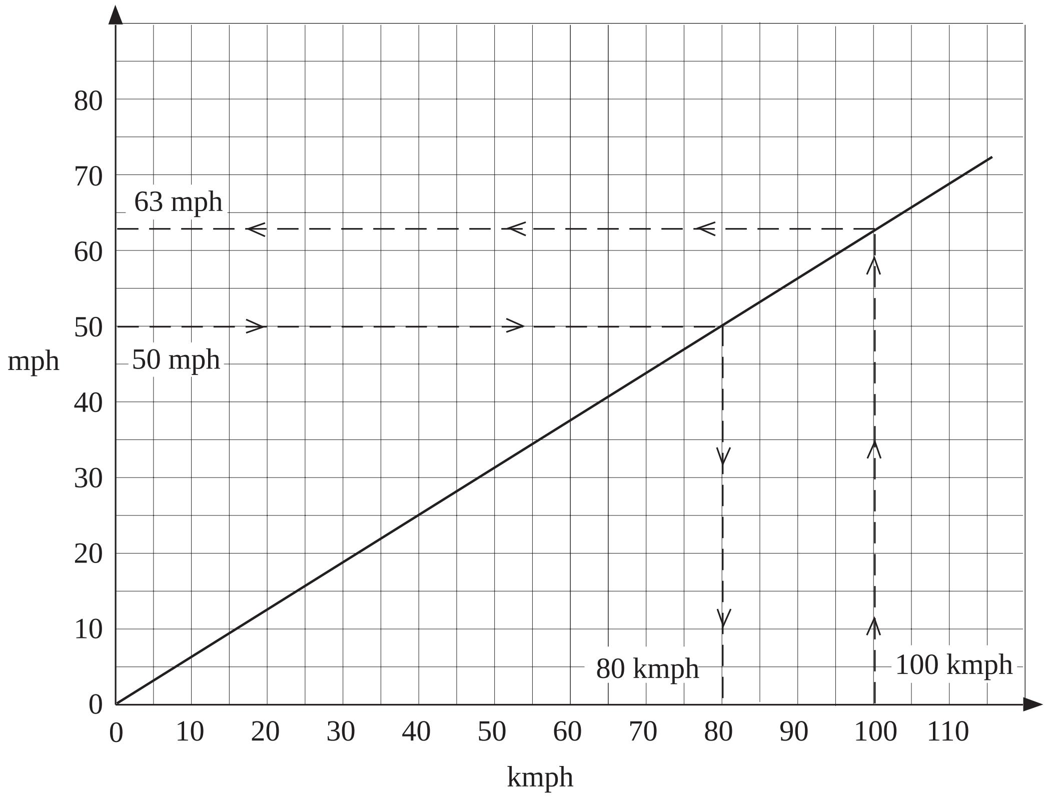
A conversion graph can be used to change one quantity to another when the units are changed.

The graph below has been used to

(a) Convert 50 mph into kmph.

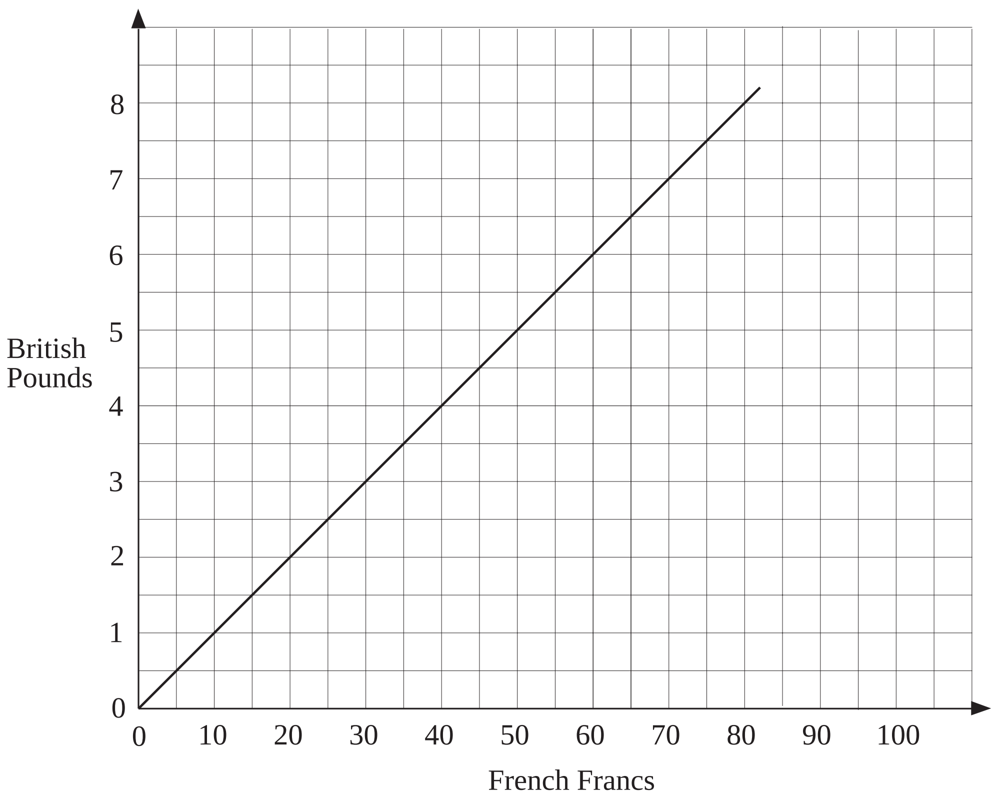
and

(b) Convert 100 kmph into mph.



Exercises

1. The graph below can be used for converting French Francs into British Pounds.



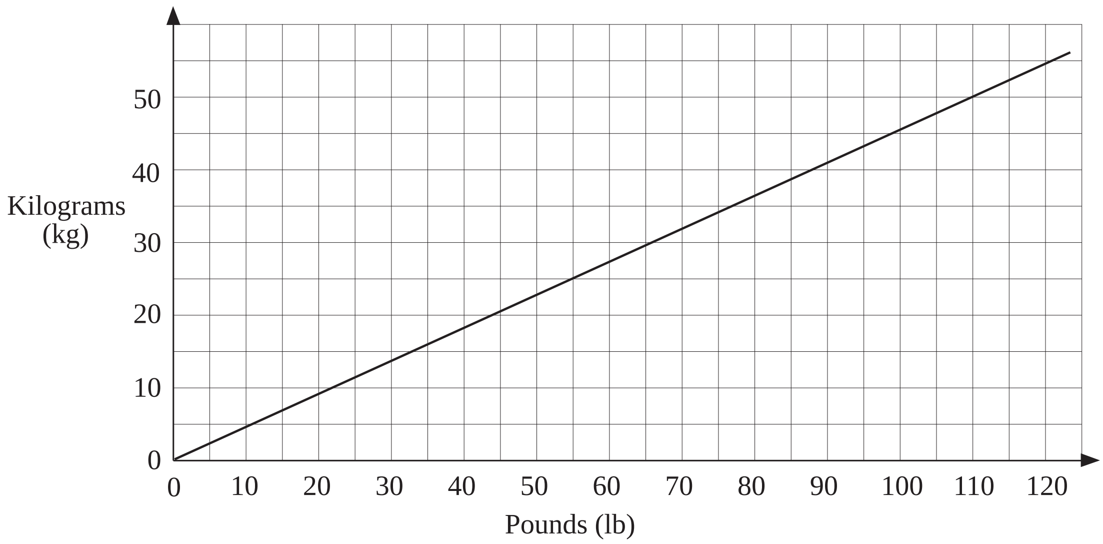
(a) Convert these amounts to French Francs.

(i) £ 3.00 (ii) £ 7.00 (iii) £ 4.50

(b) Convert these amounts to British Pounds.

(i) 80 FF (ii) 60 FF (iii) 25 FF

2. The graph below can be used for converting weight from kilograms to pounds.



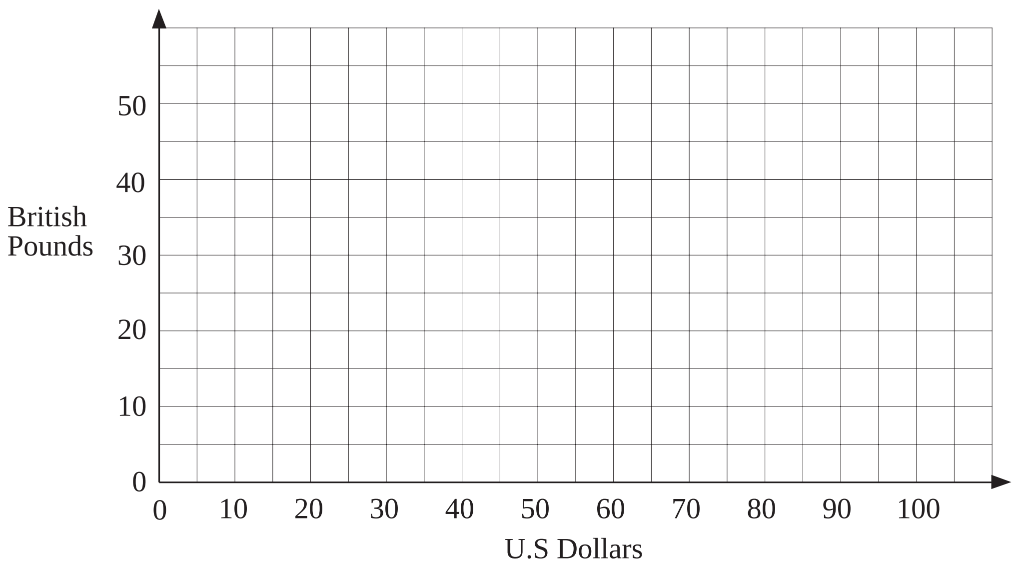
(a) Convert these weights from kilograms to pounds.

(i) 30 kg (ii) 10 kg (iii) 45 kg

(b) Convert these weights from pounds to kilograms.

(i) 110 lb (ii) 20 lb (iii) 85 lb

3. (a) Copy the set of axes shown below, ready to draw a graph for converting British Pounds to US Dollars.



(b) If £ 50 is equivalent to $ 80, plot a point on the graph and draw a straight line to use for conversions.

(c) Convert these amounts to US Dollars.

(i) £ 20 (ii) £ 35 (iii) £ 15

(d) Convert these amounts to British Pounds.

(i) $ 50 (ii) $ 45 (iii) $ 25

4. (a) If £ 100 is equivalent to 250 Australian Dollars, draw a conversion graph.

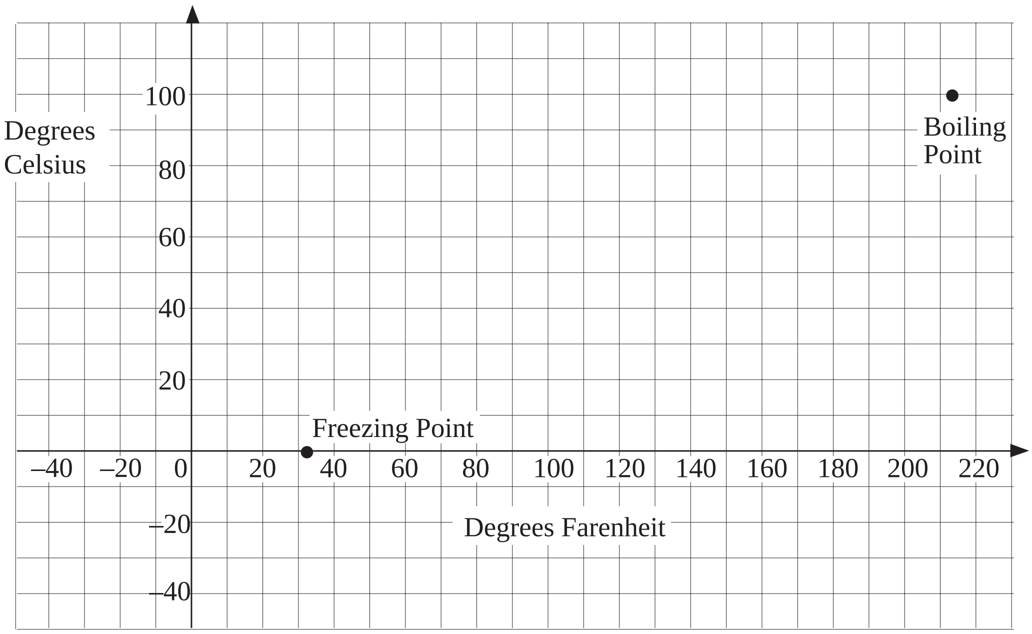
(b) Use your graph to convert these amounts to British Pounds.

(i) 200 Australian Dollars (ii) 50 Australian Dollars

(c) Use your graph to convert these amounts to Australian Dollars.

(i) £ 75 (ii) £ 80 (iii) £ 45

5. The axes below are to be used for a graph for converting temperatures between degrees Farenheit and Celsius. The two points shown on the graph represent the freezing and boiling points of water.



(a) Copy the graph and draw a line through the two points given.

(b) Use your graph to convert these temperatures to Celsius.

(i) 160°F (ii) 60°F (iii) 70°F (iv) 95°F

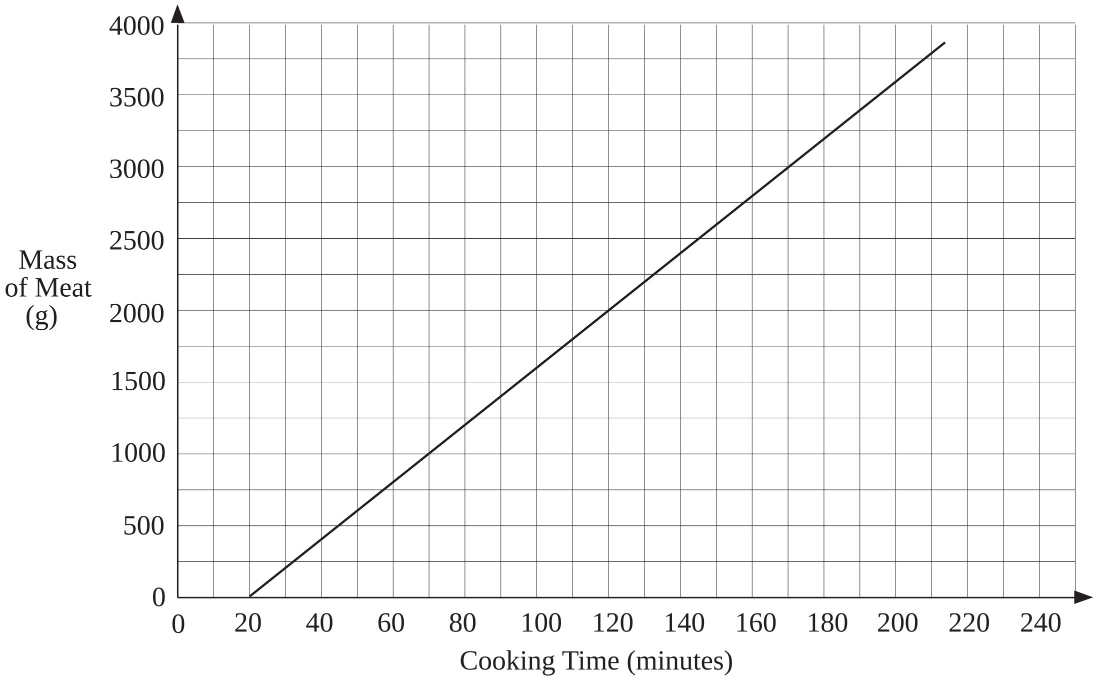
(c) Use your graph to convert these temperatures to Farenheit.

(i) 60°C (ii) 20°C (iii) 30°C (iv) 45°C

(d) Explain how to use your graph to convert 0°Farenheit to Centigrade.

(e) What temperature is the same in both Farenheit and Centigrade?

6. The graph below can be used to find the time needed to cook a piece of meat.



(a) How long is needed to cook 2000 grams of meat?

(b) How much longer does it take to cook 2500 grams than 2000 grams?

(c) What mass of meat could be cooked in $1\frac{1}{2}$ hours?

(d) The mass of a piece of meat is 1500 grams. It has been cooking for 30 minutes. How long is it until the meat will be cooked?

7. (a) If 22 gallons is equivalent to 100 litres, draw a conversion graph.

(b) Use your graph to convert 20 gallons to litres.

(c) Use your graph to convert 40 litres to gallons.

(d) John has 12 gallons of petrol in his car. He uses 4 gallons on a journey. How many litres of petrol does he have left?

(e) Rachel uses 8 litres of petrol per week going to work. How many gallons of petrol would she use in 5 weeks?

(f) Ted's car uses 30 gallons of petrol on an 800 mile journey. How many litres would be used in a 400 mile journey?

(g) A can contains $\frac{1}{4}$ of a litre of a drink. How many cans would be needed to hold 2 gallons of the drink?

(h) A modern toilet uses 8 litres of water per flush. An old toilet uses 2 gallons of water per flush.

Which toilet uses the most water?

8. If 10 metres is the same as 33 feet, draw a graph to convert between metres and feet.

Use your graph to answer the following questions.

(a) In a long jump competition Mohammed jumps 4 m and James jumps 12 feet. Who wins?

(b) Which is longer, 20 feet or 6.5 m?

(c) Philip says that 8 metres is less than 28 feet. Is he right?

(d) A rope is 9 metres long. What is the distance to the nearest foot?

(e) A new flagpole arrives at a school. It is 1 metre taller than the old one. The old flagpole was 18 feet. How tall is the new flagpole?

4 Arithmetic: Addition and Subtraction of Decimals

4.1 Addition and Subtraction

Here we first revise addition and subtraction of whole numbers.

Example 1

(a) $3 + (6 + 2) = 3 + 8$ since $6 + 2 = 8$
$= 11$

(b) $18 - (4 + 7) = 18 - 11$ since $4 + 7 = 11$
$= 7$

(c) $12 - (4 - 2) = 12 - 2$ since $4 - 2 = 2$
$= 10$

(d) $(12 - 4) - 2 = 8 - 2$ since $12 - 4 = 8$
$= 6$

Example 2

Calculate:

(a) $102.8 + 15.21$

(b) $92.69 - 10.4$

Solution

(a) To find $102.8 + 15.21$, line up the decimal points:

$$\begin{array}{r} 102.80 \\ +\ \ 15.21 \\ \hline 118.01 \\ \hline \end{array}$$

(b) To find $92.69 - 10.4$, line up the decimal points:

$$\begin{array}{r} 92.69 \\ -\ \ 10.40 \\ \hline 82.29 \\ \hline \end{array}$$

Exercises

1. Find:

(a) $3 + 5$	(b) $8 + 3$	(c) $9 + 7$
(d) $7 + 8$	(e) $7 + 6$	(f) $5 + 9$
(g) $14 + 22$	(h) $18 + 9$	(i) $16 + 15$
(j) $21 + 22$	(k) $18 + 7$	(l) $14 + 31$
(m) $47 + 9$	(n) $82 + 6$	(o) $72 + 17$

2. Is each of these statements *true* or *false*?

(a) $3 + 9 = 9 + 3$	(b) $3 - 1 = 1 - 3$
(c) $8 + 2 + 9 = 9 + 8 + 2$	(d) $14 + 7 + 6 = 7 + 20$
(e) $3 + 16 - 3 = 16$	(f) $17 - 10 = 10 - 17$
(g) $4 + 16 + 9 = 11 + 16$	(h) $14 + 8 = 8 + 14$

3. Find:

(a) $8 - 5$	(b) $9 - 7$	(c) $7 - 4$
(d) $8 - 6$	(e) $15 - 3$	(f) $18 - 5$
(g) $28 - 15$	(h) $48 - 26$	(i) $12 - 9$
(j) $16 - 7$	(k) $14 - 5$	(l) $32 - 24$
(m) $122 - 86$	(n) $92 - 47$	(o) $57 - 39$

4. Find:

(a) $3 + (6 - 2)$	(b) $5 - (8 - 7)$
(c) $(3 + 6) - 8$	(d) $15 - (4 + 2)$
(e) $(17 - 1) - 4$	(f) $23 - (4 - 2)$
(g) $5 + (14 - 7) - 3$	(h) $4 + (71 - 1) + 1$
(i) $8 - (3 - 2) + 5$	(j) $16 - (8 - 7) - 5$

5. Copy these sums and put brackets into each one, so that they are correct.

(a) $5 - 8 - 7 = 4$	(b) $6 - 3 + 2 = 1$
(c) $5 + 7 - 2 - 1 = 11$	(d) $14 - 7 - 3 - 2 = 8$

6. Find the sum of each set of numbers.

   (a) 18 and 15   (b) 6, 10 and 24

   (c) 42, 33 and 62   (d) 47, 82 and 37

7. Find the difference between each pair of numbers.

   (a) 18 and 15   (b) 22 and 47

   (c) 92 and 46   (d) 57 and 84

8. Miss Sharp teaches 2 classes in one morning. There are 32 children in the first class and 28 in the second.

   (a) How many children does she teach altogether?

   (b) How many more children are there in the first class than in the second?

9. There are 22 people on a bus.

   (a) At the next stop 5 people get off and 12 people get on. How many are now on the bus?

   (b) At the next stop nobody gets off and the bus leaves with 35 people on board. How many people got on at this stop?

10. This season Ali has scored 12 goals for his team. Last season he scored 18 goals for his team.

    (a) How many goals did he score in total in the two seasons?

    (b) What is the difference between the number of goals scored in each season?

11. Jane has 42 CDs. Sarah has 8 fewer than Jane. Nadia has 13 more than Sarah. How many CDs does Nadia have?

12. In one school there are 3 classes in Year 7. One class has 28 children, one has 29 children and the other has 31 children. How many children are there in Year 7?

13. Adam has 126 football stickers and Ben has 192. They need 250 stickers each to fill their albums.

    (a) How many more stickers does Ben need?

    (b) How many more stickers does Adam need?

    (c) How many stickers have the boys got in total?

14. There are 216 cars in a car park. In the next hour, 82 cars arrive and 73 cars leave. How many cars are in the car park at the end of the hour?

15. David buys 3 train tickets that cost £18, £46 and £78. How much does he spend altogether?

16. Alison goes on holiday on her motorbike. She keeps a record of how far she rides each day.

Day	1	2	3	4	5
Miles	120	38	59	62	119

What is the total distance she rides?

17. Use a quick method for each of these sums.

(a) $18 + 7 + 12$ (b) $108 + 19 + 12$

(c) $99 + 17 + 11$ (d) $17 + 19 + 13$

(e) $46 + 23 - 16$ (f) $128 - 15 - 13$

(g) $72 + 11 + 38$ (h) $19 + 6 - 9$

(i) $52 + 23 - 12$ (j) $16 + 18 - 6$

(k) $37 + 42 - 2$ (l) $68 + 19 + 1$

(m) $33 - 7 + 17$ (n) $67 + 18 + 13$

18. Find:

(a) $0.3 + 0.6$ (b) $0.8 + 0.1$

(c) $0.42 + 0.11$ (d) $1.2 + 3.7$

(e) $1.46 + 3.42$ (f) $5.7 + 2.4$

(g) $6.7 + 3.6$ (h) $5.12 + 8.99$

(i) $17.2 + 0.42$ (j) $5.6 + 3.21$

(k) $0.04 + 1.521$ (l) $6.3 + 4.72$

(m) $18.14 + 3.2$ (n) $16.5 + 3.218$

19. Find:

(a) $0.7 - 0.2$ (b) $0.9 - 0.6$

(c) $1.3 - 0.1$ (d) $4.2 - 3.1$

(e) $6.9 - 3.5$ (f) $8.9 - 7.3$

(g) 7.2 – 5.3 (h) 6.6 – 4.8

(i) 19.24 – 8.3 (j) 18.62 – 1.7

(k) 15.2 – 3.46 (l) 11.4 – 3.12

(m) 0.7 – 0.04 (n) 0.88 – 0.49

4.2 Dealing with Money

Example 1

Jason has a £5 note when he leaves home. He spends 27p on sweets in one shop and £3.50 on a book in another shop. How much money does he have left?

Solution

In total, Jason has spent, in £,

$$\begin{array}{r} 0.27 \\ +\ \ 3.50 \\ \hline \pounds 3.77 \\ \hline \end{array}$$

So the money he has left is

$$\begin{array}{r} 5.00 \\ -\ \ 3.77 \\ \hline \pounds 1.23 \\ \hline \end{array}$$

Exercises

TUCK SHOP PRICES	
Cans	45p
Choc-Bars	30p
Bubble-Choc	35p
Crisps	22p
Pasty	95p

1. (a) Find the cost of:

   (i) a Choc-Bar and a can of drink,

   (ii) a packet of crisps and a Bubble-Choc,

   (iii) a pasty and a can of drink.

   (b) Sarah spent exactly 67p.

   What did she buy?

2. Vijay paid for a 36p packet of sweets with a 50p coin. How much change did he get?

3. A magazine costs £2.35. How much change would you get from a £5 note if you bought the magazine?

4. Ben wants to buy a bike that costs £114.99. He has saved £98. How much more money does he need?

5. Sally buys a train ticket that costs £14.86. How much change does she get from a £20 note?

6. Halim buys a bus ticket that costs £2.80. How much change does he get from a £10 note?

7. Prakest spends £3.62 on Monday, £5.21 on Tuesday and £8.33 on Wednesday.

   (a) How much has he spent altogether?

   (b) If he had £20 to start with, how much has he got left?

8. Keith runs a take-away. This is his price list.

   (a) His first customer buys chips, a pasty and a drink. How much does this cost?

   (b) His next customer buys 2 sausages, chips and a drink. He gives Keith a £5 note. How much change does he get?

   (c) John has £1.20. He buys a pasty. How much money does he have left?

KEITH'S EATS	
Chips	75p
Burger	£1.25
Sausage	35p
Pie	92p
Pasty	79p
Drink	42p

9. Gemma goes on a diet. Her mass drops from 64.82 kg to 52.36 kg. How much mass has she lost?

10. Rachel grows sunflowers. One plant is 1.32 m tall. In the next week it grows another 19 cm.

    (a) How tall is the plant now?

    (b) How much more must it grow to be 2 m tall?

11. To go on a fairground ride you must be 140 cm tall. Emma's height is 1.24 m. How much does she need to grow before she can go on this ride?

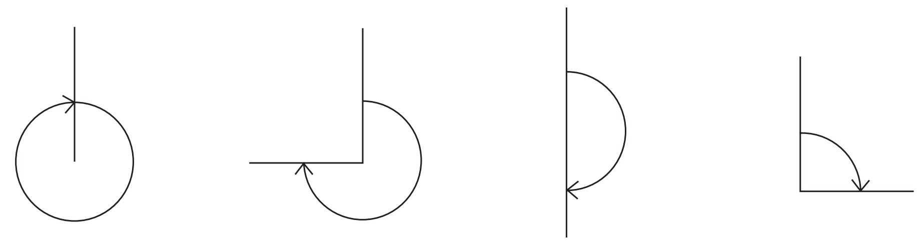
12. Karen goes to the shops twice. The first time she takes a £10 note and brings back £2.48. The second time she takes a £5 note and brings back £1.39. How much has she spent altogether?

5 Angles

Angles are an important building block in geometry and trigonometry as you will see later. In this unit you will see how turns are related to angles, how to measure them and how to work out their size in particular problems.

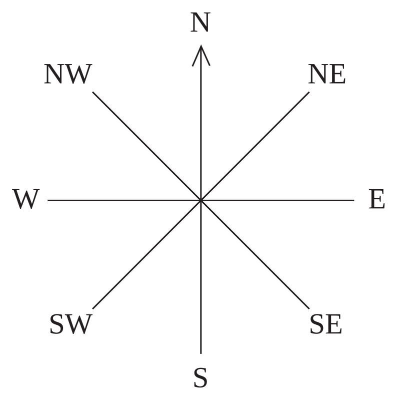
5.1 Angles and Turns

You will need to understand clearly what the terms such as *turn*, *half turn*, etc. mean in terms of angles. There are $360°$ in one complete turn, so the following are true.



1 turn	$\frac{3}{4}$ turn	$\frac{1}{2}$ turn	$\frac{1}{4}$ turn
is $360°$	is $270°$	is $180°$	is $90°$

You also need to refer to compass points: (north (N), south (S), east (E), west (W), northeast (NE), southeast (SE), southwest (SW) and northwest (NW).



Turning from N to S is $180°$ clockwise or anticlockwise

Turning from NE to SE is $90°$ clockwise (or $270°$ anti-clockwise)

Turning clockwise from NE to E is $45°$ (or $315°$ anticlockwise)

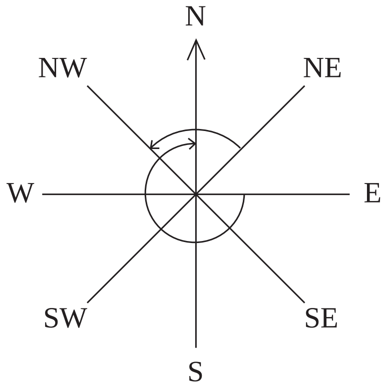
Example

What angle do you turn through if you turn

(a) from NE to NW *anticlockwise*,

(b) from E to N *clockwise*?

Solution

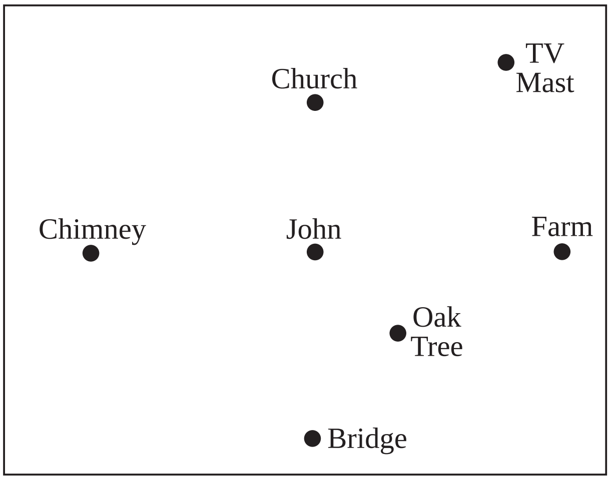


(a) You can see that this is $90\,°$ (or $\frac{1}{4}$ turn).

(b) This is a $\frac{3}{4}$ turn, i.e. $270\,°$.

Exercises

1. John is standing on a hill. The church is north of the point where he stands.



(a) In what direction is he facing if he looks at:

(i) the chimney, (ii) the bridge,

(iii) the TV mast, (iv) the farm,

(v) the oak tree?

(b) What angle does John turn through if he turns *clockwise* from looking at:

(i) the church to the farm,

(ii) the oak tree to the bridge,

(iii) the TV mast to the oak tree,

(iv) the bridge to the TV mast,

(v) the TV mast to the church?

(c) What would the angles be for question (b) if John turned *anticlockwise* instead of clockwise?

2. In a game, you spin a pointer and let it stop.

What angle does the pointer turn through if it completes:

(a) 1 turn, (b) 2 turns, (c) $\frac{3}{4}$ turn,

(d) $1\frac{1}{4}$ turns, (e) $1\frac{3}{4}$ turns, (f) $2\frac{1}{4}$ turns?

3. What angle do you turn through if you turn *clockwise* from facing:

(a) N to E, (b) W to NW, (c) SE to NW,

(d) NE to N, (e) W to NE, (f) S to SW,

(g) S to SE, (h) SE to SW, (i) E to SW?

4 What angle do you turn through if you turn *anticlockwise* from facing:

(a) N to SW, (b) S to SW,

(c) W to NW, (d) E to S?

5. In what direction will you be facing if you turn:

(a) 180 ° clockwise from NE,

(b) 180 ° anticlockwise from SE,

(c) 90 ° clockwise from SW,

(d) 45 ° clockwise from N,

(e) 225 ° clockwise from SW,

(f) 135 ° anticlockwise from N,

(g) 315° clockwise from SW?

6. Nigel stands on a low hill. The diagram on the next page shows some of the things he can see. Using information from the diagram, answer the following questions.

(a) What is NE of Nigel?

(b) What is SE of Nigel?

(c) Nigel turns from looking at the Old Fort to look at the ship. What angle does he turn through?

Explain why there is more than one answer to this question.

(d) What angle does Nigel turn through if:

(i) he turns clockwise from looking at the ship to the crane,

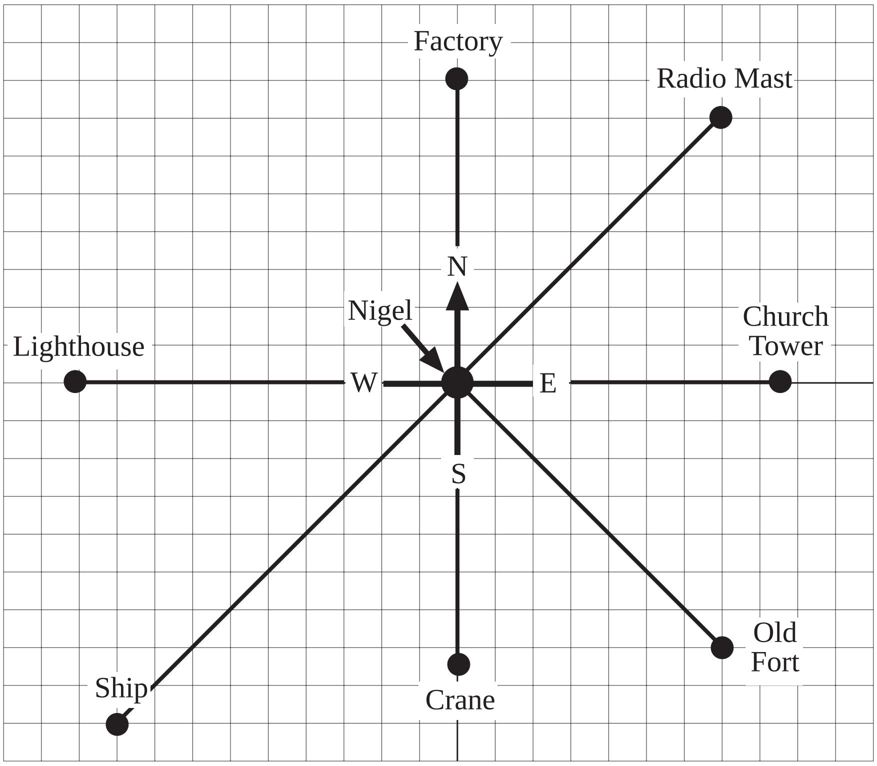
(ii) he turns anticlockwise from looking at the radio mast to the factory,

(iii) he turns anticlockwise from looking at the factory to the ship?

(e) Nigel starts looking at the factory. What does he end up looking at if he turns:

(i) 135 ° clockwise, (ii) 270 ° anticlockwise,

(iii) 225 ° clockwise, (iv) 405 ° clockwise?



7. Use the diagram on the next page to answer these questions.

(a) What is N of the shop?

(b) What is W of the church?

(c) What is E of the church?

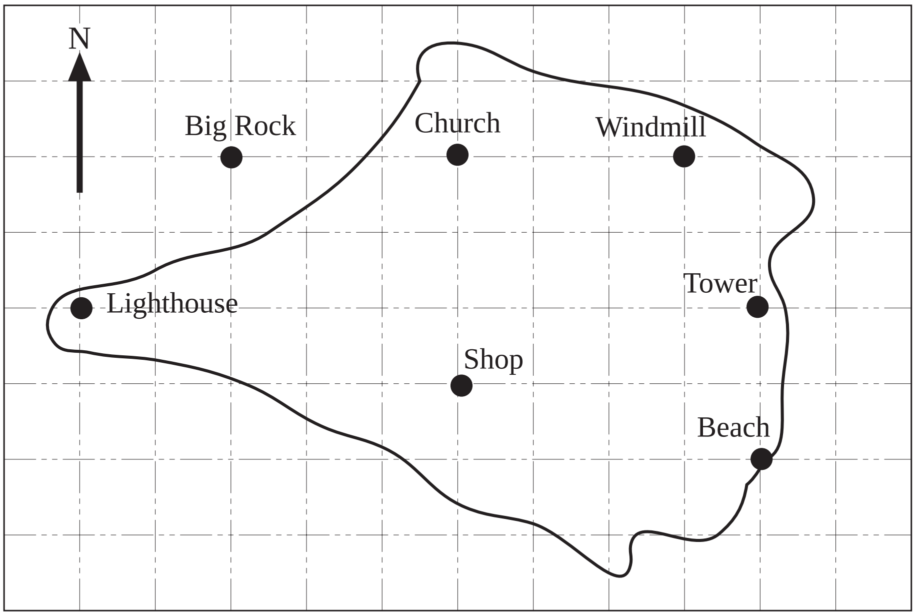
(d) What is E of Big Rock and NE of the shop?

(e) What is SW of Big Rock?

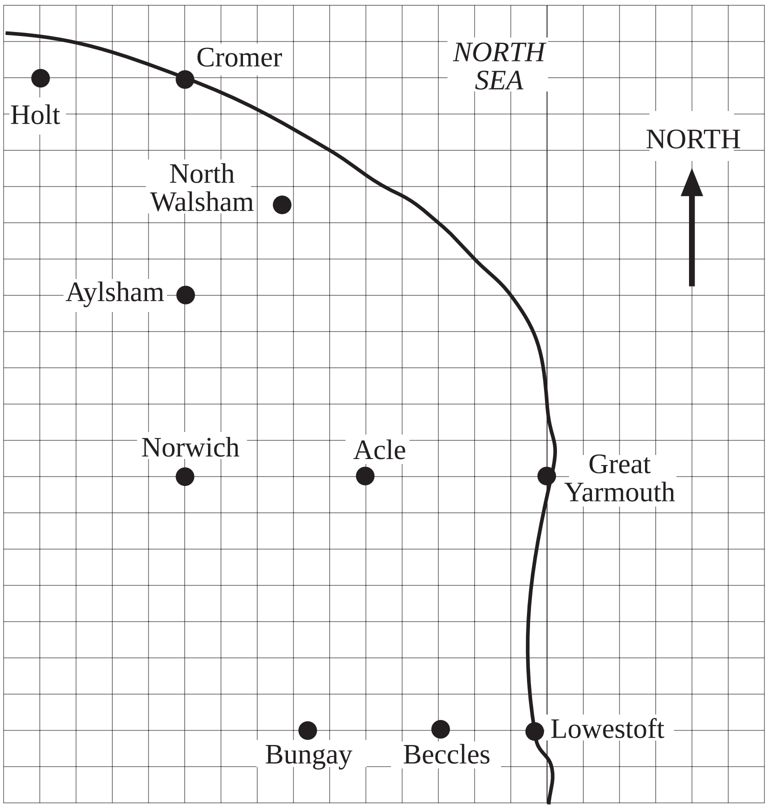
(f) In what direction should you walk from the beach to get to the tower?

(g) In what direction should you walk from the beach to get to the church?

(h) If you walk SE from the windmill, will you get to the tower? Explain your answer.



8. The map below shows part of East Anglia.



(a) What is east of Acle?

(b) What is north of Norwich and south of Cromer?

(c) What is SE of Norwich?

(d) What is NW of Acle?

(e) What is west of Cromer?

(f) What is west of Lowestoft and east of Bungay?

9. The sails of a windmill complete one full turn every 40 seconds.

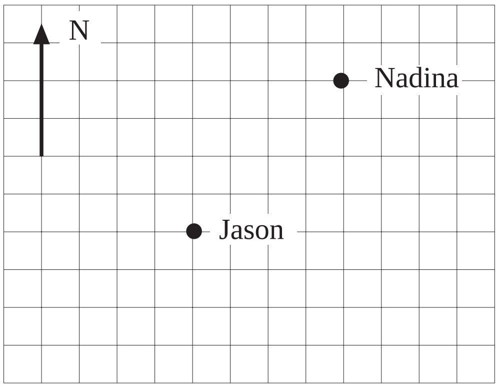
(a) How long does it take the sails to turn through:

(i) $180°$ (ii) $90°$ (iii) $45°$?

(b) What angle do the sails turn through in:

(i) 30 seconds, (ii) 15 seconds, (iii) 25 seconds?

10. The diagram shows the positions of Jason and Nadina. The arrow shows the direction of north.



(a) Copy or trace the diagram.

(b) Karen is west of Nadina and north of Jason. Mark Karen's position on your diagram.

(c) Jenny is east of Jason and southeast of Nadina. Mark Jenny's position on your diagram.

(d) Wendy is west of Jenny and southeast of Karen. Where is Wendy in relation to Nadina?

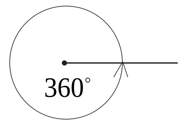
(e) Jai is north of Jason and south of Karen. Describe where he could be standing.

5.2 Measuring Angles

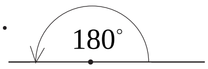
A *protractor* can be used to measure or draw angles.

Note

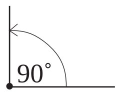
The angle around a complete circle is is $360°$.



The angle around a point on a straight line is $180°$.

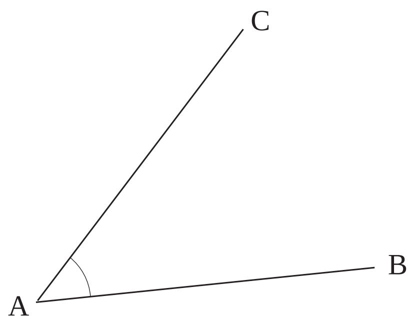


A right angle is $90°$.



Example 1

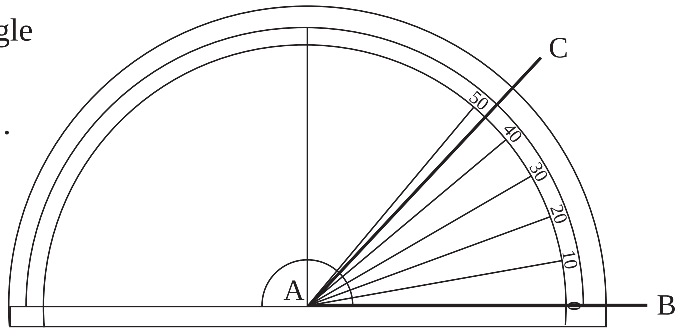
Measure the angle CAB in the triangle shown.



Solution

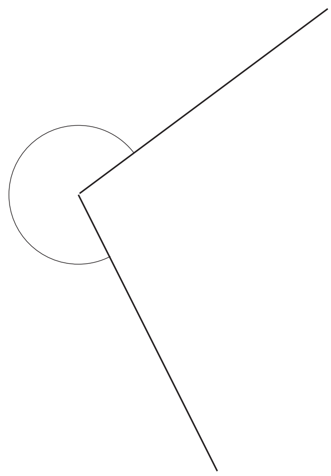
Place a protractor on the triangle as shown.

The angle is measured as $47°$.



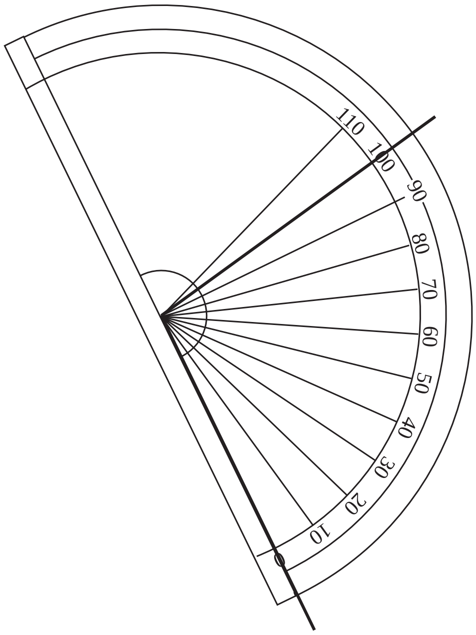
Example 2

Measure this angle.



Solution

Using a protractor, the smaller angle is measured as 100 °.



So

required angle = 360 ° − 100 °
= 260 °

Example 3

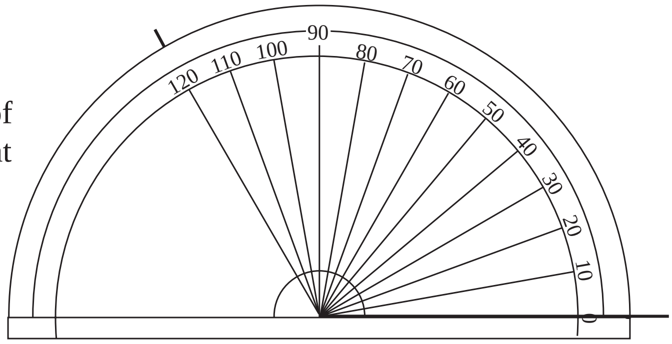
Draw angles of

(a) 120 ° (b) 330 °.

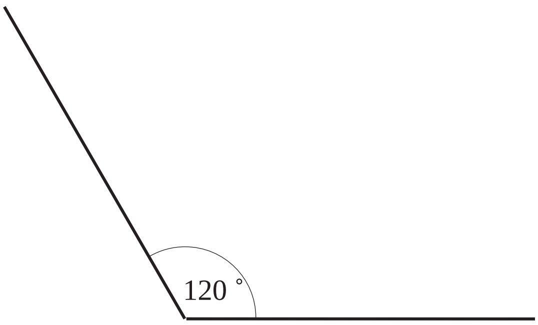
Solution

(a) Draw a horizontal line.

Place a protractor on top of the line and draw a mark at 120 °.



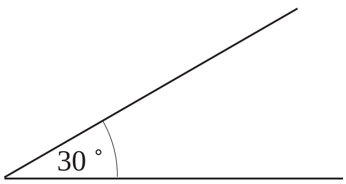
Then remove the protractor and draw the angle.



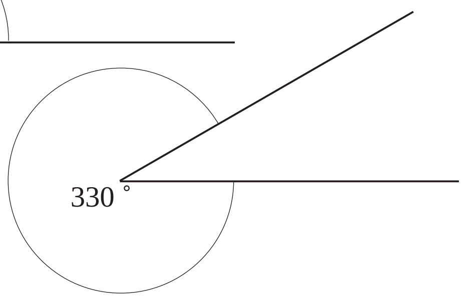
(b) To draw the angle of 330 °, first subtract 330 ° from 360 °:

360 ° − 330 ° = 30 °

Draw an angle of 30 °.



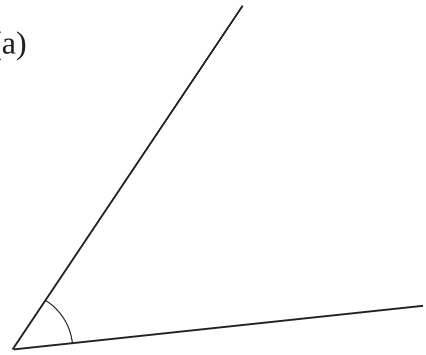
The larger angle will be 330 °.



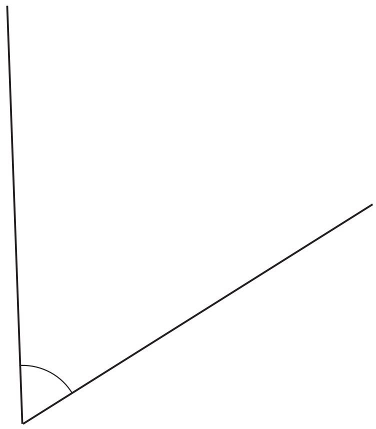
Exercises

1. For each of the following angles, first estimate the size of the angle and then measure the angle to see how good your estimate was.

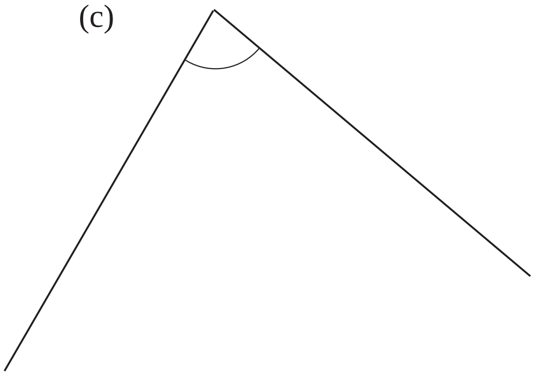
(a)



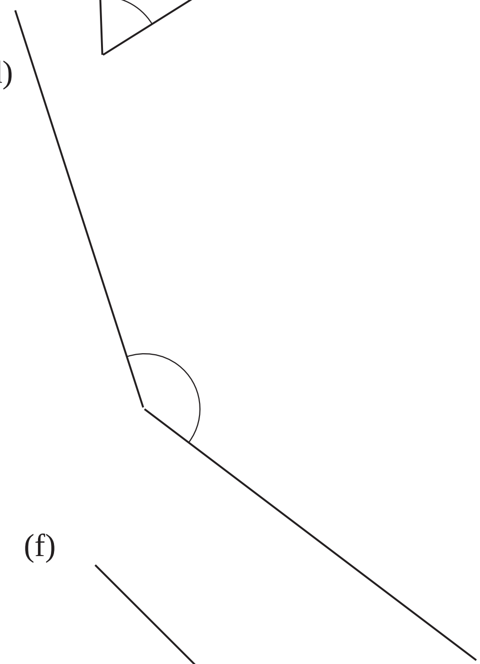
(b)



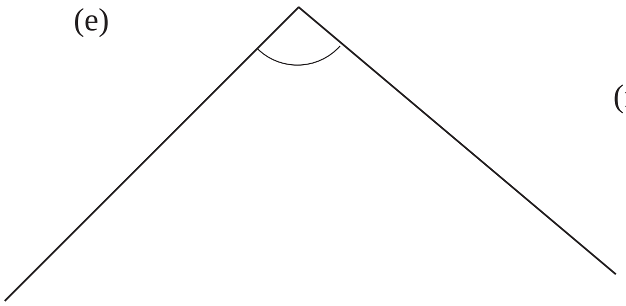
(c)



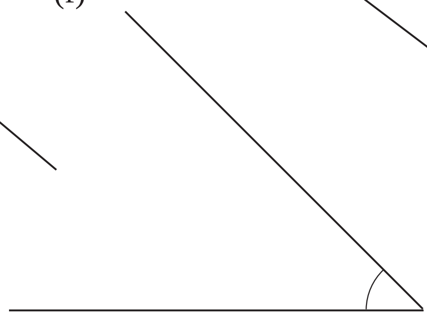
(d)



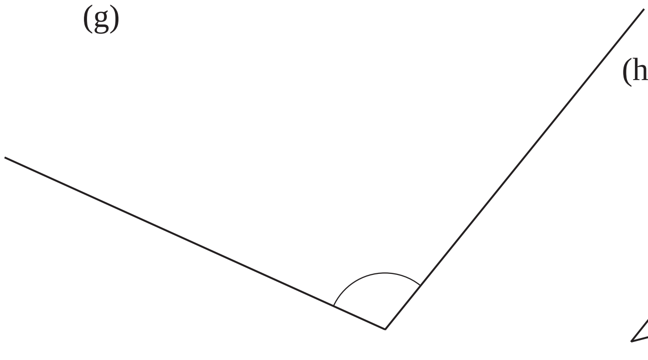
(e)



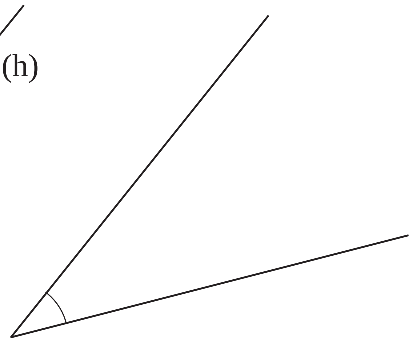
(f)



(g)

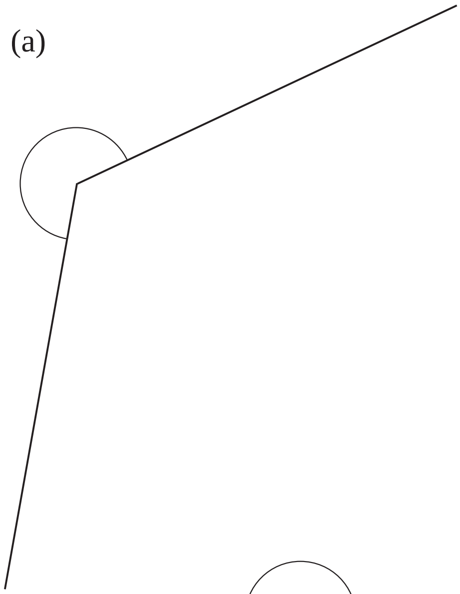


(h)

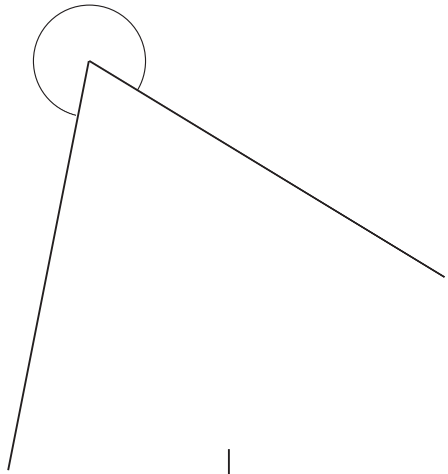


2. Estimate and measure the size of each of these reflex angles.

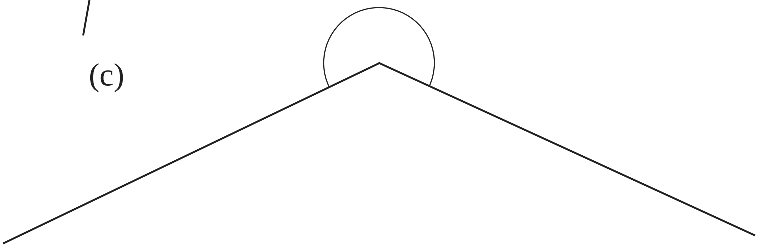
(a)



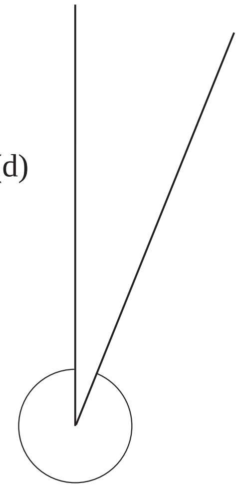
(b)



(c)



(d)



3. (a) Measure each of the angles in this pie chart.

(b) Explain how you can tell that Manchester United is the most popular of these teams.

(c) Which is the second most popular team?

4. Draw the following angles:

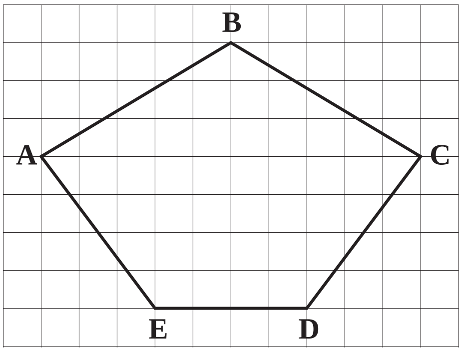
(a) 20 ° (b) 42 ° (c) 80 ° (d) 105 °

(e) 170 ° (f) 200 ° (g) 275 ° (h) 305°

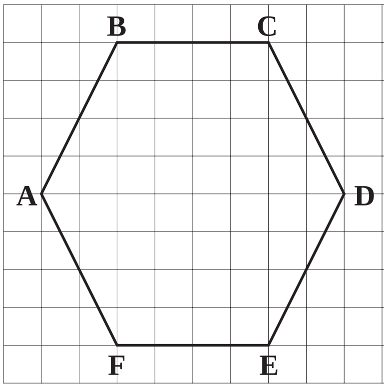
5. In which of these polygons are the angles all the same size?

Find all the angles in each polygon. *(You may need to copy the shapes into your book and extend the lines.)*

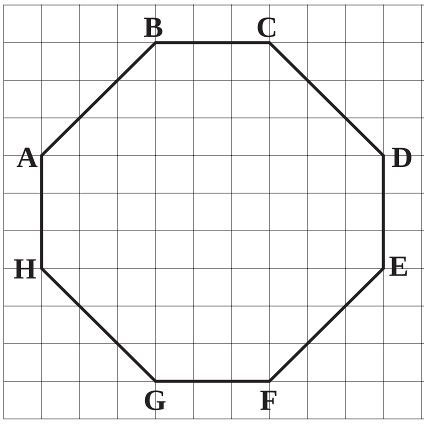
(a)



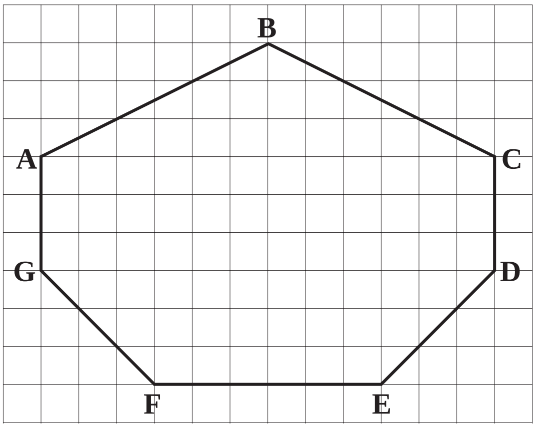
(b)



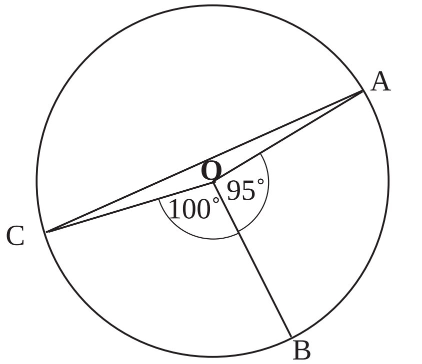
(c)



(d)



6. (a) Draw the shape below, where O is the centre of the circle. Make the radius of your circle 6 cm.



(b) Measure the distances between AB, BC and AC.

7. Ravinder finds out the favourite sports for members of his class. He works out the angles in the list shown opposite for a pie chart.

Draw the pie chart.

Sport	*Angle*
Football	110 °
Swimming	70 °
Tennis	80 °
Rugby	40 °
Hockey	30 °
Badminton	10 °
Other	20 °

5.3 Classifying Angles

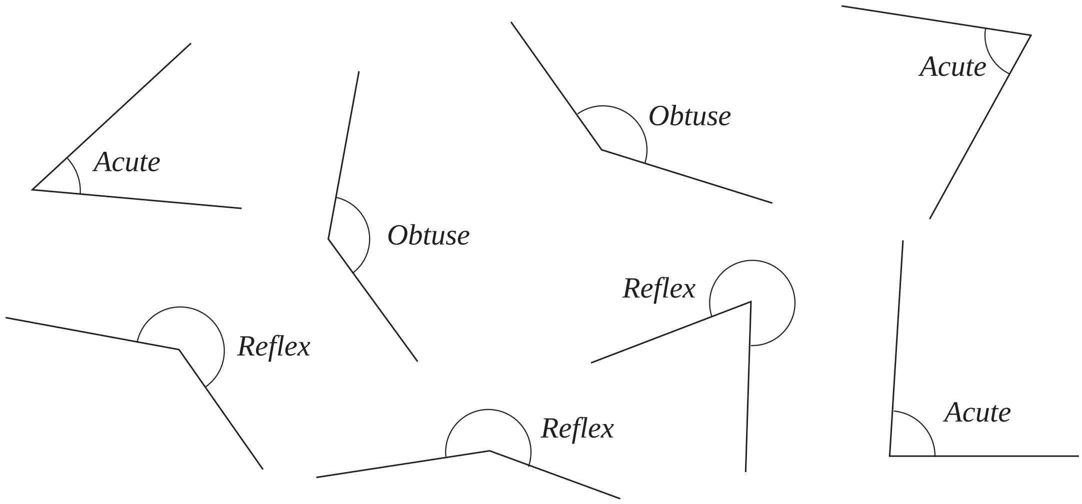
Angles of less than 90 ° are *acute* angles

Angles between 90 ° and 180 ° are *obtuse* angles

Angles between 180 ° and 360 ° are *reflex* angles

So you can easily identify the three types of angle.

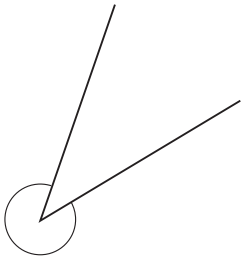
Here are some examples.



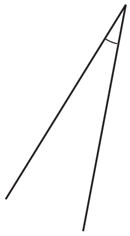
Exercises

1. Is each angle below *acute, obtuse* or *reflex*?

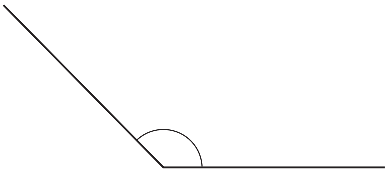
(a)



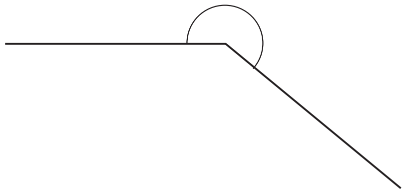
(b)



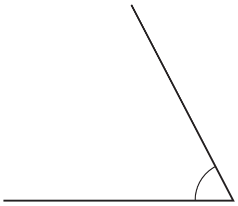
(c)



(d)



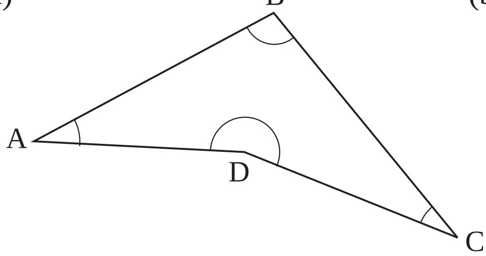
(e)



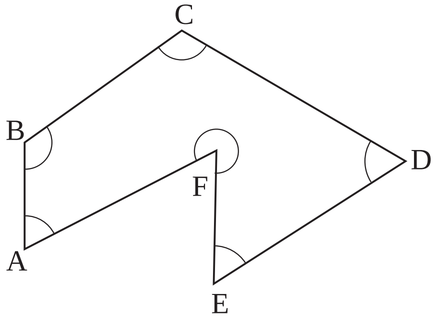
(f)

2. For each shape below state whether the angle at each corner is *acute, obtuse* or *reflex.*

(a)



(b)



3. (a) Draw a triangle with *one* obtuse angle.

   (b) Draw a triangle with *no* obtuse angles.

4. Draw a four-sided shape with:

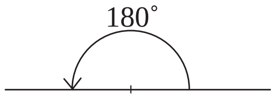
   (a) one *reflex* angle,

   (b) two *obtuse* angles.

5.4 Angles on a Line and Angles at a Point

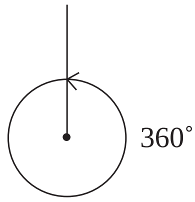
Remember that

(a) angles on a line add up to $180°$



and

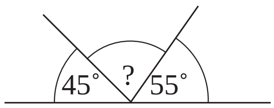
(b) angles at a point add up to $360°$.



These are two important results which help when finding the size of unknown angles.

Example 1

What is size of the angle marked ?



Solution

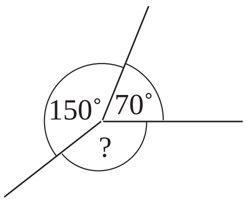
$$45° + 55° = 100°$$

So angle $= 180° - 100°$

$= 80°$

Example 2

What is the size of the angle marked ?



Solution

$$70° + 150° = 220°$$

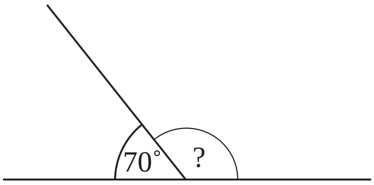
So angle $= 360° - 220°$

$= 140°$

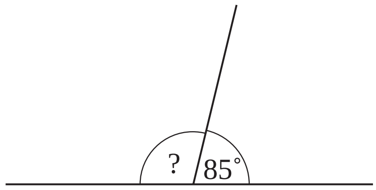
Exercises

1. Calculate the unknown angle in each of the following diagrams.

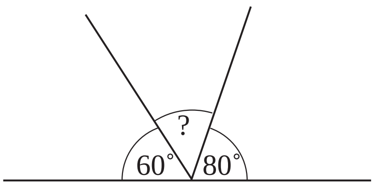
(a)



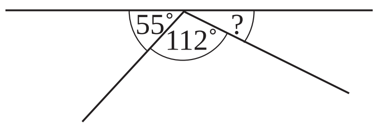
(b)



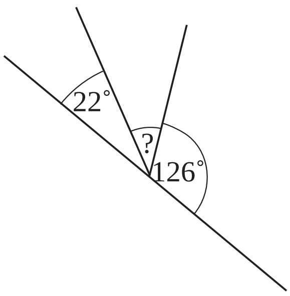
(c)



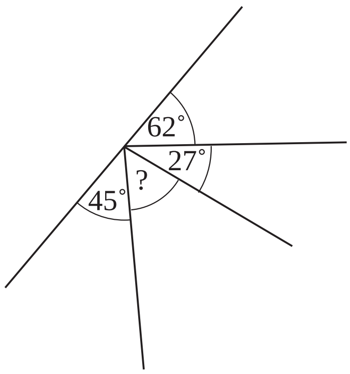
(d)



(e)

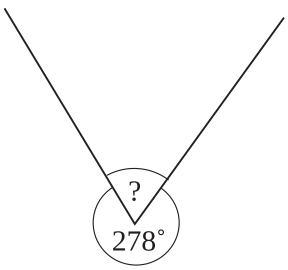


(f)

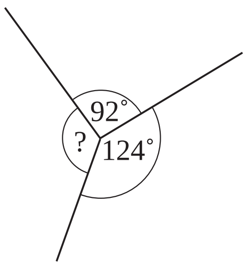


2. Calculate the unknown angle in each diagram.

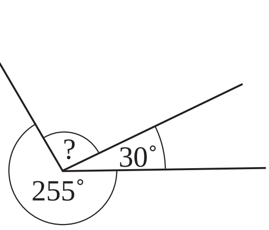
(a)



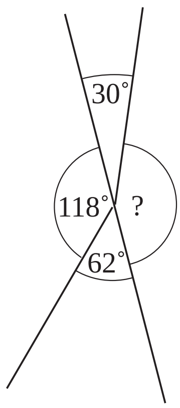
(b)



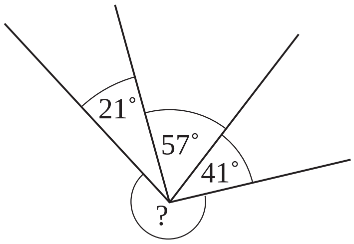
(c)



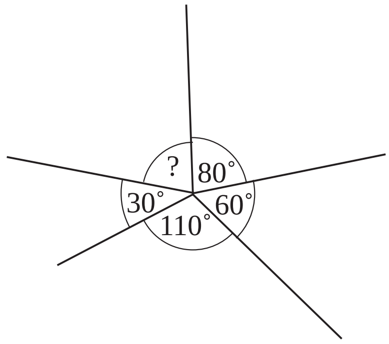
(d)



(e)

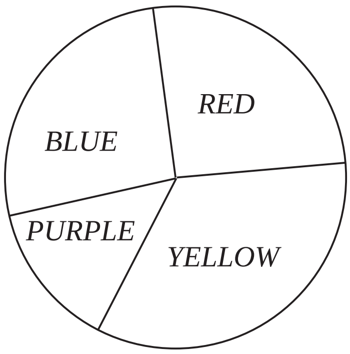


(f)



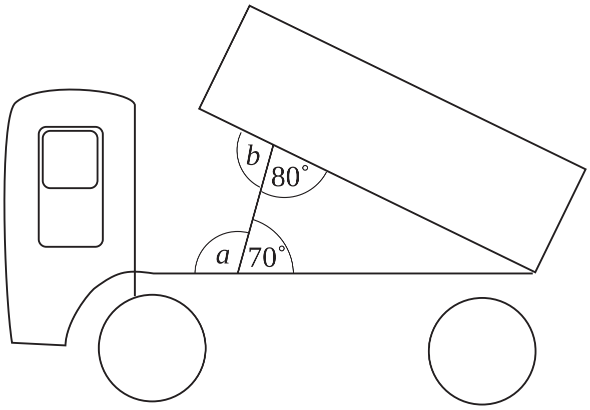
3. Some of the angles in the pie chart have been calculated:

Red	90 °
Blue	95 °
Purple	50 °



What is the angle for *yellow*?

4. The picture shows a tipper truck.



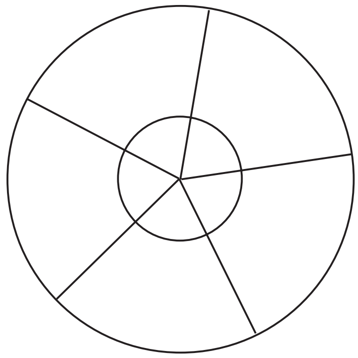
(a) Find the angles marked *a* and *b*.

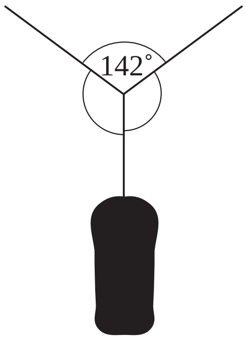
(b) The 80 ° angle decreases to 75 ° as the tipper tips further. What happens to angle *b*?

5. The diagram shows a playground roundabout viewed from above. Five metal bars are fixed to the centre of the roundabout as shown. The angles between the bars are all the same size.

   (a) What size are the angles?

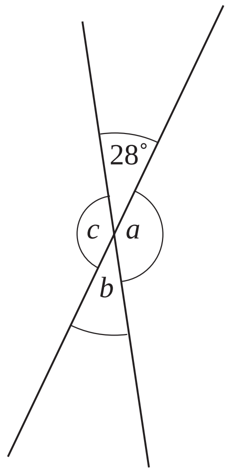
   (b) What size would the angles be if there were 9 metal bars instead of 5?



6. 

A boy hangs a punchbag on a washing line.

Find the unknown angles if both angles are the same size.



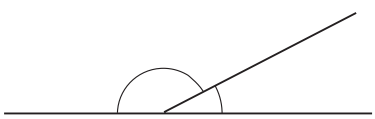
7. The diagram shows two straight lines.

   Find the angles *a*, *b* and *c*.

   What do you notice?

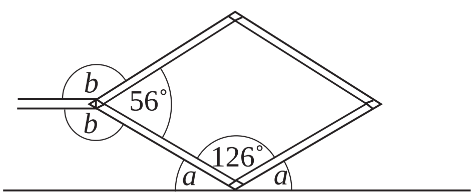
8. In the diagram the large angle is 4 times bigger than the smaller angle.

   Find the two angles.



9. The picture shown a jack, that can be used to lift up a car.

   Find the angles marked *a* and *b*.

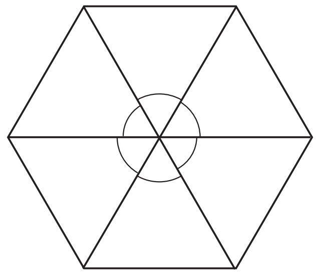


10. The diagram shows a regular hexagon.

    (a) Find the size of each of the angles marked at the centres of the hexagon.

    (b) What would these angles be if the polygon was a decagon (10 sides).

    (c) If the angles were $30°$, how many sides would the polygon have?

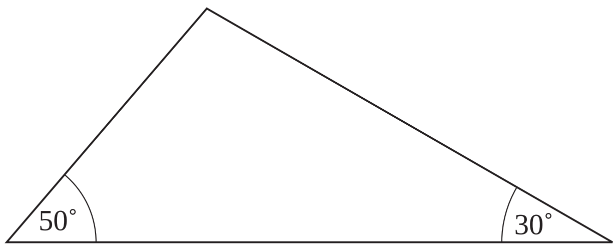


5.5 Constructing Triangles

Here you will see how to construct triangles.

Example 1

Draw this triangle and measure the unknown angle.

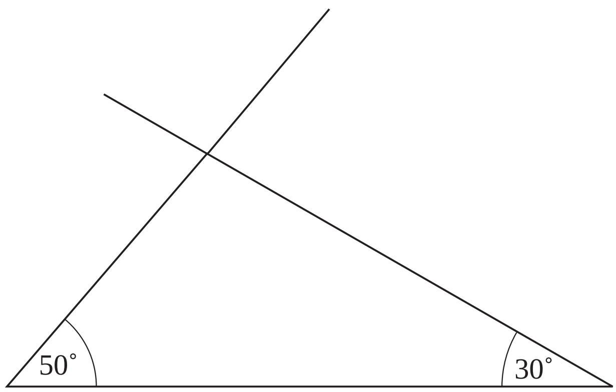


Solution

First draw the base line of 8 cm.

8 cm

At each end, use a protractor to draw lines at angles $50°$ and $30°$ to the line.

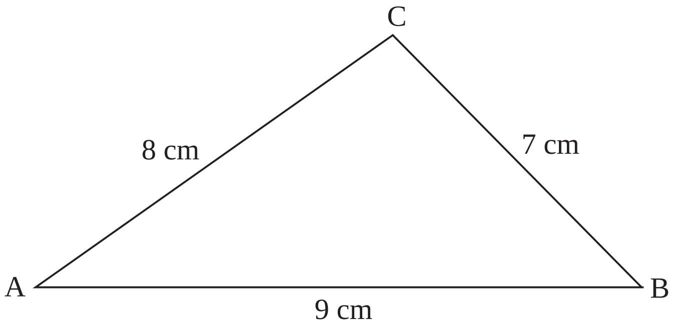


The intersection of these two lines is the third point of the triangle.

This angle measures about $100°$.

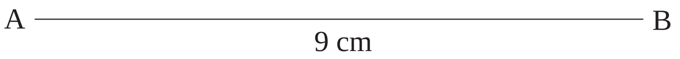
Example 2

Draw this triangle.

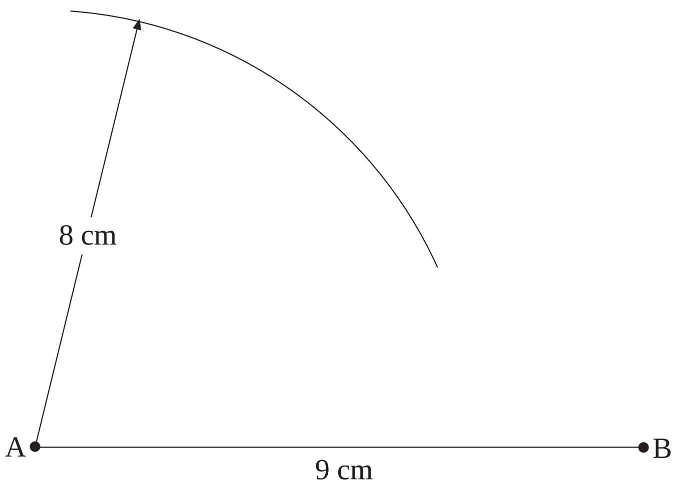


Solution

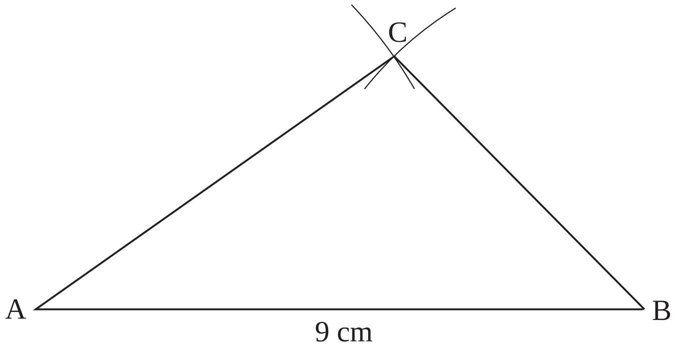
First draw the base line, AB, of length 9 cm.



Then set your compass so that the pencil tip is 8 cm from the point and draw an arc with its centre at A, as shown,



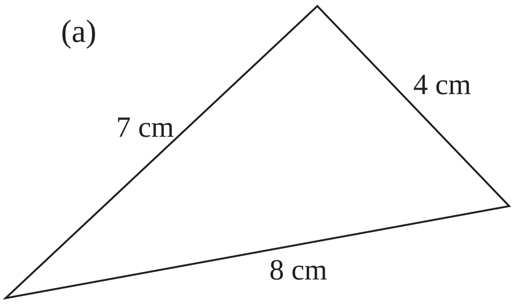
Then draw a similar arc with your compass set at 7 cm and B as the centre. The point where the two arcs cross is the third corner of the triangle.



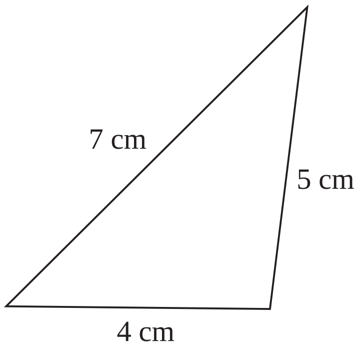
Exercises

1. Draw these triangles accurately. In each triangle, measure the angles and find their total.

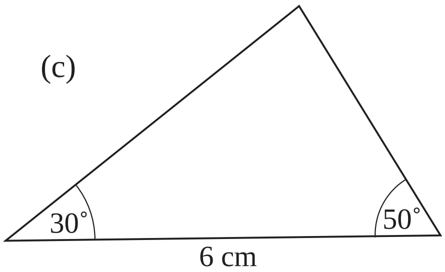
(a)



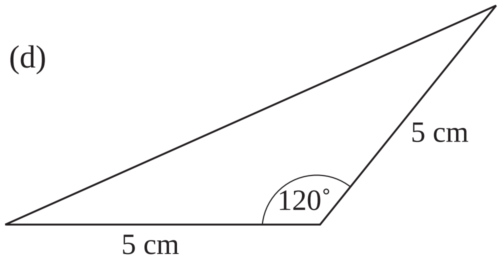
(b)



(c)



(d)

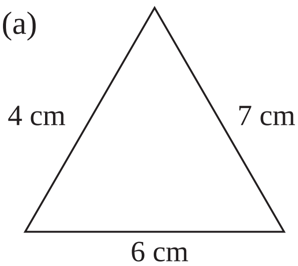


2. Compare your triangles with those drawn by other people in your class. Do your triangles look the same?

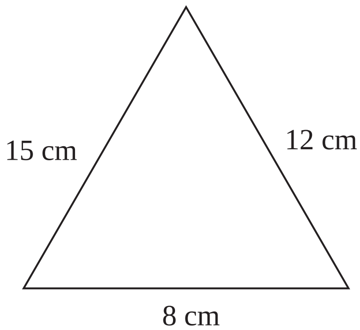
3. Explain why you cannot draw a triangle with sides of lengths 12 cm, 5 cm and 4 cm.

4. Which of these triangles can you draw?

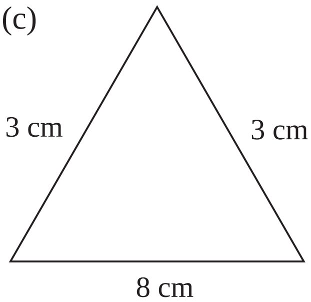
(a)



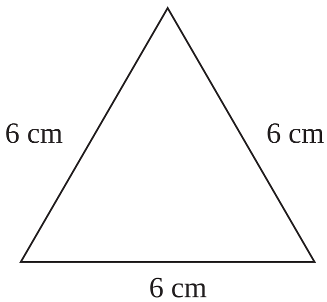
(b)



(c)

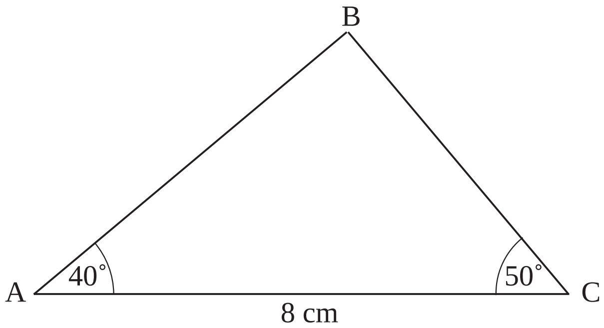


(d)



Draw those that are possible and measure the angles in them.

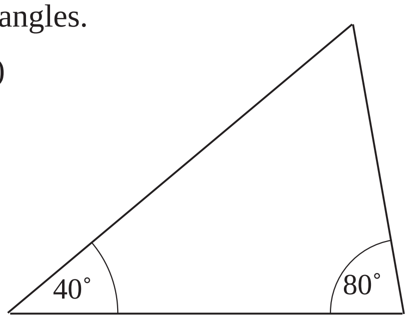
5. (a) Draw the triangle below and measure the lengths of the two sloping sides of the triangle.



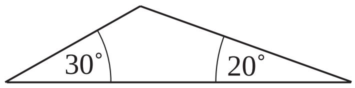
(b) Measure the third angle in the triangle.

6. Draw each triangle below and measure the third angle in each of the triangles.

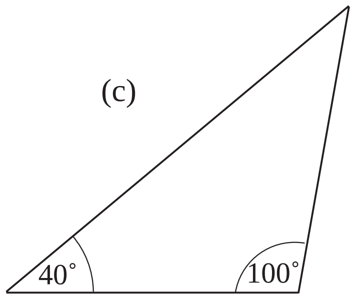
(a)



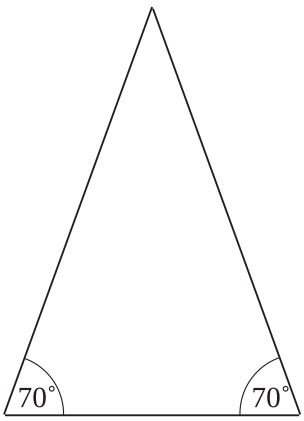
(b)



(c)



(d)

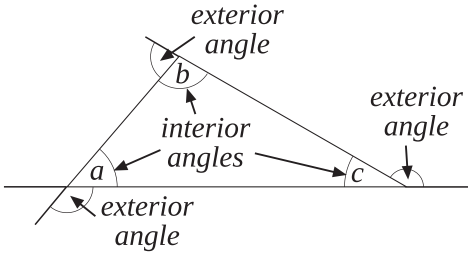


What do you notice?

5.6 Finding Angles in Triangles

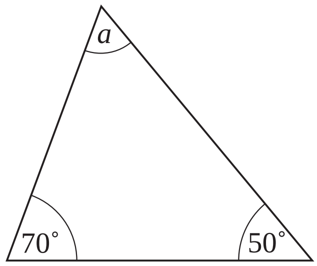
The interior angles of any triangle will always sum (add up) to 180 °.

$$a + b + c = 180°$$



Example

Find the angle marked *a* in the diagram opposite.



Solution

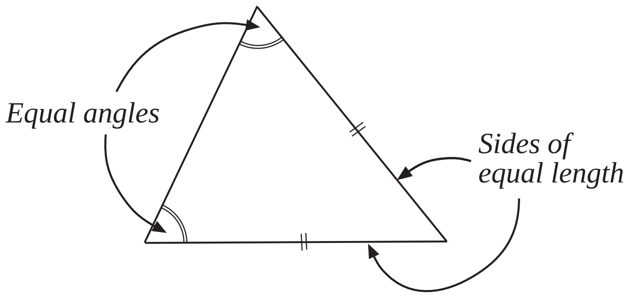
$70° + 50° = 120°$

So $180° - 120° = 60°$

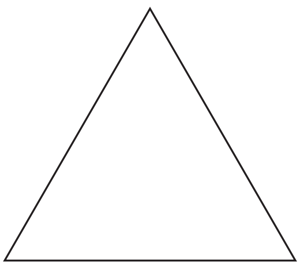
and $a = 60°$

The final part of this section deals with the classification of triangles.

ISOSCELES TRIANGLE



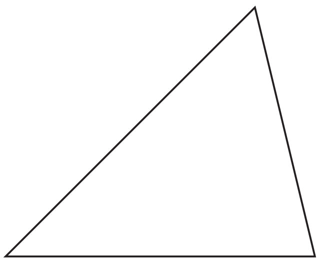
EQUILATERAL TRIANGLE



All sides are the same length

All angles are 60°

SCALENE TRIANGLE



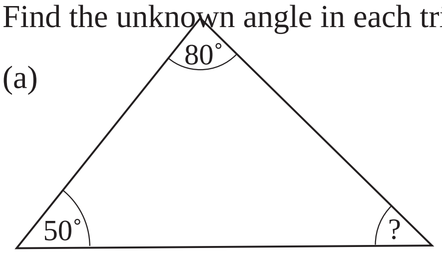
All sides have different lengths.

All angles are of different sizes.

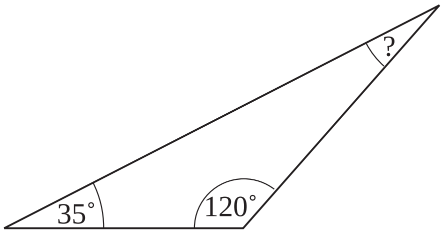
Exercises

1. Find the unknown angle in each triangle.

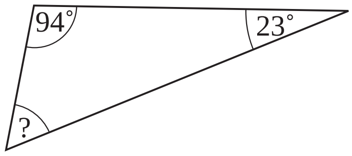
(a)



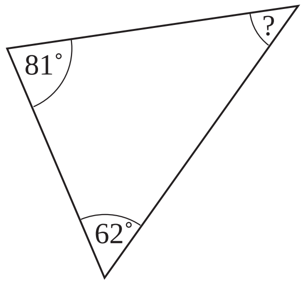
(b)



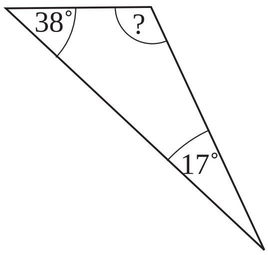
(c)



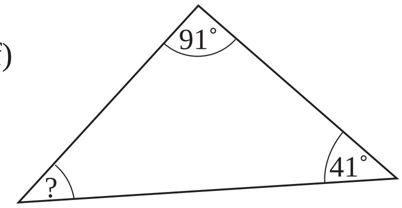
(d)



(e)

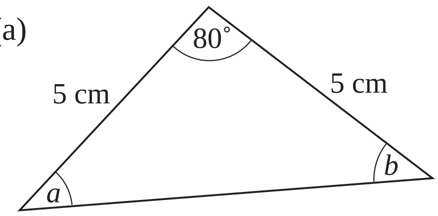


(f)

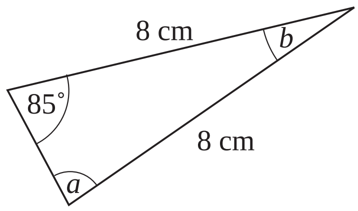


2. Find the unknown angles in each of the following triangles.

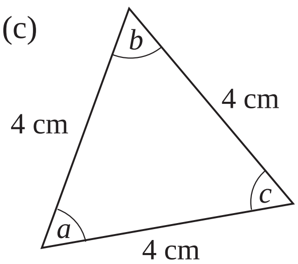
(a)



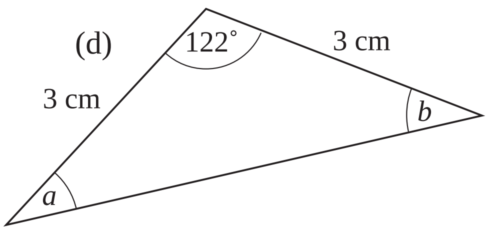
(b)



(c)

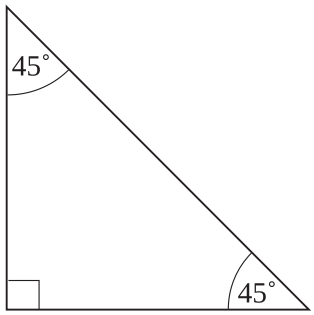


(d)

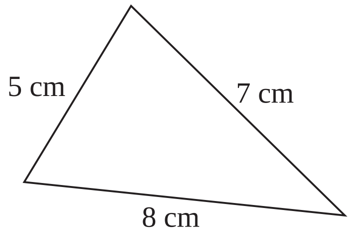


3. State whether each triangle below is *isosceles, equilateral* or *scalene.*

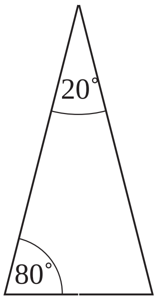
(a)



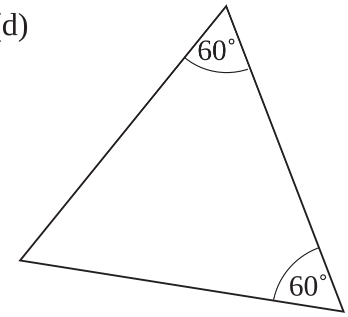
(b)



(c)

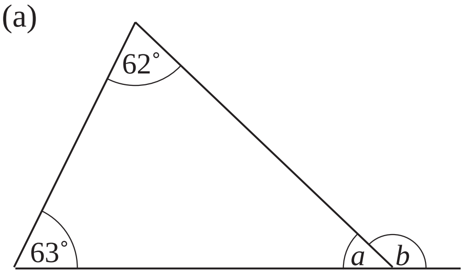


(d)

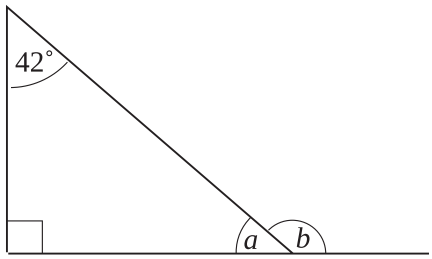


4. For each triangle below, find the unknown *interior* angle and the marked *exterior* angle.

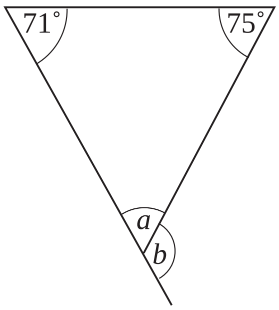
(a)



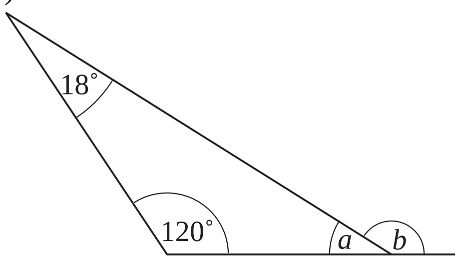
(b)



(c)



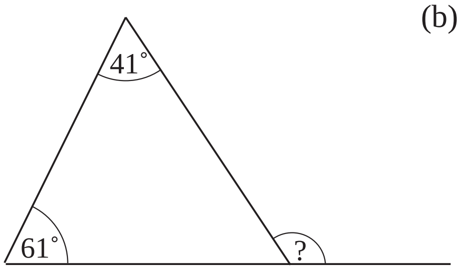
(d)



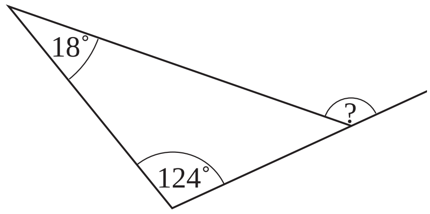
5. Explain how to find the exterior angle without having to calculate an interior angle.

Find the exterior angles marked on these triangles.

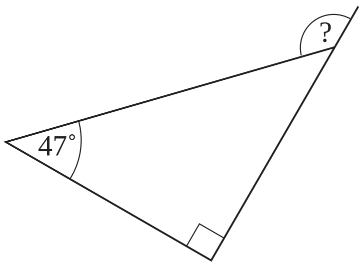
(a)



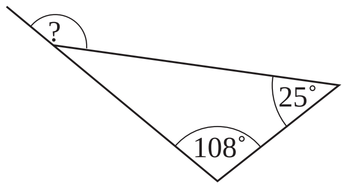
(b)



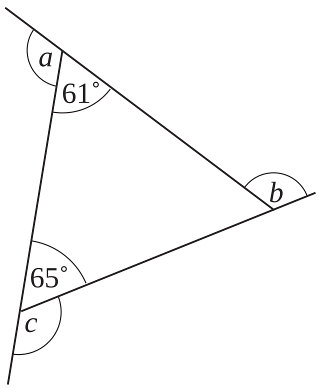
(c)



(d)



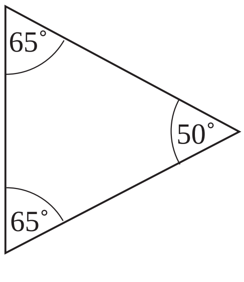
6. Find the total of the 3 exterior angles for this triangle.



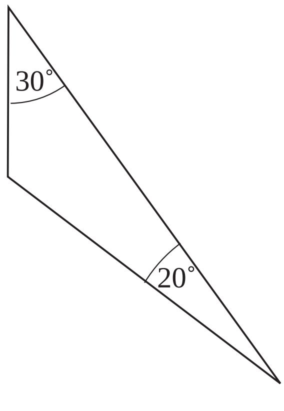
Do you think you will get the same answer for different triangles? Explain your answer.

7. For each of the following triangles, draw in the exterior angles and find their total.

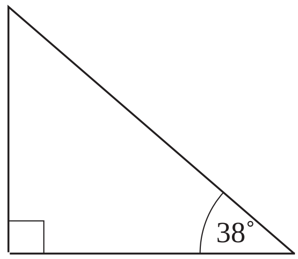
(a)



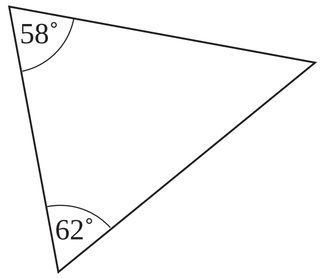
(b)



(c)



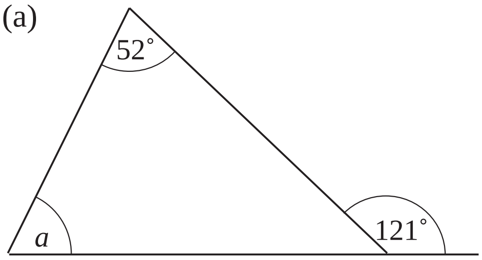
(d)



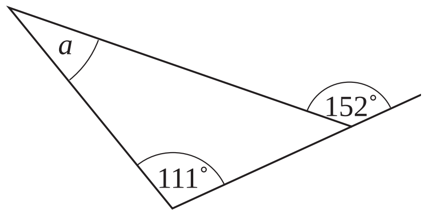
Comment on your results.

8. Find the unknown angle or angles marked in each of the following diagrams.

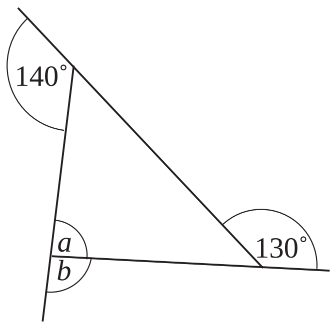
(a)



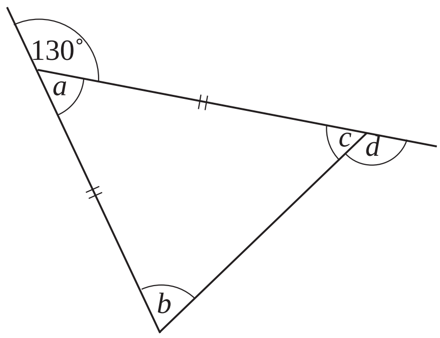
(b)



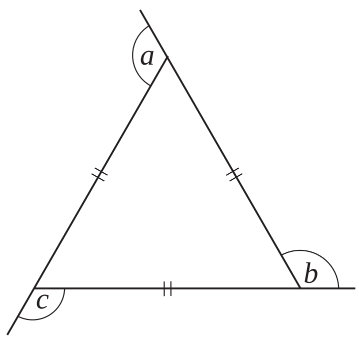
(c)



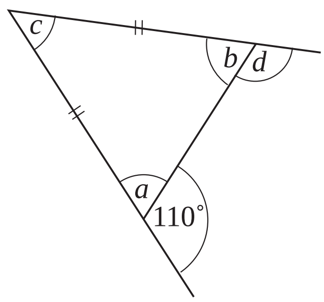
(d)



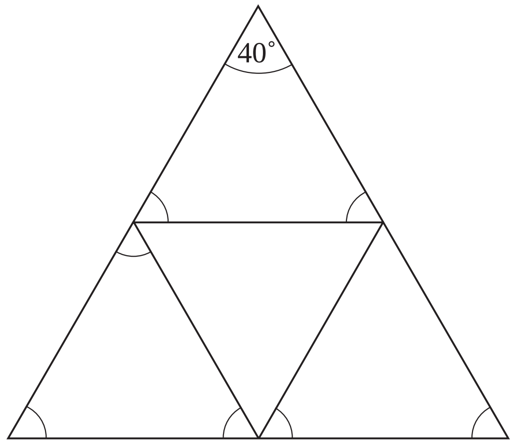
(e)



(f)



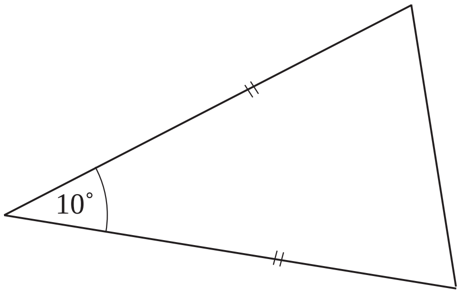
9. Part of a roof is made out of 4 similar isosceles triangles.



Copy the diagram and mark the sides that have the same lengths.

On your diagram, write in the size of all the marked angles.

10. (a) For this isosceles triangle, find the other two interior angles.



(b) Find the other angles if the $10°$ increases to $20°$ and then to $30°$.

(c) What do you think will happen if the $10°$ is increased to $40°$?

11. One angle of an isosceles triangle is $70°$. What are the other angles? (There is more than one solution!)

6 Arithmetic: Multiplication of Decimals

6.1 Multiplication of Whole Numbers

Here we start with multiplication of whole numbers, which is a useful technique for all sorts of problems.

Example

Jai spends £3 on sweets each week for 7 weeks. Calculate how much he spends altogether.

Solution

He spends (in £) $3+3+3+3+3+3+3 \; (=21)$, but it is easier to calculate

$$3 \times 7 = 21 \text{ is £21.}$$

You should know your multiplication tables up to 10×10, but for revision, we include these here.

$\times$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Exercises

1. Find

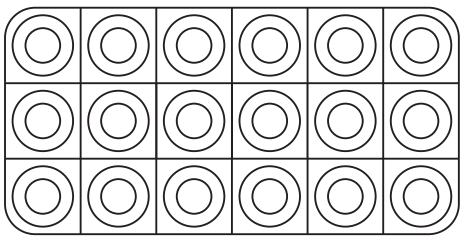
(a)	2×3	(b)	5×7	(c)	6×3
(d)	3×7	(e)	5×4	(f)	9×2
(g)	8×5	(h)	6×6	(i)	9×4
(j)	8×7	(k)	9×8	(l)	7×9
(m)	6×7	(n)	9×9	(o)	8×6

2. Is each of these statements *true* or *false*?

(a)	$5 \times 4 = 4 \times 5$	(b)	$6 \times 5 = 6 \times 7$
(c)	$8 \times 9 = 4 \times 36$	(d)	$21 \times 5 = 7 \times 15$

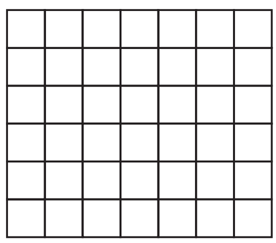
3. Jamil saves £5 per month from his pocket money.
   (a) How much does he save in 4 months?
   (b) How long will it take him to save £30?

4. How many bottles are there in this crate?



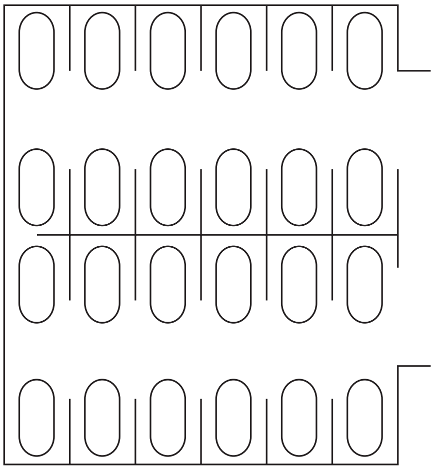
5. Emma, Rachel, Sarah and Hannah go to a disco. It costs £3 each to get in. How much do they pay altogether?

6. The picture shows the tiles on one wall in Sunnava's bathroom. How many tiles are on this wall?

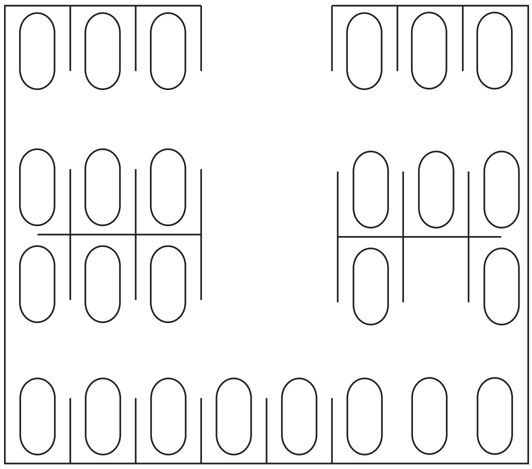


7. Packets of chewing gum are packed in a box. In a box there are 8 layers with 9 packets of chewing gum in each layer. How many packets are there in the box?

8. The picture shows the cars parked in a car park. How many cars have been parked?



9. How many cars are there in this car park?



10. A hotel has 9 floors. On each floor there are 7 windows. How many windows are there in the hotel?

6.2 Long Multiplication

You are probably familiar with long multiplication. For example, you can find 35×19 in the following way:

$$\begin{array}{rr} & 35 \\ \times & 19 \\ \hline & 350 \\ + & 315 \\ \hline & 665 \\ \hline \end{array}$$

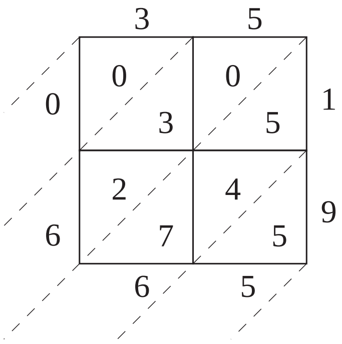
But there are many other ways of doing this sum.
For example,

(1) Napier's Method

Write the two numbers on the horizontal top and vertical side.

Multiply each digit together to give the two digit entries in the cell (write $3 \times 1 = 3$ as 03).

Now add up along the diagonals; carry digits in the usual way – this gives 0665, i.e. 665 as expected!



(2) Russian Multiplication

Write down the multiplication: 35×19

Divide left hand side by 2, ignoring remainder, and multiply right hand side (RHS) by 2

Continue in this way until 1 is reached on left hand side (LHS)

35	19
17	38
~~8~~	~~76~~
~~4~~	~~152~~
~~2~~	~~304~~
1	608

Cross out terms on RHS if there is an even number on LHS

Add up the remaining numbers on RHS: 665 (again!)

(3) Box Method

	30	5
10	$10 \times 30 = 300$	$10 \times 5 = 50$
9	$9 \times 30 = 270$	$9 \times 5 = 45$

$$\begin{array}{r} 300 \\ 270 \\ 50 \\ 45 \\ \hline 665 \\ \hline \end{array}$$

Exercises

1. Find, by any method:

(a)	3×42	(b)	8×35	(c)	6×22
(d)	9×43	(e)	12×62	(f)	15×32
(g)	84×22	(h)	19×48	(i)	62×18
(j)	43×62	(k)	172×42	(l)	461×78
(m)	184×192	(n)	392×412	(o)	494×72

2. Use Russian multiplication to find:

(a)	42×37	(b)	62×81	(c)	14×93
(d)	27×43	(e)	82×29	(f)	38×46
(g)	57×37	(h)	29×49	(i)	33×28

3. Use the box method or Napier's method to find:

(a)	12×15	(b)	32×21	(c)	89×42
(d)	45×57	(e)	62×91	(f)	112×428

6.3 Multiplying with Decimals

We now extend our multiplication to decimals.

Example

You know that $35 \times 19 = 665$.

Deduce the value of

(a) 3.5×19 (b) 3.5×1.9 (c) 350×1.9 (d) 350×190

Solution

(a)
$$3.5 \times 19 = \frac{35}{10} \times 19 = \frac{35 \times 19}{10} = \frac{665}{10} = 66.5$$

(b) $3.5 \times 1.9 = \dfrac{35}{10} \times \dfrac{19}{10}$

$$= \frac{35 \times 19}{100}$$

$$= \frac{665}{100}$$

$$= 6.65$$

(c) $350 \times 1.9 = (35 \times 10) \times \dfrac{19}{10}$

$$= \frac{35 \times 10 \times 19}{10}$$

$$= 665$$

(d) $350 \times 190 = (35 \times 10) \times (19 \times 10)$

$$= (35 \times 19) \times (10 \times 10)$$

$$= 665 \times 100$$

$$= 66\ 500$$

Of course, in practice you do not need to write out the calculations in full like this, but simply write down the answers.

Exercises

1. Find:

(a)	3×0.8	(b)	5×0.7	(c)	3×2.6
(d)	9×1.2	(e)	6×1.5	(f)	8×7.9
(g)	2.1×3.2	(h)	5.6×7.2	(i)	8.4×2.1
(j)	9.2×1.8	(k)	1.2×6.2	(l)	15×7.3
(m)	22×9.4	(n)	62×7.1	(o)	74×5.3

2. Work out the following, using a quick method if possible.

(a)	6×10	(b)	0.7×10
(c)	12.2×100	(d)	112×10
(e)	$2 \times 3.2 \times 5$	(f)	$2 \times 62 \times 50$
(g)	1.47×1000	(h)	18.41×10
(i)	365×100	(j)	$200 \times 7200 \times 5$

3. Find:

(a) 2.47×1.6 (b) 3.25×11.1 (c) 3.42×6.19

(d) 7.24×5.16 (e) 8.21×15.1 (f) 32.1×0.47

6.4 Problems Involving Multiplication

We now see how multiplication helps when solving problems in context.

Example 1

In a train there are 6 coaches each with 68 seats and two coaches each with 42 seats. What is the total seating capacity of the train?

Solution

$$\begin{aligned} \text{The total number of seats} &= 6 \times 68 + 2 \times 42 \\ &= 408 + 84 \\ &= 492 \text{ seats} \end{aligned}$$

Example 2

Find the cost of 12 lunches, each costing £3.29.

Solution

You can use long multiplication to get the answer.

$$\begin{array}{rr} & 3.29 \\ & \times 12 \\ \hline & 3290 \\ + & 658 \\ \hline & £39.48 \\ \hline \end{array}$$

Exercises

1. It costs £9 to go on a school trip. A class of 28 children all go on the trip. How much do they pay in total?

2. Chocolate bars are packed in boxes. Each box contains 24 bars. Mrs Patel buys 8 boxes for the tuck shop. How many bars does she buy?

3. A train has 8 carriages. There are 52 seats in each carriage. How many seats are there on the train?

4. A milk crate contains 24 bottles of milk. There are 32 crates on a milk float. How many bottles are there on the milk float?

5. Matthew organises a trip to a concert. He buys 32 tickets which cost £35 each. How much does he spend on the tickets?

6. Shamil helps his parents build a patio. It is rectangular. There are 12 slabs along one side and 18 along the other side. How many slabs are there in the patio?

7. A burger costs £1.29. Find the cost of 10 burgers.

8. Alex earns £2.54 each day for his paper round. How much does he earn in 6 days?

9. A meal for an adult costs £4.99 and a meal for a child costs £2.25. Find the total cost of 2 adult and 4 child meals.

10. Rope is sold for £1.28 per metre. Find the cost of 10 metres of rope.

11. The price of a carpet is £4.99 per square metre. Find the cost of 8 square metres of carpet.

12. Chain is sold for £2.44 per metre. Find the cost of 3.2 metres of chain.

13. Apples are sold for £1.06 per kilogram. Find the cost of 2.4 kilograms of apples.

14. A factory makes 260 televisions in a day. How many televisions are made at the factory in a whole year? Give 3 possible answers, explaining each one.

7 Number Patterns and Sequences

In this unit we consider how number patterns arise, how to find particular patterns and finding the formula for a general term in a sequence. Again, this topic is an important building block in mathematical understanding.

7.1 Multiples

We start by looking at a sequence formed by taking multiples of a particular number. For example,

$$3, \quad 6, \quad 9, \quad 12, \quad 15, \; \ldots, \; \ldots$$

which are the multiples of 3.

Example

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This square shows the multiples of a number. What is this number?

Write down the numbers that should go in each of these boxes. The number square will help you with some of them.

(a) The 5th multiple of ☐ is ☐.

(b) The ☐ th multiple of ☐ is 36.

(c) The 12th multiple of ☐ is ☐.

(d) The 20th multiple of ☐ is ☐.

(e) The ☐ th multiple of ☐ is 96.

(f) The 100th multiple of ☐ is ☐.

Solution

The number is 4, and

(a) the 5th multiple of 4 is 20,

(b) the 9th multiple of 4 is 36,

(c) the 12th multiple of 4 is 48,

(d) the 20th multiple of 4 is 80,

(e) the 24th multiple of 4 is 96,

(f) the 100th multiple of 4 is 400.

Exercises

1. On a number square like this one, shade all the multiples of 6. Then answer the questions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) What is the 4th multiple of 6?

(b) What is the 10th multiple of 6?

(c) What is the 12th multiple of 6?

(d) What is the 100th multiple of 6?

2. The multiples of a number have been shaded on this number square. What is the number?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Copy each statement about these multiples and write down the numbers that should go in the boxes.

(a) The 3rd multiple of ☐ is ☐ .

(b) The 9th multiple of ☐ is ☐ .

(c) The 200th multiple of ☐ is ☐ .

(d) The ☐ th multiple of ☐ is 66.

(e) The ☐ th multiple of ☐ is 330.

3. (a) Write down the first 8 multiples of 8.
   (b) Write down the first 8 multiples of 6.
   (c) What is the smallest number that is a multiple of both 6 and 8?
   (d) What are the next two numbers that are multiples of both 6 and 8?

4. (a) Write down the first 6 multiples of 12.
   (b) What is the 10th multiple of 12?
   (c) What is the 100th multiple of 12?
   (d) What is the 500th multiple of 12?
   (e) If 48 is the *n*th multiple of 12, what is *n*?
   (f) If 96 is the *n*th multiple of 12, what is *n*?

5. (a) What multiples have been shaded in this number square?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   (b) What is the first multiple *not* shown in the number square?

6. (a) Explain why 12 is a multiple of 6 and 4.
   (b) Is 12 a multiple of any other numbers?

7. The number 24 is a multiple of 2 and a multiple of 3. What other numbers is it a multiple of?

8. Two multiples of a number have been shaded on this number square. What is the number?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

9. Two multiples of a number have been shaded on this number square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(a) What is the number?

(b) What is the 19th multiple of this number?

10. Three multiples of a number are 34, 170 and 255. What is the number.

11. Three multiples of a number are 38, 95 and 133. What is the number?

12. Four multiples of a number are 49, 77, 133 and 203. What is the number?

7.2 Finding the Next Term

Here we use the given numbers of the sequence to deduce the pattern and hence find the next term

Example

What are the next 3 numbers in the sequences:

(a) 12, 17, 22, . . .

(b) 50, 47, 44, 41, 38, . . .

Solution

(a) To spot the pattern, it is usually helpful to first find the differences between each term; i.e.

12 17 22

5 5

So the next term is found by adding 5 to the previous term; this gives 27, 32, 37.

(b) Again we find the difference:

50 47 44 41 38

–3 –3 –3 –3

So the next term is found by taking away 3 from the previous term, giving 35, 32, 29.

Exercises

1. Copy each of the sequences below and write in the next 3 numbers in each sequence. Complete the working that is shown.

   (a) 1, 4, 7, 10, 13, . . .

   3 3 3

   (b) 3, 5, 7, 9, 11, . . .

   (c) 5, 8, 11, 14, 17, . . .

   (d) 6, 8, 10, 12, 14, . . .

   (e) 20, 19, 18, 17, 16, . . .

   (f) 6, 9, 12, 15, 18, . . .

   (g) 22, 20, 18, 16, 14, . . .

2. Copy each sequence and fill in the missing number.

   (a) 4, 7, $\square$, 13, 16, . . .

   (b) 7, $\square$, 15, 19, 23, . . .

   (c) 8, 14, 20, $\square$, 32, . . .

   (d) 3, 11, $\square$, 27, 35, . . .

   (e) 15, $\square$, 27, 33, 39, . . .

3. Copy and continue each sequence, giving the next three numbers.

   (a) 18, 30, 42, 54, 66, . . .

   (b) 4.1, 4.7, 5.3, 5.9, 6.5, . . .

   (c) 14, 31, 48, 65, 82, . . .

   (d) 101, 119, 137, 155, 173, . . .

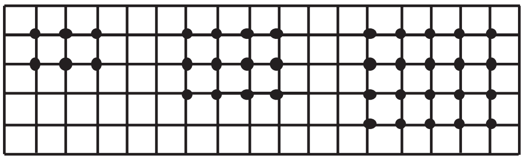
   (e) 3.42, 3.56, 3.70, 3.84, 3.98, . . .

   (f) 10, 9.5, 9, 8.5, 8, 7.5, . . .

   (g) $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, . . .

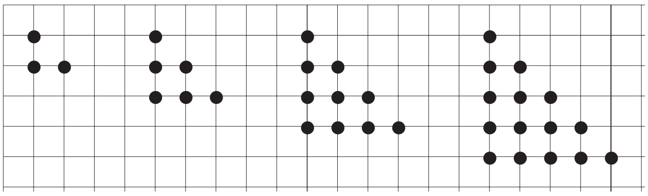
4. For each sequence of patterns, draw the next two shapes and find the next 3 numbers in the sequence.

(a)



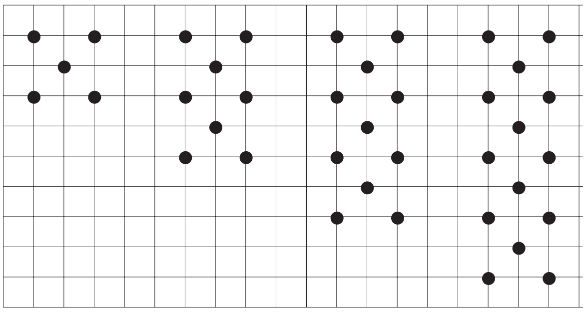
6, 12, 20, . . .

(b)



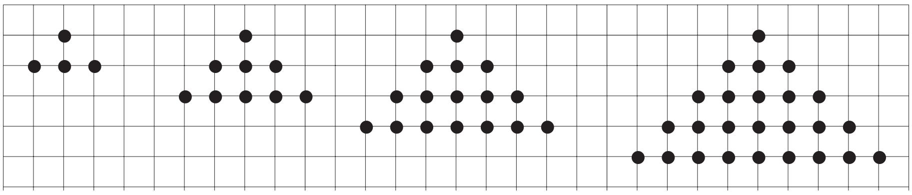
3, 6, 10, 15, . . .

(c)



5, 8, 11, 14, . . .

(d)



4, 9, 16, 25, . . .

5. Find the first number in each of the sequences.

(a) ☐, 6, 11, 16, 21, . . .

(b) ☐, 7, 9, 11, 13, . . .

(c) ☐, 6, 5, 4, 3, . . .

(d) ☐, 19, 28, 37, 46, . . .

(e) ☐, 12, 9, 6, 3, . . .

6. Copy each sequence and write in the next three terms.

(a) 1, 4, 9, 16, 25, . . .

(b) 2, 5, 10, 17, 26, . . .

(c) 0, 3, 7, 12, 18, . . .

(d) 6, 12, 20, 30, 42,

(e) 0.5, 2.0, 4.5, 8.0, 12.5, . . .

7. Copy each sequence and fill in the missing numbers.

(a) 2, 4, ☐, 16, 32, . . .

(b) 100, 81, 64, ☐, 36, . . .

(c) 6, 9, ☐, 21, 30, . . .

(d) 0, 1.5, 4, ☐, 12, . . .

(e) 1, 7, 17, ☐, 49, . . .

8. Write down the next two terms in each sequence.

(a) $\frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4}, \quad \frac{4}{5}, \quad \ldots$

(b) $\frac{9}{11}, \quad \frac{8}{12}, \quad \frac{7}{13}, \quad \frac{6}{14}, \quad \ldots$

(c) $\frac{3}{6}, \quad \frac{5}{7}, \quad \frac{7}{8}, \quad \frac{9}{9}, \quad \ldots$

(d) $\frac{2}{1}, \quad \frac{3}{4}, \quad \frac{4}{9}, \quad \frac{5}{16}, \quad \ldots$

(e) $\frac{0}{2}, \quad \frac{3}{5}, \quad \frac{8}{10}, \quad \frac{15}{17}, \quad \ldots$

7.3 Generating Number Sequences

Here we introduce the concept of the general terms of a sequence. For example, the formula $5n$, with $n = 1, 2, 3, 4, \ldots$, generates the sequence

$$5 \times 1, \quad 5 \times 2, \quad 5 \times 3, \quad 5 \times 4, \quad \ldots$$

that is $\quad 5, \quad 10, \quad 15, \quad 20, \quad \ldots$

Similarly, $5n+1$ gives, in the same way,

$$(5\times 1)+1,\ (5\times 2)+1,\ (5\times 3)+1,\ (5\times 4)+1,\ \ldots$$

that is $\quad 6,\ 11,\ 16,\ 21,\ \ldots$

Example 1

What sequence is generated by the formulae

(a) $5n-1$ (b) $6n+2$?

Solution

(a) Putting $n=1,\ 2,\ 3,\ 4,\ \ldots$ gives

$4,\ 9,\ 14,\ 19,\ \ldots$

(b) Putting $n=1,\ 2,\ 3,\ 4,\ \ldots$ gives

$8,\ 14,\ 20,\ 26,\ \ldots$

Example 2

What is the formula for this sequence

$11,\ 21,\ 31,\ 41,\ 51,\ 61$?

Solution

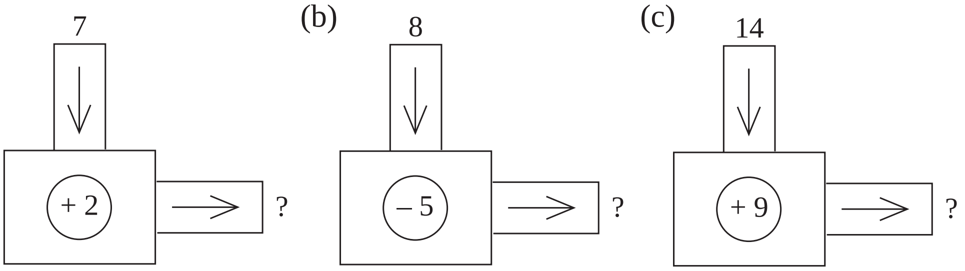
As you are starting with 11, and $11=10+1$, and you continue to add 10 each time, the formula will be

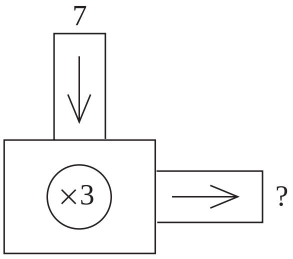
$$10n+1$$

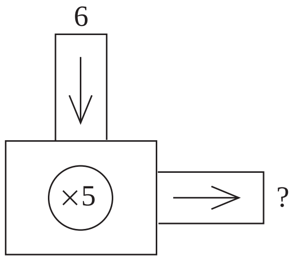
Exercises

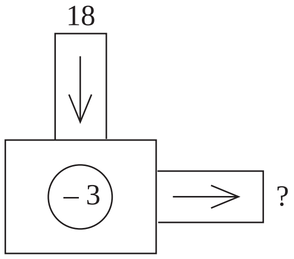
1. What number comes out of each of these number machines?

(a) (b) (c)

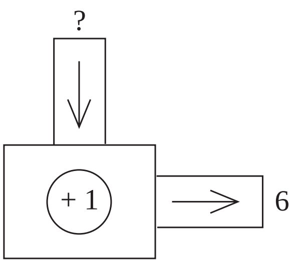


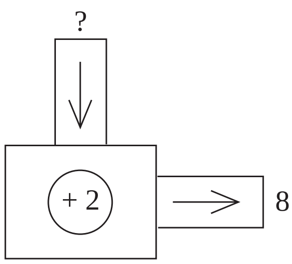
(d) 

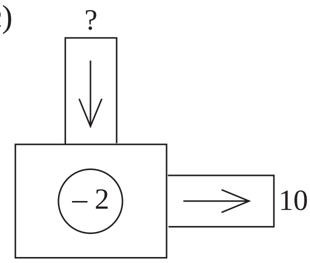
(e) 

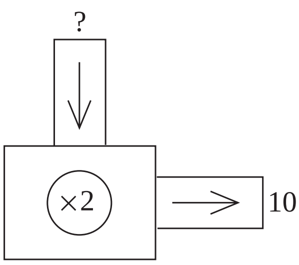
(f) 

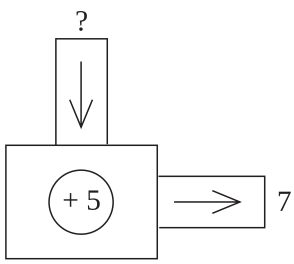
2. What number was put into each of these number machines?

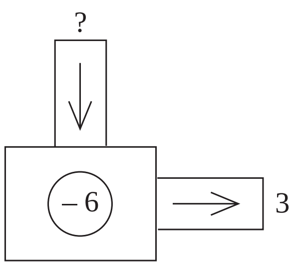
(a) 

(b) 

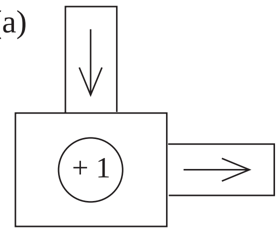
(c) 

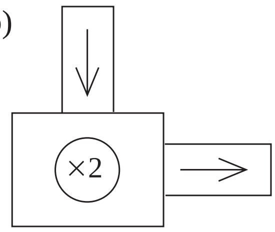
(d) 

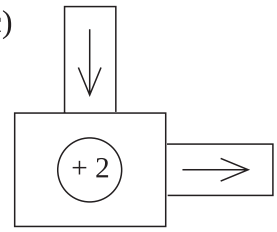
(e) 

(f) 

3. The sequence 1, 2, 3, 4, 5, . . . is put into each of these machines. Write down the first 5 terms of the sequence that comes out of each machine.

(a) 

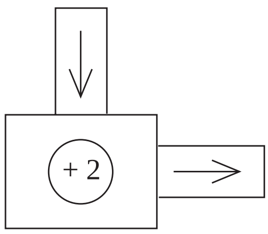
(b) 

(c) 

Which machine gives the *even* numbers?

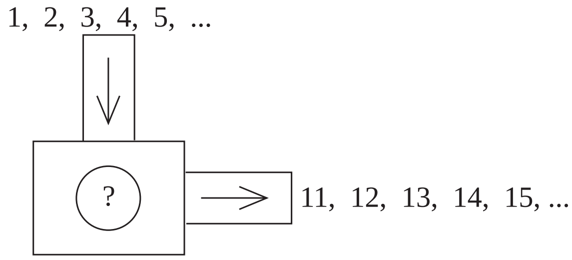
4. (a) Write down the first 5 multiples of 2.

(b) What happens if you put these multiples of 2 into this machine?

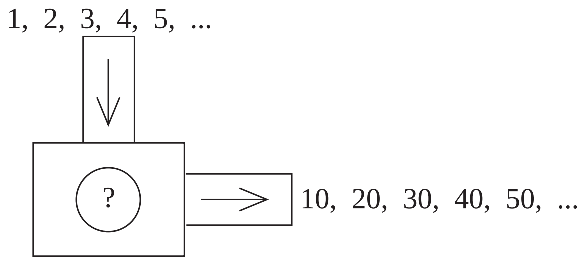


5. The sequence 1, 2, 3, 4, 5, . . . is put into each number machine. What does each machine do?

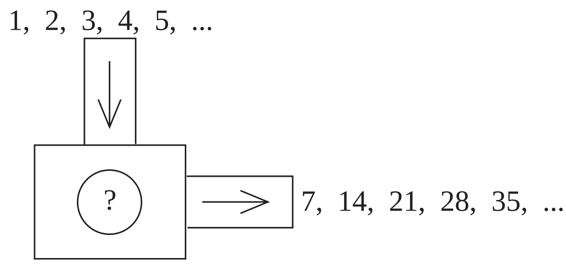
(a)



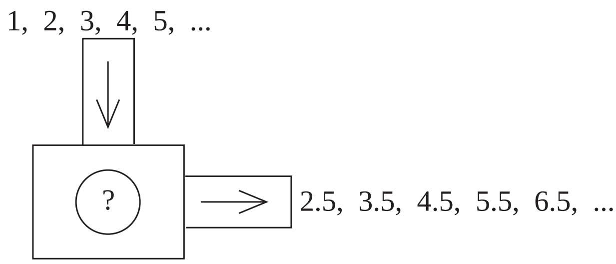
(b)



(c)



(d)



6. Write down the first 5 terms of the sequence given by each of these formulae:

(a) $3n$ (b) $3n+1$ (c) $3n+4$

(d) $3n-1$ (e) $3n-2$

7. Write down the first 5 terms of the sequence given by each of these formulae:

(a) $9n$ (b) $12n$ (c) $2n+1$

(d) $3n+2$ (e) $5n-2$ (f) $7n-1$

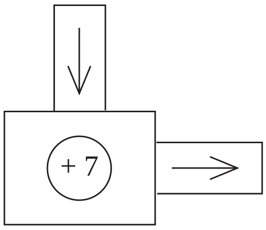
8. (a) What is the 10th term of the sequence $2n + 1$?

   (b) What is the 8th term of the sequence $3n + 6$?

   (c) What is the 5th term of the sequence $4n + 1$?

   (d) What is the 7th term of the sequence $5n - 1$?

9. When the sequence 1, 2, 3, 4, 5, . . . is put into the machine:

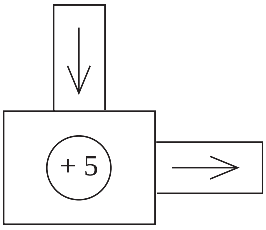


it creates the sequence with formula $n + 7$.

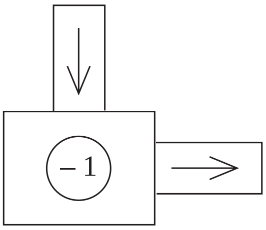
(a) Write down the first 6 terms of the sequence with formula $n + 7$.

(b) What happens if the sequence 1, 2, 3, 4, 5, . . . is put into these machines? Write down the formula for the sequence you get.

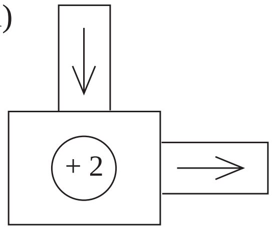
(i)



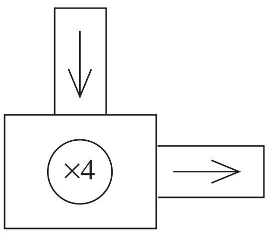
(ii)



(iii)



10. You need this machine to get the sequence with formula $4n$ from the sequence 1, 2, 3, 4, 5, . . .

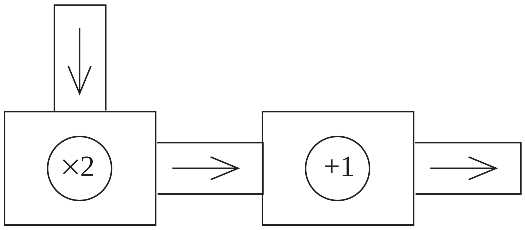


(a) Write down the first 5 terms of this sequence.

(b) Draw the machine you would need to get
6, 12, 18, 24, 30 from 1, 2, 3, 4, 5, . . .

(c) Draw the machine you would need to get the sequence with formula $7n$ from 1, 2, 3, 4, 5, ...

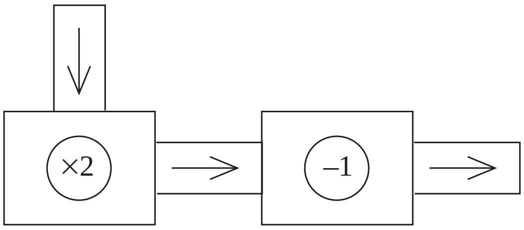
11. Two machines can be put together like this, to make a double machine.



(a) What do you get if you put 1, 2, 3, 4, 5, . . . into this double machine? What is the formula for the sequence you get?

(b) Repeat part (a) for each of these double machines.

(i)



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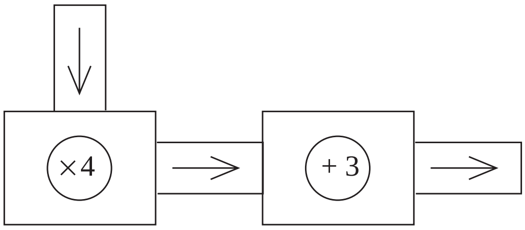

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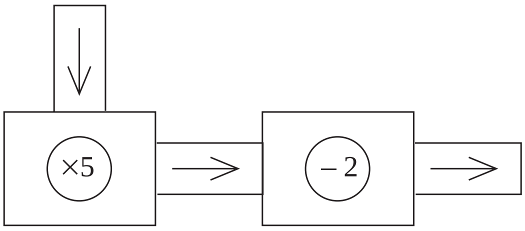

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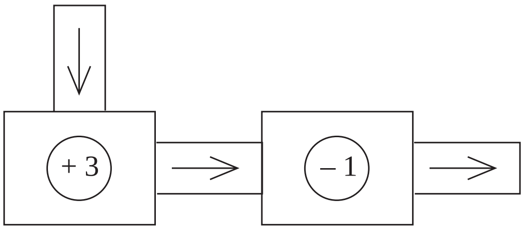


(c) What single machine has the same effect as the double machine in part (b)(iv)? What is the formula for the single machine?

12. Draw double machines that could be used to get each of these sequences from 1, 2, 3, 4, 5, . . .

    Also write down the formula for each sequence.

    (a) 5, 9, 13, 17, 21, . . .

    (b) 2, 5, 8, 11, 14, . . .

    (c) 6, 11, 16, 21, 26, . . .

    (d) 4, 9, 14, 19, 24, . . .

    (e) 102, 202, 302, 402, 502, . . .

7.4 Formulae for General Terms

It is very helpful not only to be able to write down the next few terms in a sequence, but also to be able to write down, for example, the 100th or even the 1000th term!

Example

For the sequence

3, 7, 11, 15, . . . , . . .

find (a) the next *three* terms,

(b) the 100th term,

(c) the 1000th term.

Solution

(a) Looking at the differences,

3, 7, 11, 15

4 4 4

we can see that 4 is added each time to get the next term.

So we obtain 19, 23, 27.

(b) To find the 100th term, starting at 3 (the first term), you add on 4, ninety-nine times, giving

$$\begin{aligned} 3 + 4 \times 99 &= 3 + 396 \\ &= 399 \end{aligned}$$

(c) Similarly, the 1000th term is

$$\begin{aligned} 3 + 4 \times 999 &= 3 + 3996 \\ &= 3999 \end{aligned}$$

Note

You can go one stage further and write down the formula for a general term, i.e. the *n*th term.

This is

$$3 + 4 \times (n - 1) = 3 + 4n - 4$$
$$= 4n - 1 \qquad \text{(Check the previous answers.)}$$

Exercises

1. For the sequence:

   2, 5, 8, 11, 14, . . .

   (a) What is the difference between each term?

   (b) Explain why the formula for the *n*th term is $3n - 1$.

2. For the sequence

   6, 8, 10, 12, 14, . . .

   (a) find the difference between each term,

   (b) explain why the formula for the *n*th term is $2n + 4$.

3. For each sequence, write down the difference between each term and the formula for the *n*th term.

   (a) 3, 5, 7, 9, 11, . . .

   (b) 5, 11, 17, 23, 29, . . .

   (c) 4, 7, 10, 13, 16, . . .

   (d) 2, 5, 8, 11, 14, . . .

   (e) 6, 10, 14, 18, 22, . . .

4. (a) What formula gives the sequence

   4, 8, 12, 16, 20, . . .

   (b) What formula gives the sequence that is the multiples of 5?

5. (a) What is the formula for the *n*th term of this sequence?

   7, 14, 21, 28, 35, . . .

   (b) How can you get this sequence from the sequence in (a)?

   8, 15, 22, 29, 36, . . .

   (c) What is the formula for the *n*th term of the sequence in (b)?

6. (a) Write down the first 6 multiples of 11.

   (b) What is the formula for the *n*th term of the sequence of the multiples of 11?

   (c) What is the formula for the *n*th term of this sequence?

$$10, \quad 21, \quad 32, \quad 43, \quad 54, \quad \ldots$$

7. Write down the formula for the *n*th term of each of these sequences.

   (a) 3, 6, 9, 12, 15, ...

   (b) 5, 12, 19, 26, 33, ...

   (c) 21, 29, 37, 45, 53, ...

   (d) 8, 11, 14, 17, 20, ...

   (e) 1, 4, 7, 10, 13, ...

   (f) 103, 106, 109, 112, 115, ...

8. (a) Explain why the formula for the *n*th term of this sequence,

$$\frac{1}{2}, \quad \frac{1}{4}, \quad \frac{1}{6}, \quad \frac{1}{8}, \quad \frac{1}{10}, \quad \ldots$$

   is $\frac{1}{2n}$.

   (b) What is the formula for the *n*th term of this sequence?

$$\frac{1}{3}, \quad \frac{1}{5}, \quad \frac{1}{7}, \quad \frac{1}{9}, \quad \frac{1}{11}, \quad \ldots$$

9. Find formulae for the *n*th term of each of these sequences.

   (a) $\frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4}, \quad \frac{4}{5}, \quad \frac{5}{6}, \quad \ldots$

   (b) $\frac{1}{4}, \quad \frac{2}{5}, \quad \frac{3}{6}, \quad \frac{4}{7}, \quad \frac{5}{8}, \quad \ldots$

   (c) $\frac{1}{10}, \quad \frac{2}{11}, \quad \frac{3}{12}, \quad \frac{4}{13}, \quad \frac{5}{14}, \quad \ldots$

   (d) $\frac{2}{8}, \quad \frac{4}{9}, \quad \frac{6}{10}, \quad \frac{8}{11}, \quad \frac{10}{12}, \quad \ldots$

   (e) $\frac{3}{5}, \quad \frac{6}{6}, \quad \frac{9}{7}, \quad \frac{12}{8}, \quad \frac{15}{9}, \quad \ldots$

10. The formula for the nth term of this sequence is n^2.

$$1, \quad 4, \quad 9, \quad 16, \quad 25, \quad \ldots$$

What is the formula for the nth term of the following sequences?

(a) 0, 3, 8, 15, 24, . . .

(b) 10, 13, 18, 25, 34, . . .

(c) 2, 8, 18, 32, 50, . . .

(d) 1, 8, 27, 64, 125, . . .

8 Arithmetic: Division of Decimals

8.1 Mental Division of Whole Numbers

The process of division is multiplication in reverse. So, since $4 \times 3 = 12$, then $12 \div 4 = 3$ and $12 \div 3 = 4$. You also need to remember the order in which operations must be carried out, which can be summarised by BODMAS:

Brackets first
O
Divide
Multiply
Add
Subtract

Example

Calculate (a) $16 \times 2 + 3$, (b) $16 \times (2 + 3)$.

Solution

(a) $16 \times 2 + 3 = 32 + 3$ (**M** before **A**)
$= 35$

(b) $16 \times (2 + 3) = 16 \times 5$ (**B** before **M**)
$= 80$

Exercises

1. Find

(a)	$16 \div 4$	(b)	$12 \div 6$	(c)	$15 \div 5$
(d)	$20 \div 4$	(e)	$18 \div 9$	(f)	$40 \div 8$
(g)	$36 \div 9$	(h)	$15 \div 3$	(i)	$64 \div 8$
(j)	$42 \div 7$	(k)	$24 \div 6$	(l)	$32 \div 8$
(m)	$49 \div 7$	(n)	$56 \div 8$	(o)	$45 \div 5$

2. Is each of these statements *true* or *false*?

(a) $10 \div 2 = 2 \div 10$ (b) $12 + 8 \div 2 = 10$
(c) $3 + 12 \div 4 = 6$ (d) $6 \div 2 + 3 = 6$

3. Find:

(a) $3 + 4 \times 8$	(b) $8 + 3 \times 6$	(c) $8 \times 6 - 4$
(d) $12 \div 2 + 5$	(e) $5 - 12 \div 3$	(f) $14 \div 2 + 8$
(g) $3 \times 2 + 8 \div 4$	(h) $3 \times 6 - 15 \div 3$	(i) $42 \div 7 + 3$
(j) $16 \div 4 + 24 \div 6$	(k) $8 \times 2 + 5 \times 3$	(l) $8 \times 6 - 45 \div 5$

4. A pupil works out $200 \div 4$ by this method:

$$200 \div 2 = 100$$

$$100 \div 2 = 50$$

Use similar methods to find:

(a) $500 \div 4$	(b) $52 \div 4$	(c) $68 \div 4$
(d) $128 \div 4$	(e) $224 \div 4$	(f) $104 \div 8$
(g) $80 \div 16$	(h) $112 \div 16$	(i) $128 \div 8$

8.2 Division Methods for Whole Numbers and Decimals

Care must be taken when handling divisions, particularly when they involve decimals.

Example

Find

(a) $1300 \div 100$

(b) $1.75 \div 5$

(c) $6.31 \div 4$

Solution

(a) $1300 \div 100 = \dfrac{1300}{100}$

$= 13$

(b) $1.75 \div 5 = 0.35$ since

$$\begin{array}{r} 0.35 \\ 5\,\overline{)\,1.7^{2}5} \end{array}$$

(c) 631 ÷ 4 gives $\begin{array}{r} 157 \\ 4\overline{)\,6^{2}3^{3}1} \end{array}$ r 3, i.e. 157 with remainder 3

Alternatively, to get the answer in decimal form, write

$$\begin{array}{r} 157.75 \\ 4\overline{)\,6^{2}3^{3}1.^{3}0^{2}0} \end{array} \qquad \text{i.e. } 157.75$$

Exercises

1. Find:

(a) 12 ÷ 10 (b) 4200 ÷ 10

(c) 600 000 ÷ 10 (d) 3714 ÷ 10

(e) 5728 ÷ 10 (f) 6000 ÷ 100

(g) 7000 ÷ 1000 (h) 75 000 ÷ 100

(i) 750 ÷ 100 (j) 3714 ÷ 100

(k) 8412 ÷ 100 (l) 642 130 ÷ 10 000

2. Carry out the following divisions.

(a) 69 ÷ 3 (b) 4545 ÷ 9 (c) 6612 ÷ 3

(d) 2947 ÷ 7 (e) 7404 ÷ 6 (f) 37 050 ÷ 5

(g) 2208 ÷ 12 (h) 13 488 ÷ 24 (i) 1792 ÷ 32

(j) 10 530 ÷ 45 (k) 4284 ÷ 18 (l) 10 496 ÷ 41

3. Carry out the following divisions.

(a) 2.54 ÷ 2 (b) 21.63 ÷ 3 (c) 10.24 ÷ 4

(d) 87.5 ÷ 5 (e) 918.4 ÷ 7 (f) 49.24 ÷ 4

(g) 388.5 ÷ 15 (h) 123.84 ÷ 12 (i) 714.84 ÷ 6

4. Carry out the following divisions, giving your answers as decimals.

(a) 21 ÷ 4 (b) 81 ÷ 2 (c) 162 ÷ 4

(d) 263 ÷ 4 (e) 84 ÷ 8 (f) 241 ÷ 8

8.3 Division Problems

As with multiplication, division is often needed in practical problems.

Example

45 sweets are divided equally between 9 children. How many do they each get?

Solution

Each child gets $45 \div 9 = 5$ sweets.

Exercises

1. A mini chocolate bar costs 8p. How many bars can be bought with 72p?

2. A multistorey car park has 4 levels, each taking the same number of cars. When full it holds 124 cars. How many cars can park at each level?

3. A train can carry 384 passengers. If has 8 carriages, each with the same seating capacity. How many people can each carriage hold?

4. Rafiq borrows £50 from his Dad. He pays it back in 10 equal weekly instalments. How much does he pay back each week?

5. £375.69 is raised at a jumble sale. This is divided equally between 3 charities. How much does each of the charities get?

6. Grace and her 3 brothers are given £37 to share equally between them. How much do they get each?

7. Charlotte has 24 sweets. She shares them out equally between herself and her 3 friends. How many sweets do they get each?

8. Three children are paid £15 for working in a garden. They share the money equally between them. How much do they get each?

9. Karen buys 6 tickets, each costing the same, for the theatre. She pays a total of £54 for the tickets. How much does each ticket cost?

10. A rope is 22.48 m long. It is cut into 4 parts of equal length. How long is each part?

11. A baker mixes 1944 grams of dough. It is used to make 12 small loaves of equal weight. How much dough is used in each loaf?

12. Rachel, Ben, Emma and Hannah are given £5.50 to share equally between them. Describe the problem they have.

13. 40 children want to go on a school trip to Wimbledon. They will be taken in minibuses that each hold 13 passengers. How many minibuses will be needed for the trip?

14. How many chocolate bars costing 23p each can I buy with £2?

15. The 'Oblivion' ride at Alton Towers takes 16 people each time it goes around. How many times must it go around if 70 people want to have a go?

16. A teacher has 240 grams of clay. She cuts off lumps of mass 35 grams each.
    (a) How many lumps can she make?
    (b) How much clay is left over?

17. John sees some cassette tapes that cost 85p each. He has £5.
    (a) How many tapes can he buy?
    (b) If he buys as many tapes as he can, how much change will he have?

18. A text book costs £7.50. A teacher has £149 to spend on books. How many copies of this text book can she buy?

9 Areas and Perimeters

This is is our next key Geometry unit. In it we will recap some of the concepts we have met before. We will also begin to develop a more algebraic approach to finding areas and perimeters.

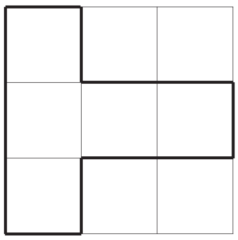
9.1 Area

The easiest method to find an area of a shape, particularly if it is a simple shape made up of straight lines, is to count the squares inside it.

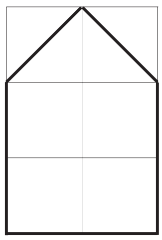
Example 1

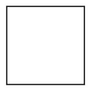
Find the area of each of these shapes in terms of the square shown.

(a)



(b)

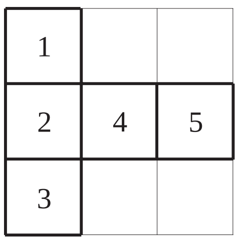




One square

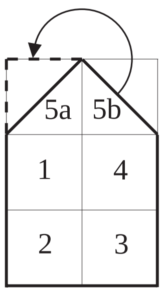
Solution

(a)



This can be divided into 5 of the squares, so its area is 5 square units.

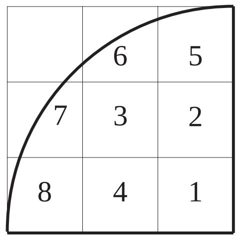
(b)



We have 4 squares (labelled 1, 2, 3 and 4), and the two triangles (labelled 5a and 5b) can be joined together to form another square. So, in total, we have an area of 5 square units.

Example 2

Estimate the area of this shape.



Solution

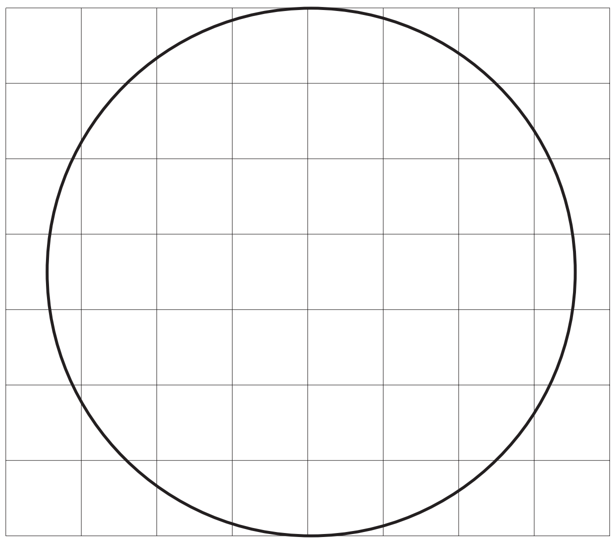
There are 4 complete squares (labelled 1, 2, 3 and 4). Region 5 and 6 together make up about $1\frac{1}{2}$ squares, as do regions 7 and 8. So we have another 3 squares giving a total of 7 square units (plus a little bit more!).

Exercises

1. Draw around your hand on squared paper and find its area.

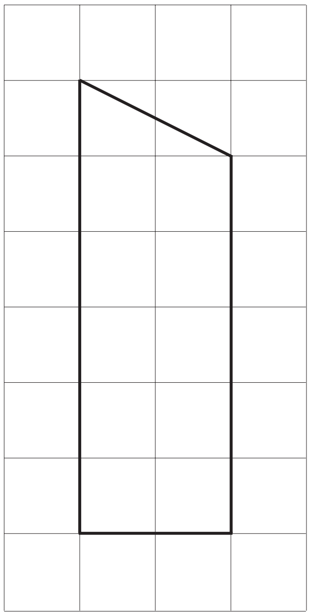
   Who has the largest hand in your class?

2. Find the area of this circle by counting squares.

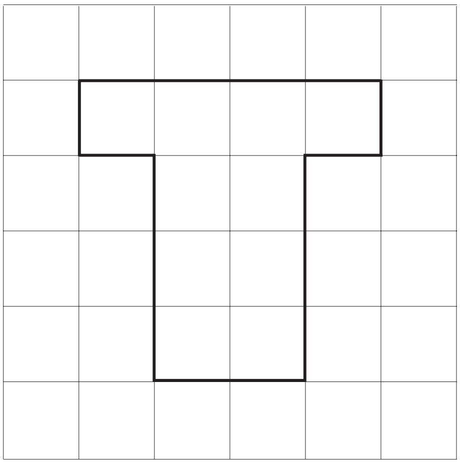


3. Find the areas of these shapes by counting squares.

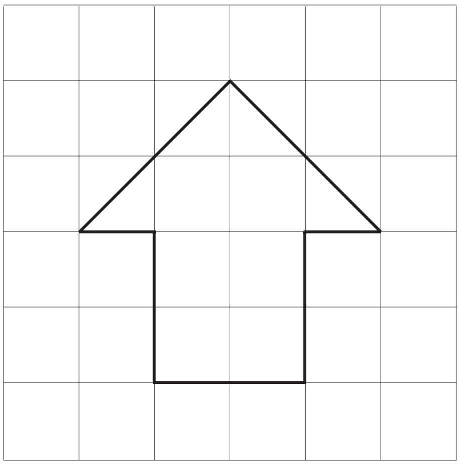
(a)



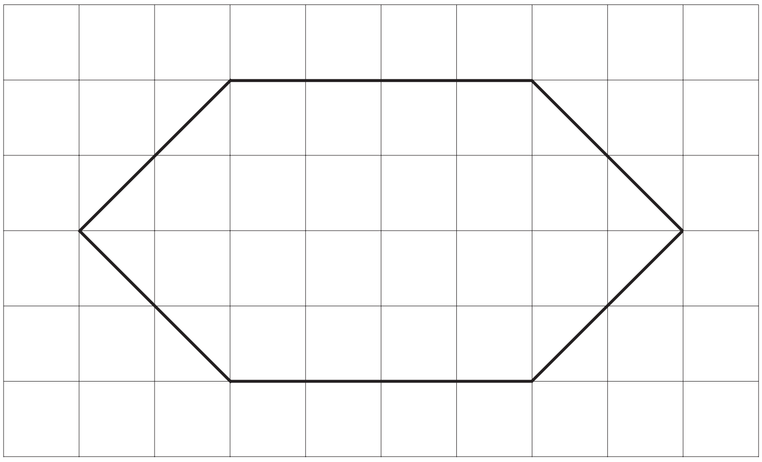
(b)



(c)



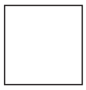
(d)



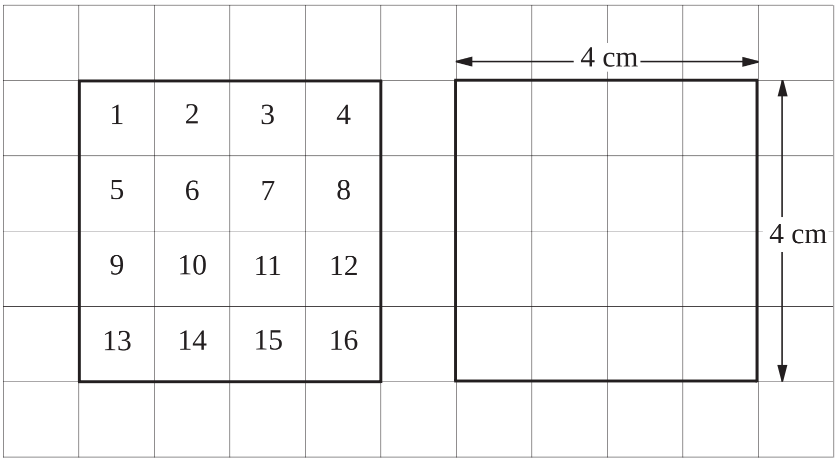
9.2 Area and Perimeter of a Square

We now bring in standard units for measuring area and perimeter. You should always put units in your answers.

The area of a square can be found by counting squares or multiplying the length of the sides. The area of a square with sides 1 cm is 1 cm^2.



Area = 1 cm^2

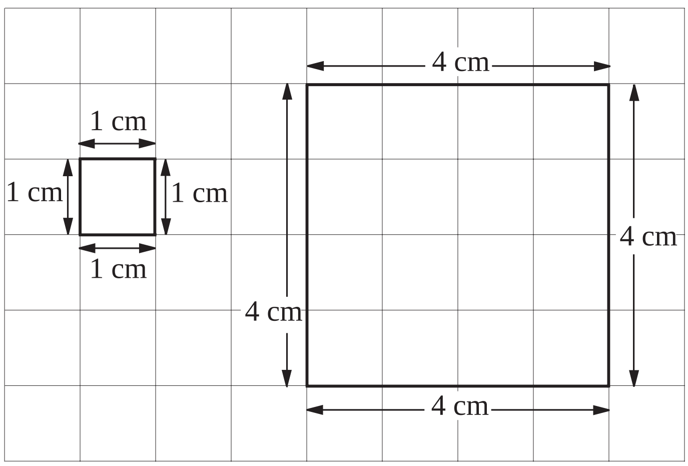


Area = 16 cm^2

Area = 4×4

= 16 cm^2

The perimeter of a square is the total length of the four sides.



Perimeter = 1 + 1 + 1 + 1
= 4 cm

Perimeter = 4 + 4 + 4 + 4
= 16 cm

Note

Note also that:

1 m	=	100 cm
1 cm	=	10 mm

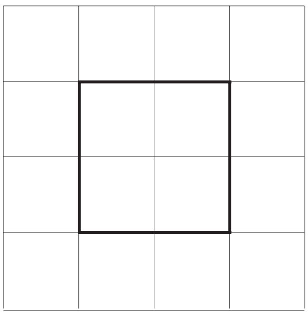
So that, for example,

25 mm	=	2.5 cm
8 mm	=	0.8 cm
261 cm	=	2.61 m
32 cm	=	0.32 m
6 cm	=	0.06 m

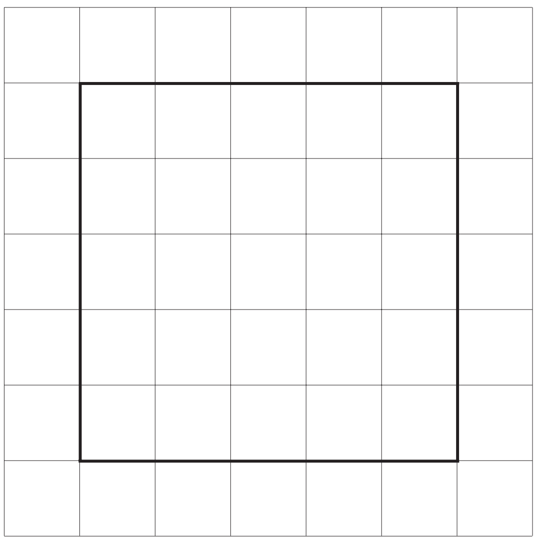
Exercises

1. Find the area and perimeter of each of these squares.

(a)



(b)



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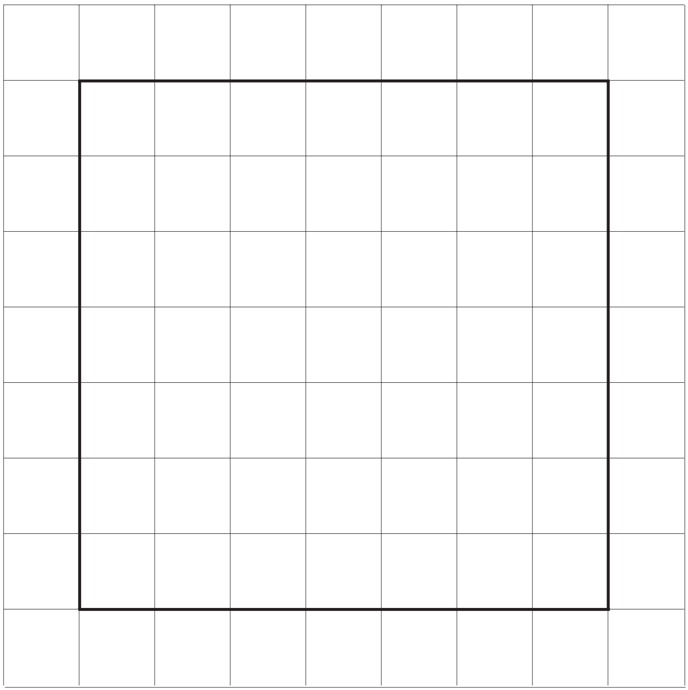


2. Find the area of squares with sides of length:

(a)	10 cm	(b)	12 cm	(c)	8 cm
(d)	9 cm	(e)	15 cm	(f)	20 cm

3. Find the perimeter of squares with sides of length:

(a)	13 cm	(b)	8 cm	(c)	16 cm
(d)	19 cm	(e)	9 cm	(f)	18 cm

4. Copy and complete each of these statements.

(a) 3.2 cm = ☐ mm (b) 10.3 cm = ☐ mm

(c) 28 mm = ☐ cm (d) 216 mm = ☐ cm

(e) 152 cm = ☐ m (f) 84 cm = ☐ m

(g) 1.62 m = ☐ cm (h) 1.7 m = ☐ cm

(i) 0.82 m = ☐ cm (j) 0.07 m = ☐ cm

5. A square has sides of length 20 mm. Find the area of the square in:

(a) mm^2 (b) cm^2

6. The perimeter of a square is 40 cm. How long are its sides?

7. The area of a square is 36 cm^2. How long are its sides?

8. The perimeter of a square is 44 cm. What is its area?

9. The area of a square is 144 cm^2. What is its perimeter?

10. For a 2 cm square the perimeter is 8 cm and the area is 4 cm^2. The perimeter is twice the area.

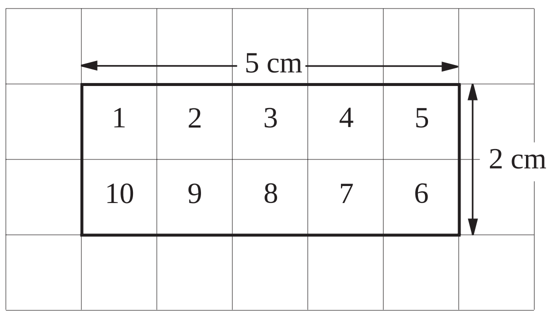
    What are the lengths of the sides of a square for which the perimeter is

    (a) equal to the area;

    (b) half of the area?

9.3 The Area and Perimeter of a Rectangle

For a rectangle, say 5 cm by 2 cm, we can proceed either by counting squares or multiplying the lengths. So for example,



the area of this rectangle is 10 cm^2 from counting squares or, alternatively;

$$\text{Area} = 5 \times 2$$
$$= 10 \text{ cm}^2$$

Note also that 1 cm is the same as 10 mm,

so that a 1 cm square has an area of 1 cm^2 and this can also be written as

$$1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm}$$

i.e $\boxed{1 \text{ cm}^2 = 100 \text{ mm}^2}$

Example

What is 1 m^2 in terms of cm^2?

Solution

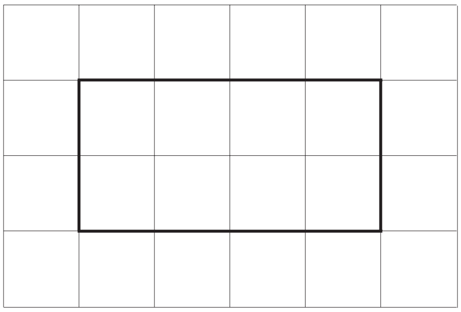
$$1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm}$$

i.e. $1 \text{ m}^2 = 10000 \text{ cm}^2$

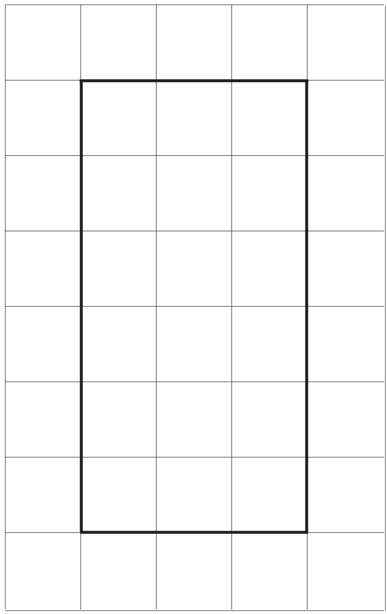
Exercises

1. Find the area of these rectangles in cm^2.

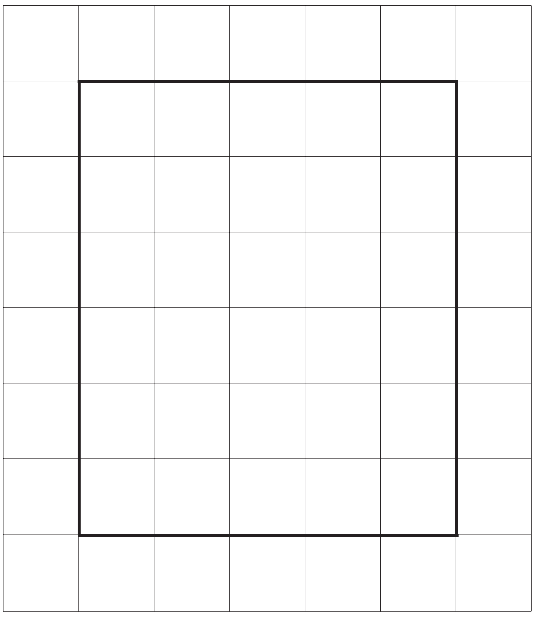
(a)



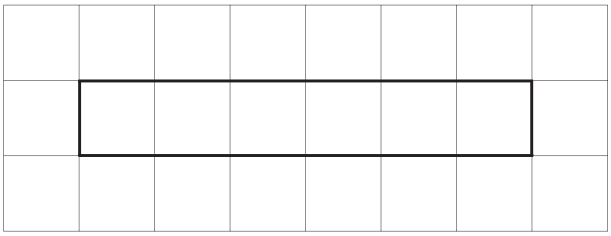
(b)



(c)



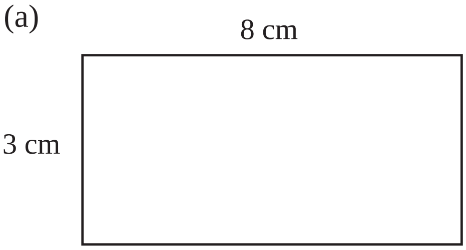
(d)



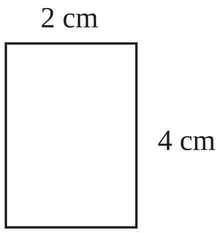
2. Find the perimeter of the rectangles in question 1.

3. Find the area of these rectangles in suitable units. The diagrams have not been drawn accurately.

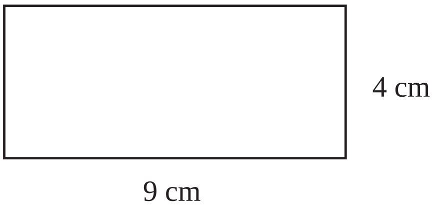
(a)



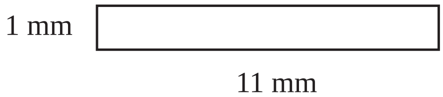
(b)



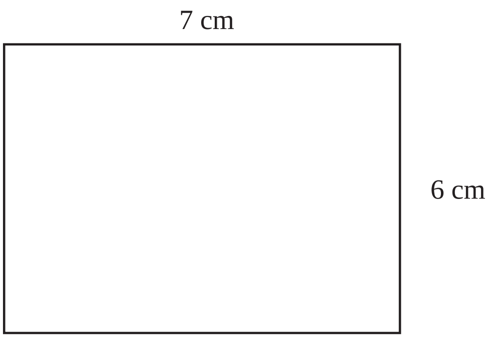
(c)



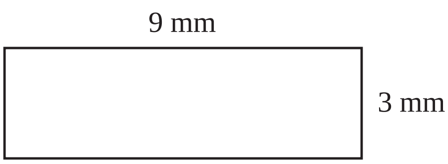
(d)



(e)



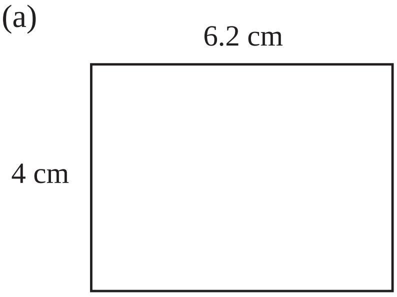
(f)



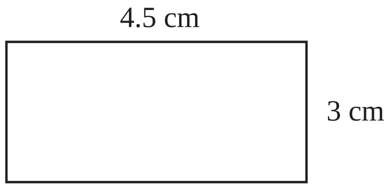
4. Find the perimeter of the rectangles in question 3.

5. Find the area and perimeter of these rectangles.

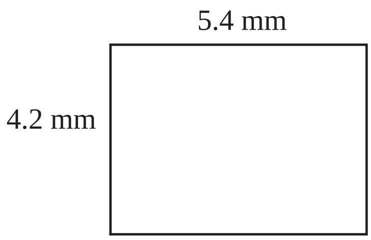
(a)



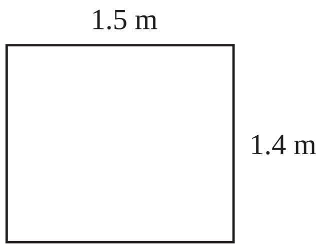
(b)



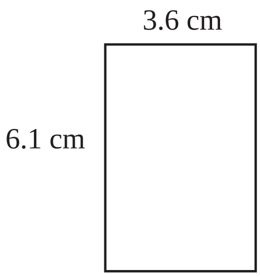
(c)



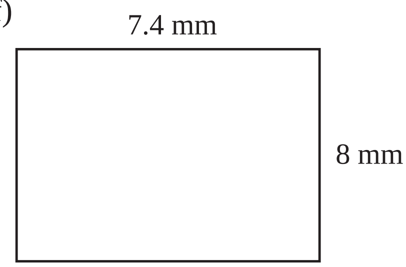
(d)



(e)



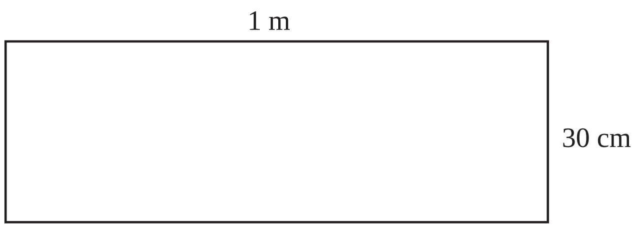
(f)



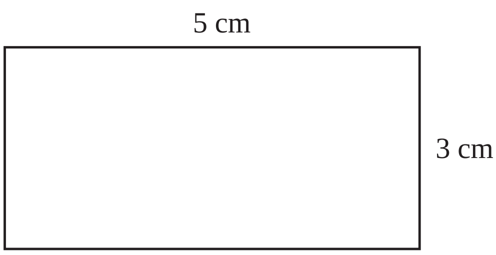
6. Find the area and perimeter of this rectangle

(a) in cm^2 and cm

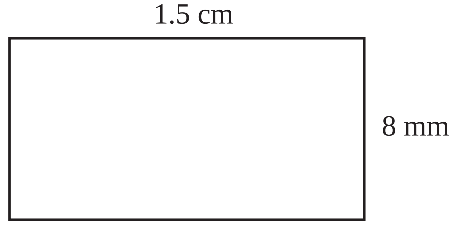
(b) in m^2 and m.



7. Find the area of this rectangle in mm^2 and cm^2.



8. Find the perimeter and area of this rectangle making clear which units you have decided to use.



9. A rectangle has an area of 48 cm^2. The length of one side is 6 cm.

   Find the perimeter of the rectangle.

10. A rectangle has a perimeter of 24 cm and an area of 32 cm^2.

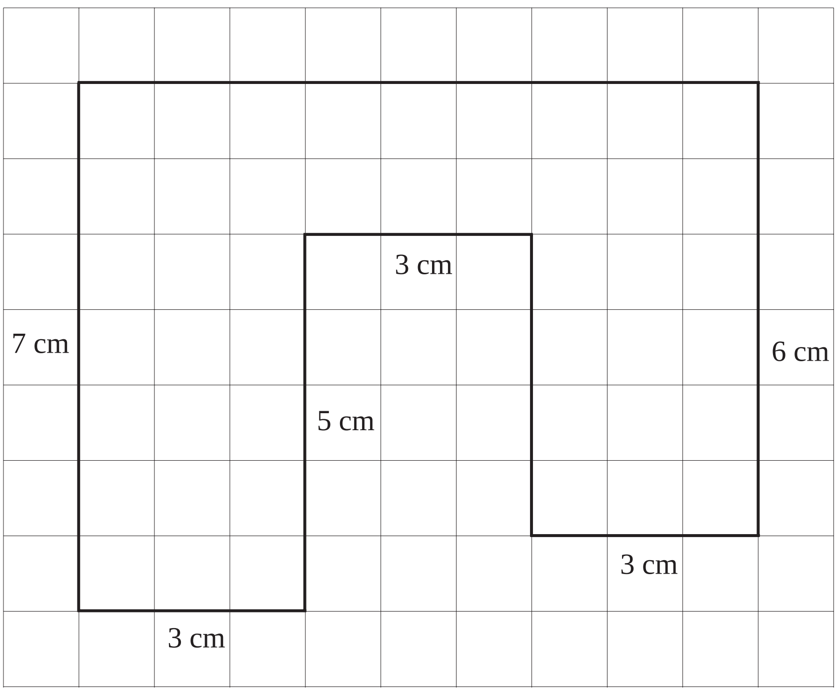
    What are the lengths of the sides of the rectangle?

9.4 Area of Compound Shapes

We illustrate this method with an example.

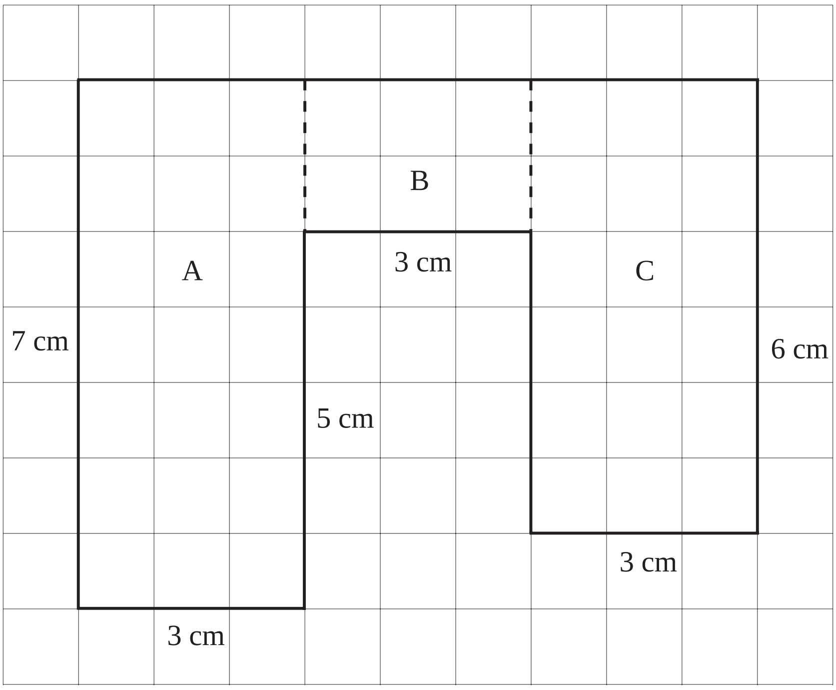
Example

Find the area of the shape shown below.



Solution

Divide the shape into rectangles; one way is shown below.



$$\text{Area of A} = 3 \times 7 = 21 \text{ cm}^2$$

$$\text{Area of B} = 3 \times 2 = 6 \text{ cm}^2$$

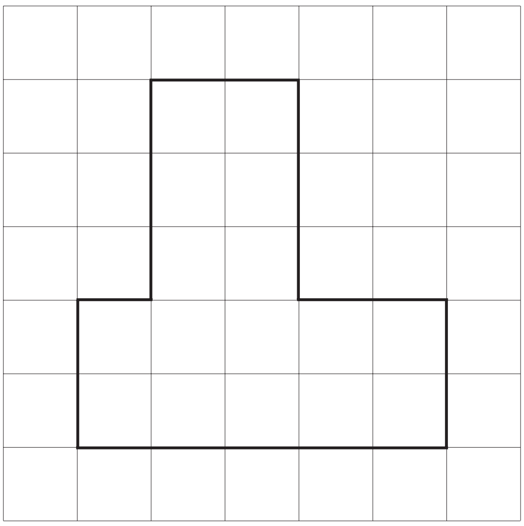
$$\text{Area of C} = 3 \times 6 = 18 \text{ cm}^2$$

$$\textit{Total Area} = 21 + 6 + 18 = 45 \text{ cm}^2$$

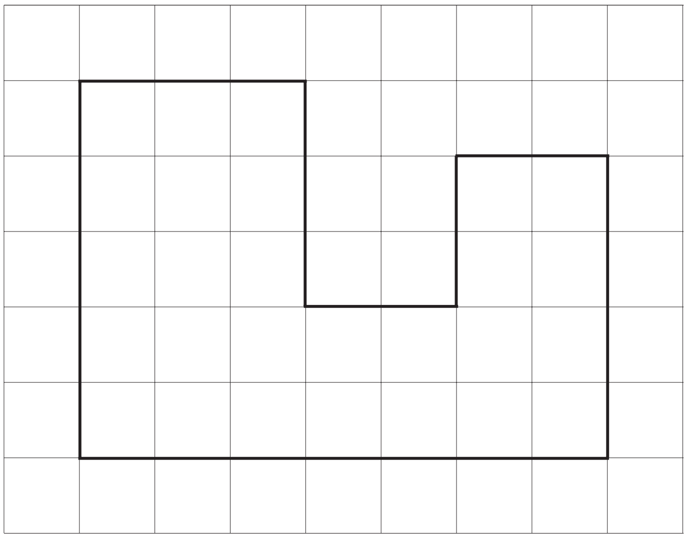
Exercises

1. Find the area of these shapes.

   (a)

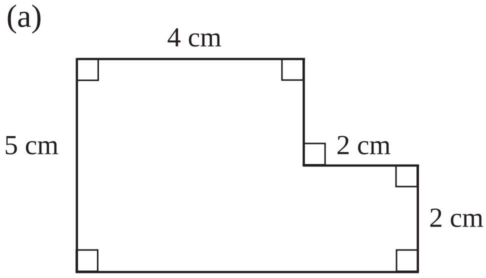


(b)

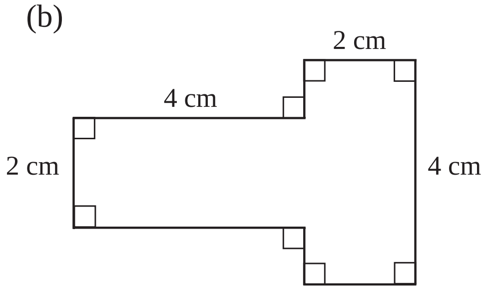


2. Find the area of each of these shapes. The diagrams have not been drawn accurately.

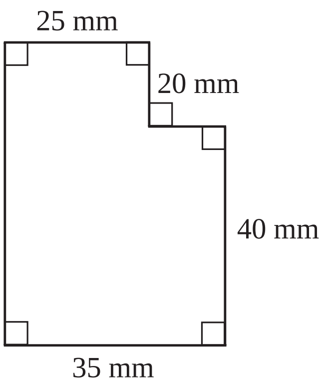
(a)



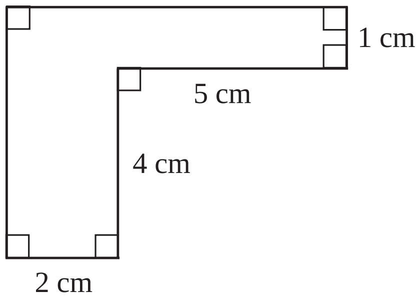
(b)



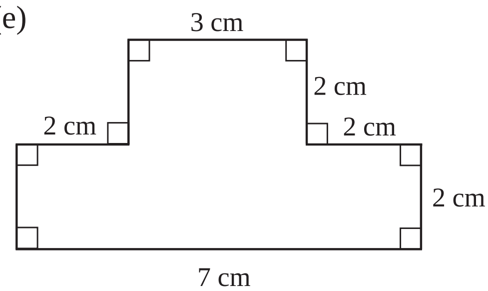
(c)



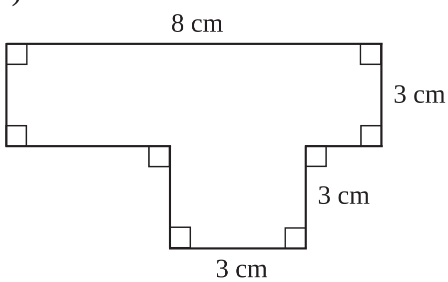
(d)



(e)



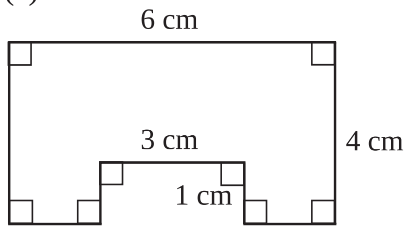
(f)



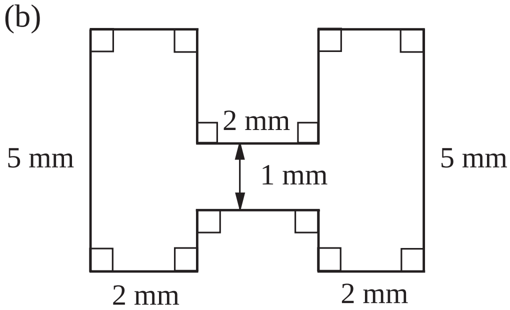
3. Find the perimeter of the shapes in question 2.

4. Find the area and perimeter of these shapes.

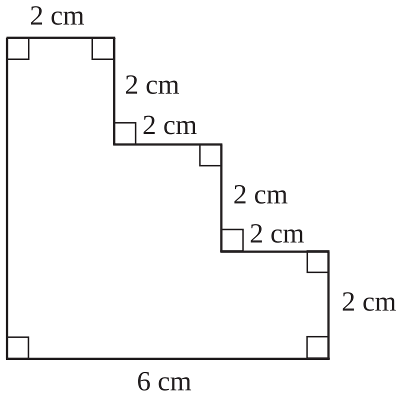
(a)



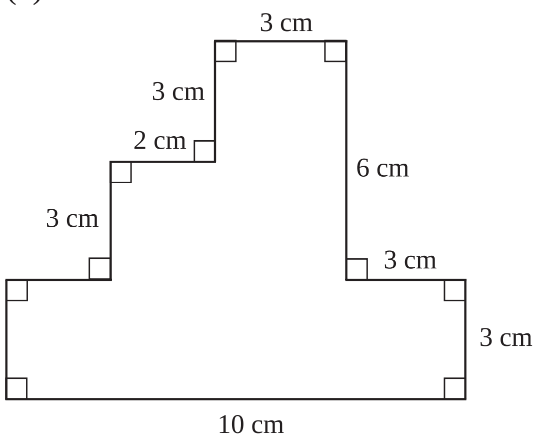
(b)



(c)

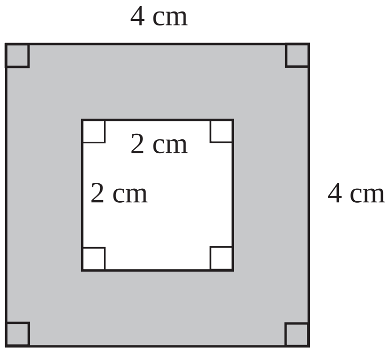


(d)

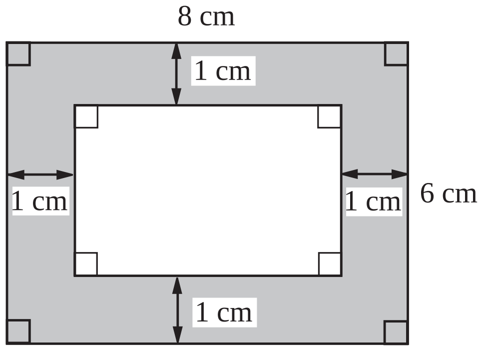


5. Find the shaded area in each of the diagrams below.

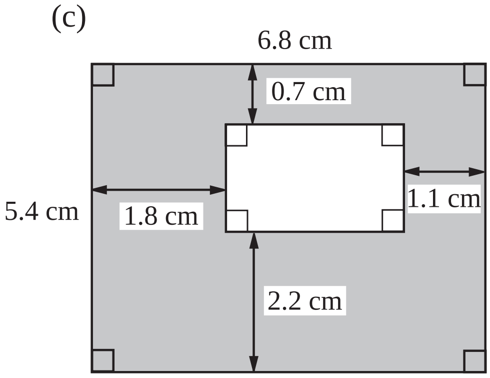
(a)



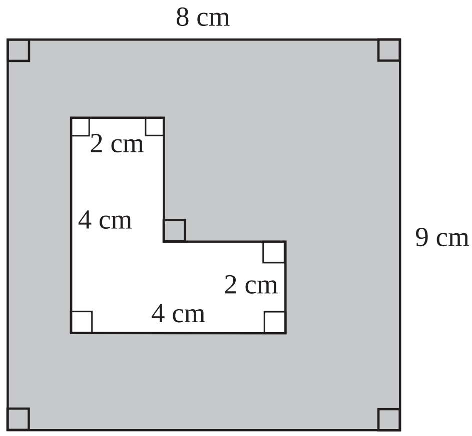
(b)



(c)

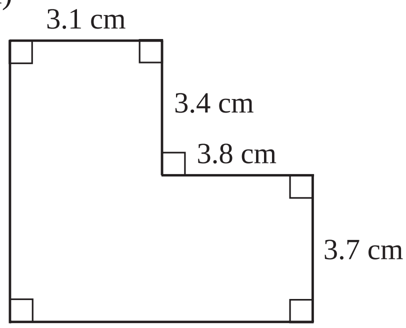


(d)

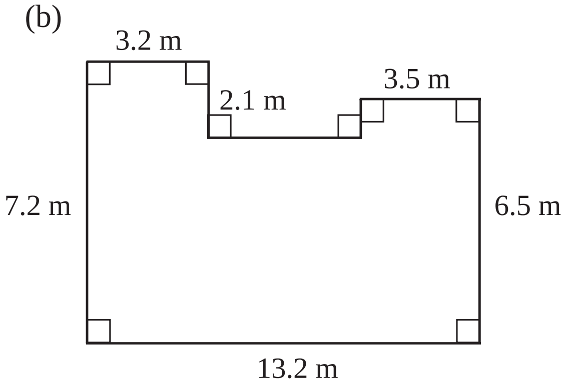


6. Find the area and perimeter of these shapes.

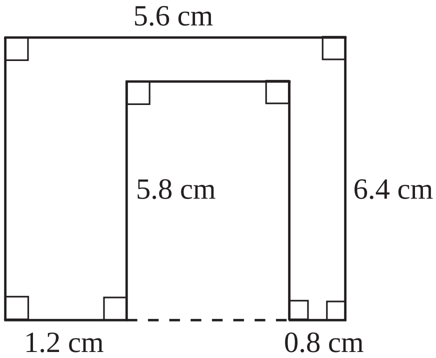
(a)



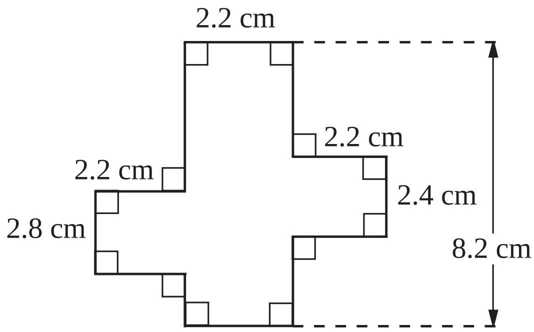
(b)



(c)

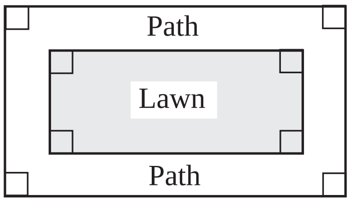


(d)

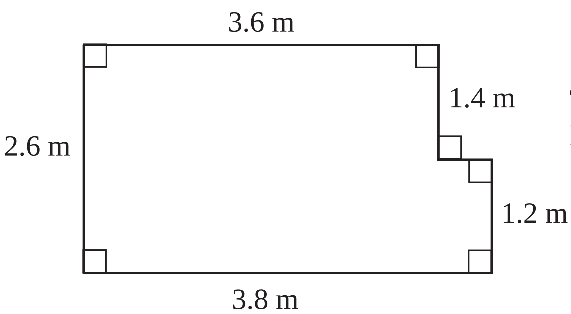


7. A lawn is 3 m by 5 m. A path 1 m wide is laid around the lawn.

Find the area of the path.



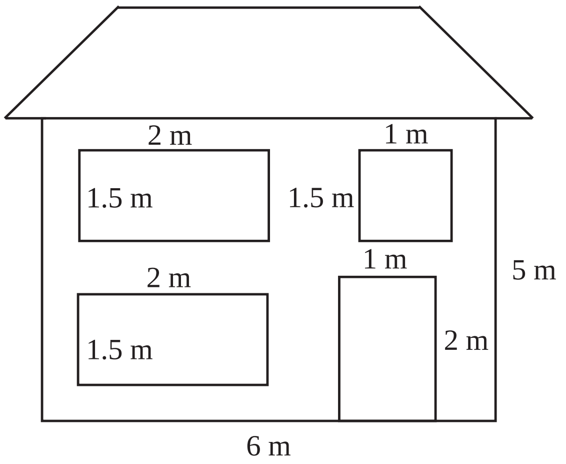
8. Ben's dad buys a carpet that is 3 m wide and 4 m long.



This is the shape of Ben's room

How much carpet is wasted?

9. Wendy is going to paint the front of the house for her mum and dad.

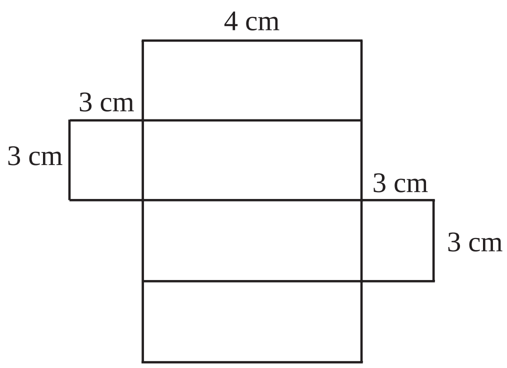


(a) Find the area that needs to be painted.

(b) Wendy gets 50p for every 1 m^2 that she paints. How much money does Wendy get?

10. This shape can be cut out of card and folded to form a box.

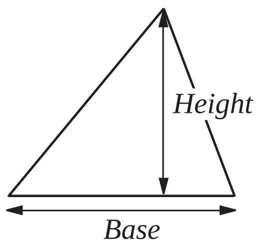
How much card is wasted if this shape is cut out of a sheet 15 cm by 20 cm?



9.5 The Area of a Triangle

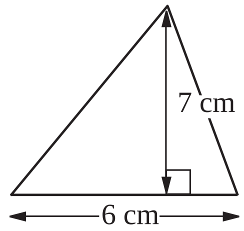
For a triangle,

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$



Example 1

Find the area of the triangle shown.

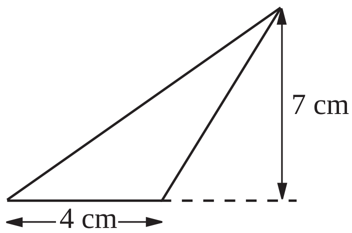


Solution

$$\text{Area} = \frac{1}{2} \times 6 \times 7$$
$$= 21 \text{ cm}^2$$

Example 2

Find the area of the triangle shown.



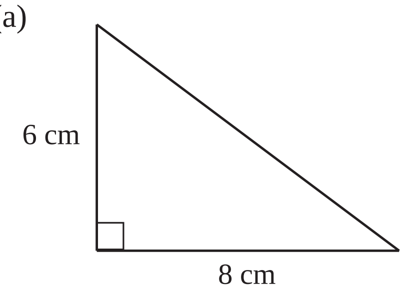
Solution

$$\text{Area} = \frac{1}{2} \times 4 \times 7$$
$$= 14 \text{ cm}^2$$

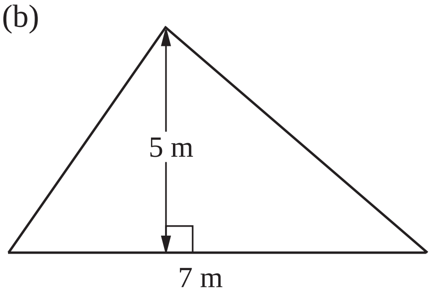
Exercises

1. Find the area of each of these triangles.

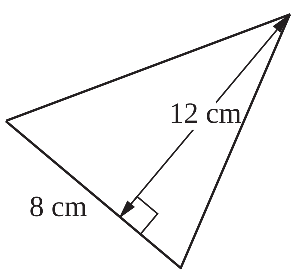
(a)



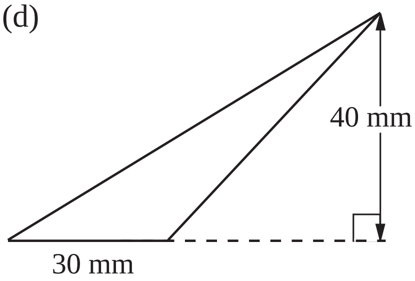
(b)



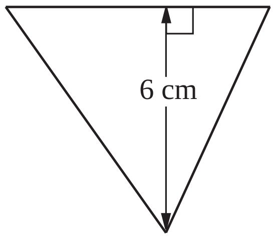
(c)



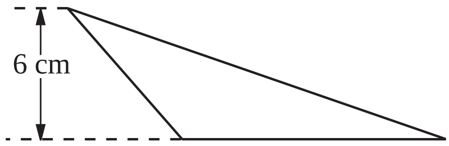
(d)



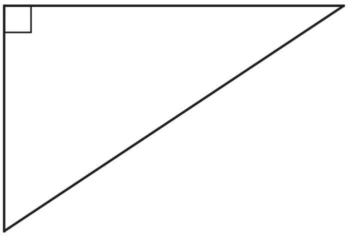
(e)



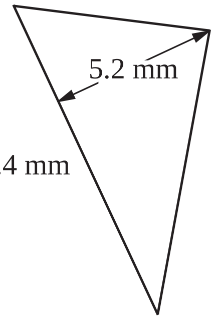
(f)



(g)

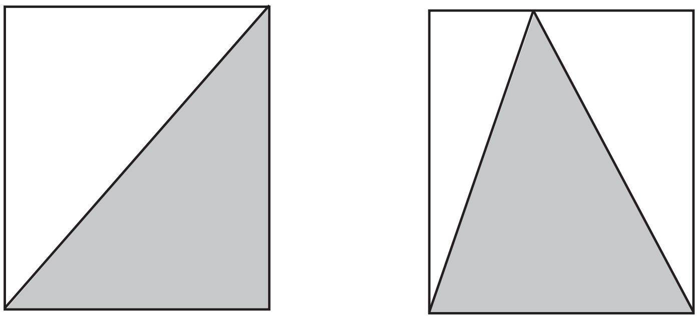


(h)

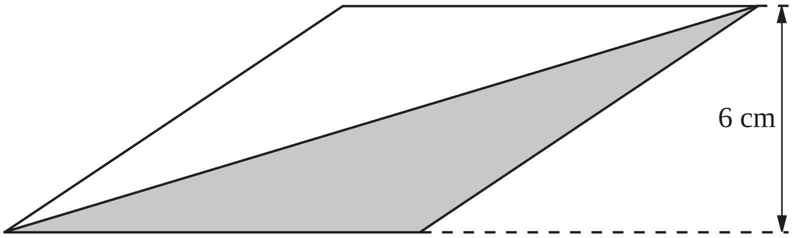


2. The diagrams show two triangles drawn inside a rectangle.

   Explain why the two shaded triangles have the same area.

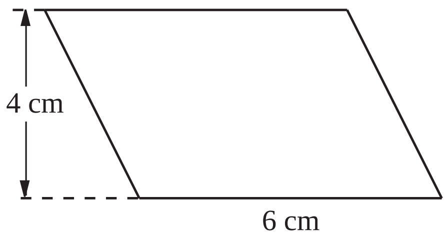


3. (a) Find the area of the shaded triangle.



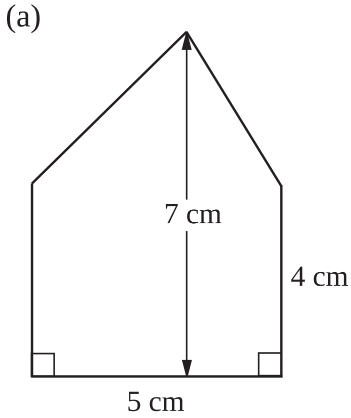
   (b) What is the area of the parallelogram?

4. Find the area of this parallelogram.

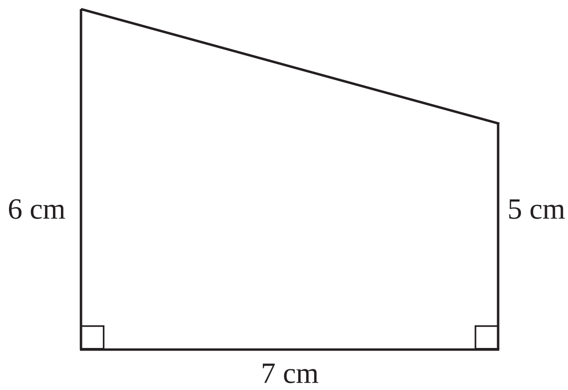


5. Find the area of each of these shapes. They have not been drawn accurately.

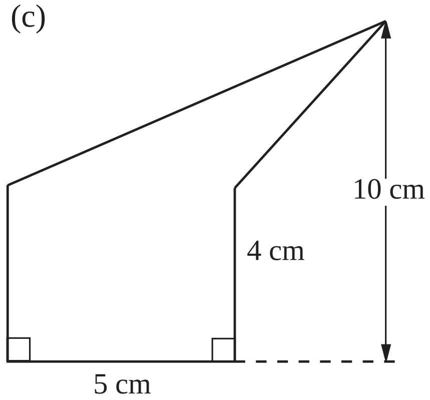
(a)



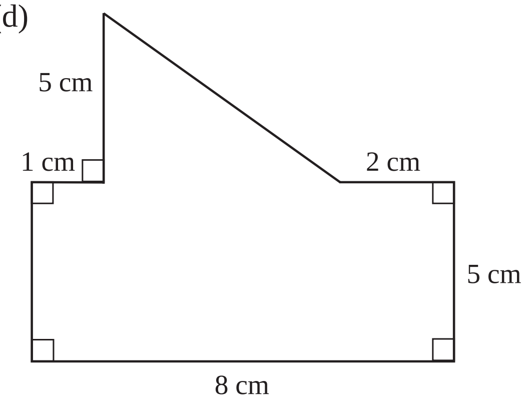
(b)



(c)

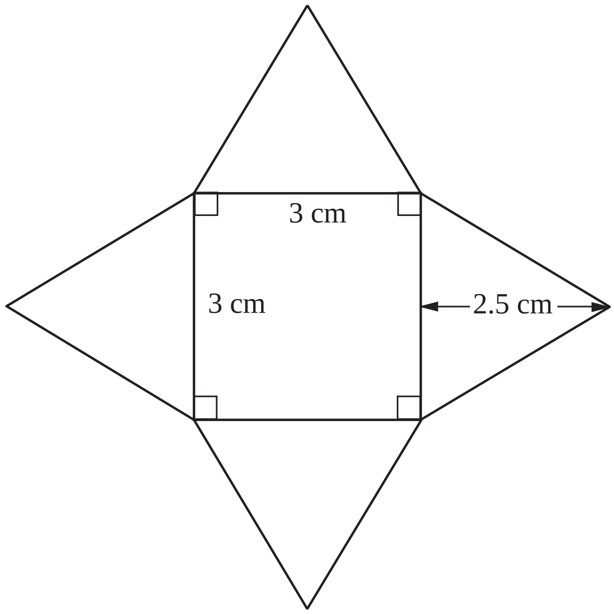


(d)



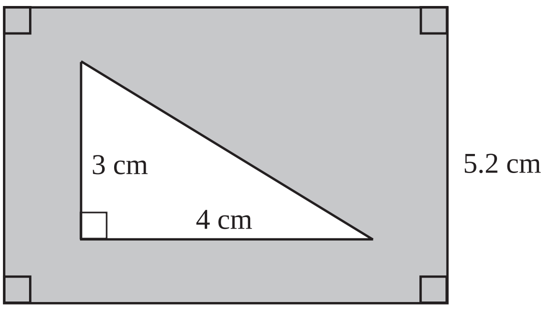
6. A pyramid can be made by folding up the shape below.

   What is the area of this shape?

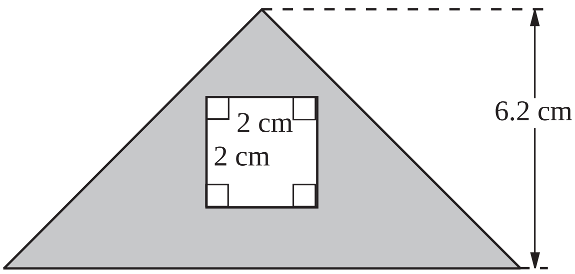


7. Find the shaded areas.

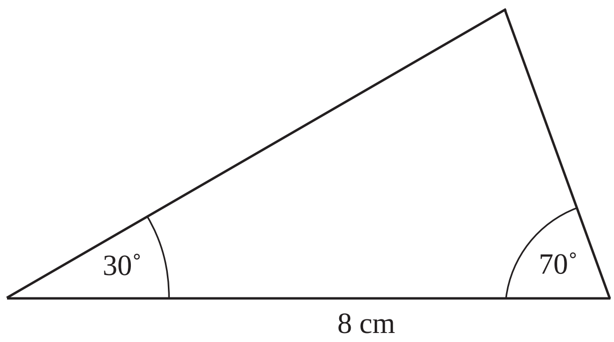
(a)



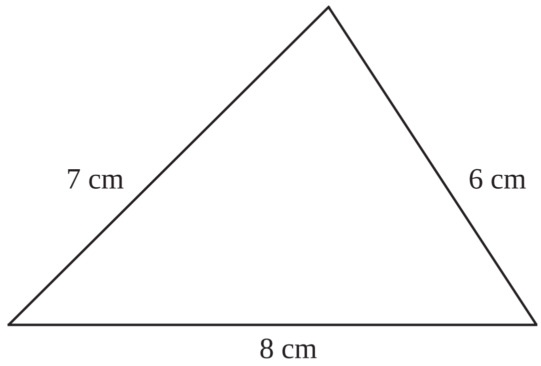
(b)



8. Draw this triangle. Find its area to the nearest 0.1 cm^2.



9. Find the area of this triangle to the nearest 0.1 cm^2.



10. Find the area of an equilateral triangle with sides of length 4 cm, giving your answer correct to 1 decimal place.

11. What is the area of the biggest triangle that can be drawn inside a parallelogram with sides of length 10 cm and 12 cm?

10 Arithmetic: Fractions

10.1 Fractions

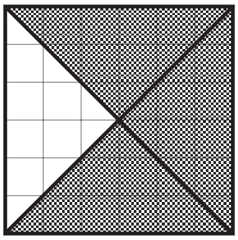
This unit deals with the important topic of fractions. These are numbers of the form $\dfrac{a}{b}$ when a and b are whole numbers and $b \neq 0$.

We first see what is meant by a fraction when related to geometrical shapes.

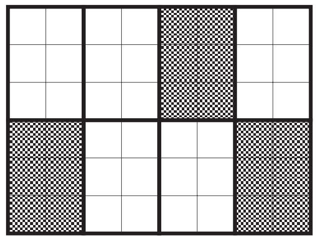
Example

What fraction of each shape is shaded?

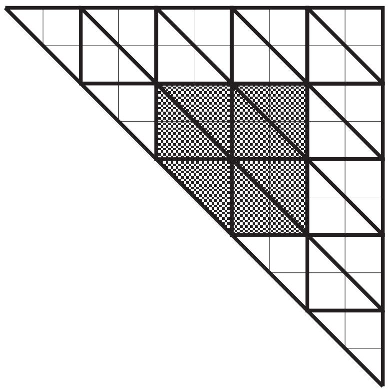
(a)



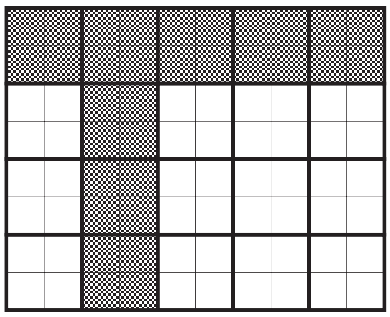
(b)



(c)



(d)



Solution

(a) $\dfrac{3}{4}$

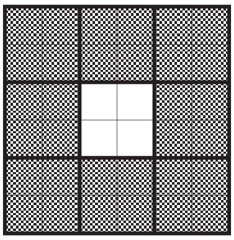
(b) $\dfrac{3}{8}$

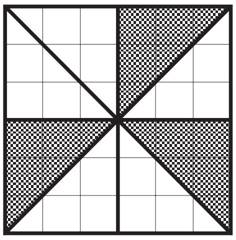
(c) $\dfrac{7}{25}$ (as there are 7 shaded triangles and 25 in total)

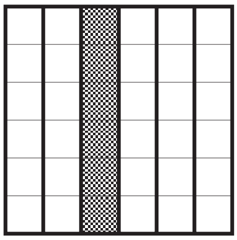
(d) $\dfrac{8}{20}$, but this is the same as $\dfrac{4}{10}$ or $\dfrac{2}{5}$.

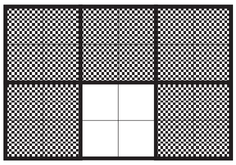
Exercises

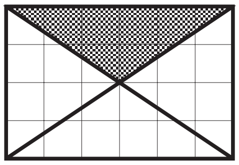
1. What fraction of each of the following shapes is shaded?

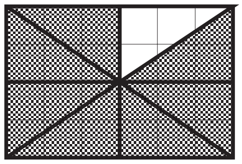
(a) 

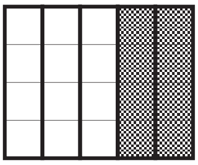
(b) 

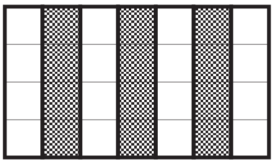
(c) 

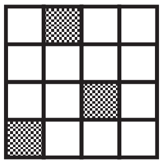
(d) 

(e) 

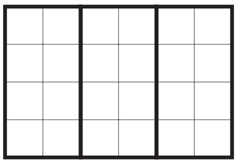
(f) 

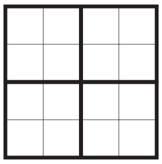
(g) 

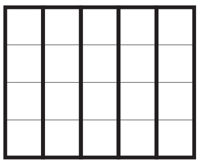
(h) 

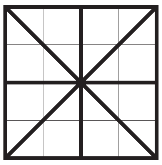
(i) 

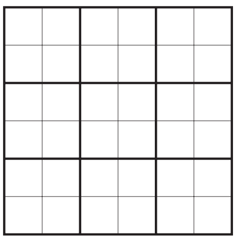
2. Copy each of these shapes and shade the fraction stated.

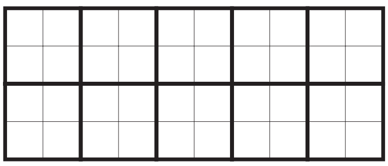
(a)  $\frac{1}{3}$

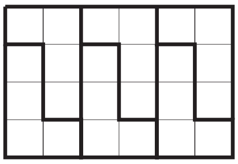
(b)  $\frac{3}{4}$

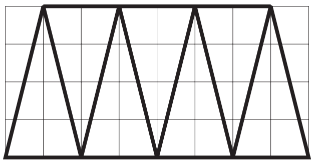
(c)  $\frac{3}{5}$

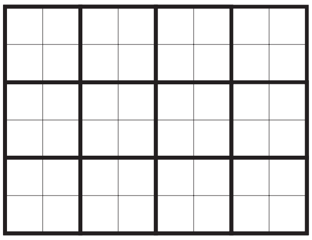
(d)  $\frac{5}{8}$

(e)  $\frac{7}{9}$

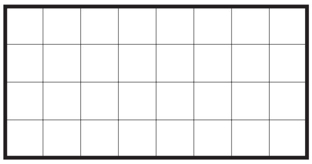
(f)  $\frac{3}{10}$

(g)  $\frac{5}{6}$

(h)  $\frac{5}{7}$

(i)  $\frac{5}{12}$

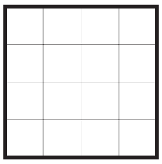
3. (a) On a copy of this rectangle, draw lines to divide it into 8 equal parts.

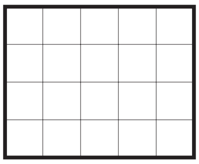


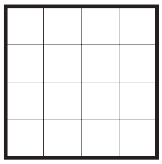
(b) Shade $\frac{1}{8}$ of the rectangle.

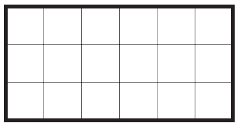
(c) What fraction has *not* been shaded?

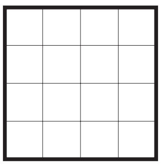
4. Copy each shape and shade the fraction stated.

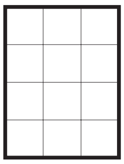
(a)  $\frac{1}{4}$

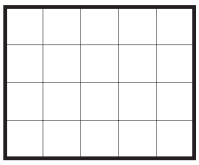
(b)  $\frac{4}{5}$

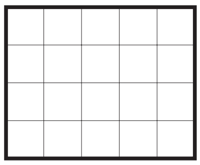
(c)  $\frac{7}{8}$

(d)  $\frac{2}{3}$

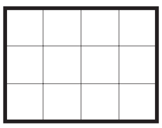
(e)  $\frac{7}{16}$

(f)  $\frac{9}{12}$

(g)  $\frac{7}{10}$

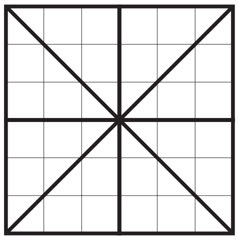
(h)  $\frac{13}{20}$

5. On separate diagrams, shade the stated fraction of this shape.



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$ (e) $\frac{1}{12}$

6. (a) Copy this shape.



(b) Shade $\frac{3}{8}$ of the shape.

(c) Shade another $\frac{2}{8}$ of the shape.

(d) What is the total fraction now shaded?

(e) How much is left unshaded?

7. Sarah shades $\frac{3}{7}$ of a shape. What fraction of the shape is left unshaded?

8. A cake is divided into 12 equal parts. John eats $\frac{3}{12}$ of the cake and Kate eats another $\frac{1}{12}$. What fraction of the cake is left?

9. A car park contains 20 spaces. There are 17 cars parked in the car park.

(a) What fraction of the car park is full?

(b) What fraction of the car park is empty?

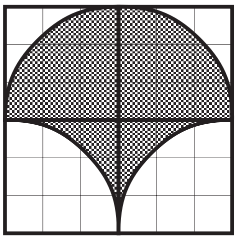
10. Ali eats $\frac{3}{10}$ of the sweets in a packet.

Tariq eats another $\frac{4}{10}$ of the sweets.

(a) What fraction of the sweets has been eaten?

(b) What fraction of the sweets is left?

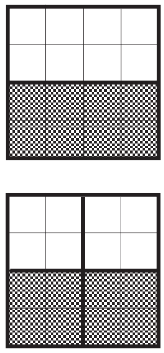
11. Draw as many ways as you can of shading $\frac{1}{2}$ of a shape.



Be as imaginative as you can.

Here is an idea to get you thinking.

10.2 Equivalent Fractions

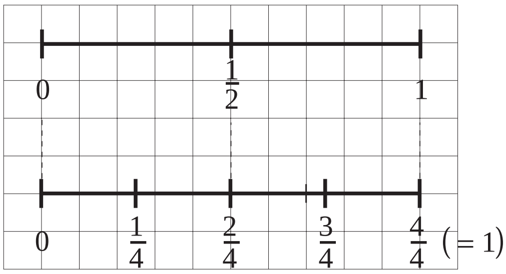


It is easy to see from the diagram opposite that

$$\frac{1}{2} \text{ and } \frac{2}{4}$$

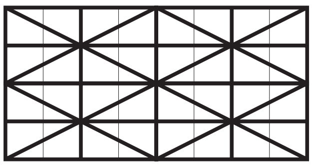
are *equivalent fractions*, i.e. they both have the same value.

Another way of looking at this is to note that the fractions are at the same place on a number line.



Example

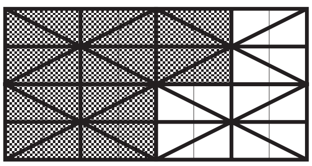
Use the diagram below to shade $\frac{5}{8}$ of the rectangle in different ways.



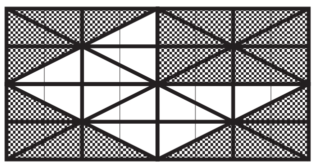
Solution

Here are two possible answers.

(a)



(b)

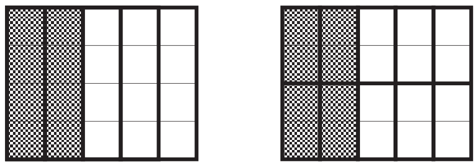


In fact, shading any 20 of the small triangles (out of 32) gives the fraction $\frac{5}{8}$

(which is equivalent to $\frac{10}{16}$ or $\frac{20}{32}$, etc.)

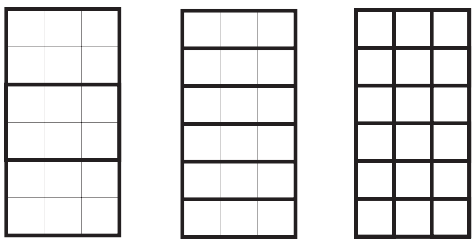
Exercises

1. The diagrams show that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$.

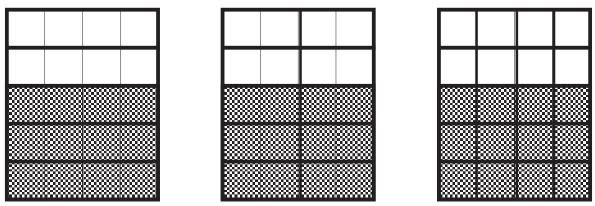


Draw a diagram to show that $\frac{2}{5}$ is also equivalent to $\frac{8}{20}$.

2. Use the diagrams below to show that $\frac{2}{3} = \frac{4}{6} = \frac{12}{18}$.



3. Write down the equivalent fractions shown by these diagrams.



4. Use the diagram on the next page to fill in the missing numbers.

(a) $\frac{1}{2} = \frac{?}{4} = \frac{?}{6} = \frac{?}{8} = \frac{?}{10} = \frac{?}{12} = \frac{?}{14} = \frac{?}{16}$

(b) $\frac{1}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12} = \frac{?}{15}$

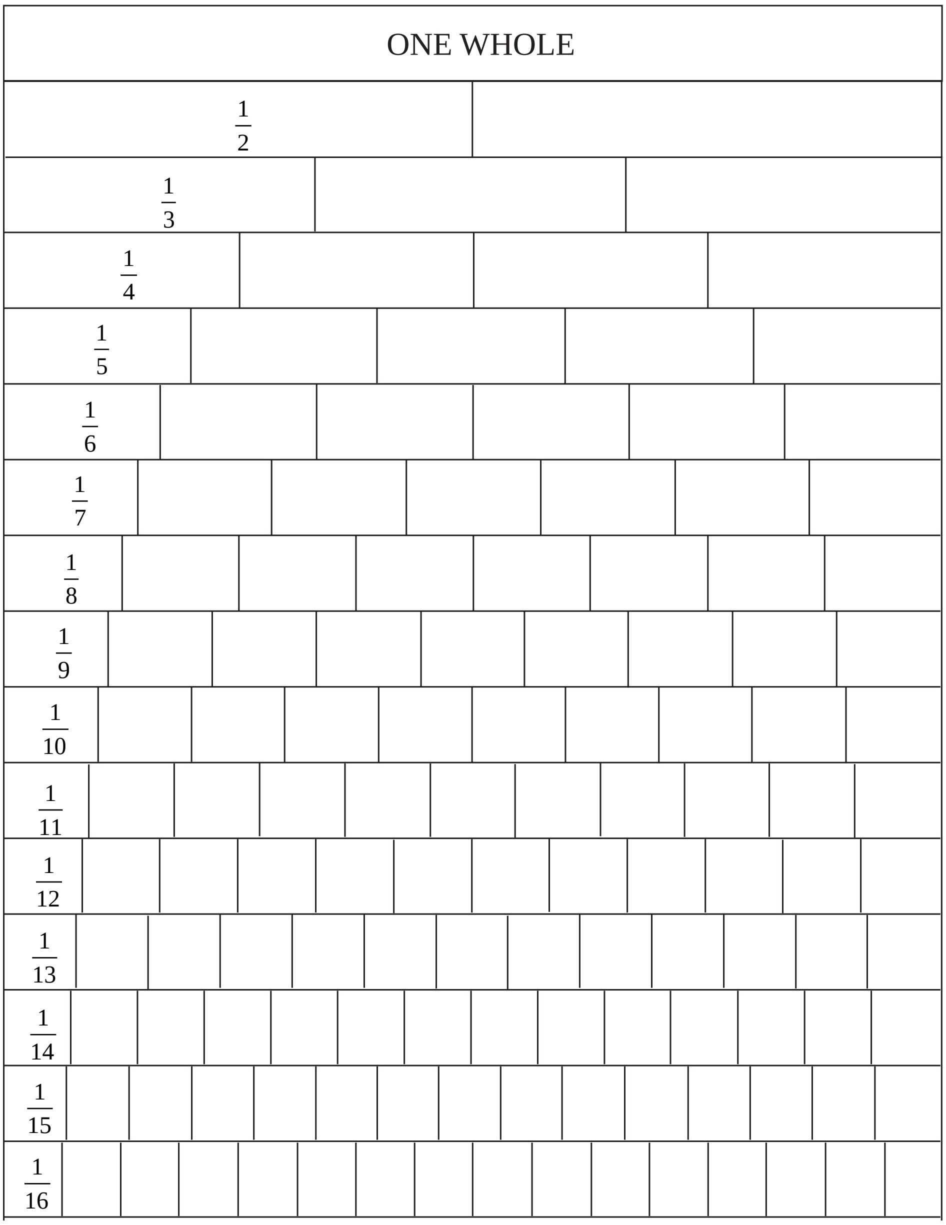
(c) $\frac{1}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16}$

(d) $\frac{1}{5} = \frac{?}{10} = \frac{?}{15}$

(e) $\frac{1}{6} = \frac{?}{12}$

(f) $\dfrac{1}{7} = \dfrac{?}{14}$

(g) $\dfrac{1}{8} = \dfrac{?}{16}$



5. Write down the missing numbers.

(a) $\dfrac{3}{4} = \dfrac{?}{8}$ (b) $\dfrac{2}{5} = \dfrac{?}{10}$ (c) $\dfrac{5}{7} = \dfrac{?}{14}$

(d) $\dfrac{3}{4} = \dfrac{?}{12}$ (e) $\dfrac{2}{5} = \dfrac{?}{15}$ (f) $\dfrac{2}{3} = \dfrac{?}{9}$

(g) $\dfrac{3}{8} = \dfrac{?}{16}$ (h) $\dfrac{2}{7} = \dfrac{?}{14}$ (i) $\dfrac{2}{3} = \dfrac{?}{12}$

(j) $\frac{3}{4} = \frac{?}{20}$ (k) $\frac{3}{5} = \frac{?}{20}$ (l) $\frac{5}{7} = \frac{?}{21}$

6. Write out each of these pairs of fractions, inserting either a $<$ or a $>$ into each one to make it correct.

(a) $\frac{1}{2} \quad \frac{1}{3}$ (b) $\frac{1}{4} \quad \frac{1}{5}$ (c) $\frac{1}{6} \quad \frac{1}{7}$

(d) $\frac{1}{10} \quad \frac{1}{9}$ (e) $\frac{1}{2} \quad \frac{2}{3}$ (f) $\frac{3}{4} \quad \frac{2}{3}$

(g) $\frac{2}{5} \quad \frac{1}{2}$ (h) $\frac{7}{10} \quad \frac{7}{8}$ (i) $\frac{5}{7} \quad \frac{3}{5}$

(j) $\frac{5}{6} \quad \frac{5}{7}$ (k) $\frac{2}{3} \quad \frac{5}{7}$ (l) $\frac{4}{5} \quad \frac{5}{6}$

7. Write down the missing numbers.

(a) $\frac{15}{30} = \frac{?}{2}$ (b) $\frac{6}{9} = \frac{?}{3}$ (c) $\frac{9}{12} = \frac{?}{4}$

(d) $\frac{3}{12} = \frac{?}{4}$ (e) $\frac{8}{18} = \frac{?}{9}$ (f) $\frac{16}{40} = \frac{?}{5}$

(g) $\frac{30}{50} = \frac{?}{5}$ (h) $\frac{14}{21} = \frac{?}{3}$ (i) $\frac{16}{24} = \frac{?}{3}$

(j) $\frac{17}{51} = \frac{?}{3}$ (k) $\frac{144}{200} = \frac{?}{25}$ (l) $\frac{132}{216} = \frac{?}{18}$

8. Write each set of fractions in *increasing* order.

(a) $\frac{1}{7}, \quad \frac{1}{9}, \quad \frac{1}{3}, \quad \frac{1}{10}, \quad \frac{1}{4}$

(b) $\frac{2}{3}, \quad \frac{3}{5}, \quad \frac{4}{7}, \quad \frac{2}{9}$

(c) $\frac{3}{5}, \quad \frac{2}{7}, \quad \frac{5}{6}, \quad \frac{4}{9}$

(d) $\frac{2}{5}, \quad \frac{3}{5}, \quad \frac{2}{7}, \quad \frac{3}{7}, \quad \frac{5}{7}$

(e) $\frac{3}{7}, \quad \frac{1}{9}, \quad \frac{5}{7}, \quad \frac{5}{9}, \quad \frac{7}{9}$

9. Write each fraction in its simplest form.

(a) $\frac{8}{24}$ (b) $\frac{15}{20}$ (c) $\frac{20}{25}$

(d) $\frac{9}{13}$ (e) $\frac{21}{28}$ (f) $\frac{40}{64}$

(g) $\frac{9}{36}$ (h) $\frac{80}{200}$ (i) $\frac{132}{150}$

10. State whether each of the following is *true* or *false*. If *false*, explain why.

(a) $\frac{3}{7} > \frac{3}{5}$ (b) $\frac{3}{8} = \frac{36}{88}$ (c) $\frac{11}{44} = \frac{1}{4}$

(d) $\frac{5}{8} > \frac{1}{2}$ (e) $\frac{3}{8} > \frac{1}{2}$ (f) $\frac{1}{6} < \frac{1}{7}$

(g) $\frac{8}{9} > \frac{7}{8}$ (h) $\frac{9}{10} < \frac{10}{11}$ (i) $\frac{44}{99} = \frac{4}{11}$

10.3 Fractions of Quantities

Now we start to use fractions in a practical way.

Example

(a) Find $\frac{1}{5}$ of £30. (b) Find $\frac{4}{5}$ of £30

Solution

You can, of course, do this practically, but it is much easier to work out

(a)
$$\frac{1}{5} \times £30 = £30 \div 5$$
$$= £\frac{30}{5}$$
$$= £6$$

(b)
$$\frac{1}{5} \times £30 = £6$$
$$\frac{4}{5} \times £30 = £\ 4 \times 6$$
$$= £24$$

Exercises

1. Find:

(a) $\frac{1}{2}$ of 12 (b) $\frac{1}{4}$ of 8 (c) $\frac{1}{5}$ of 15

(d) $\frac{1}{3}$ of 12 (e) $\frac{1}{5}$ of 30 (f) $\frac{1}{4}$ of 40

(g) $\frac{1}{7}$ of 14 (h) $\frac{1}{8}$ of 64 (i) $\frac{1}{8}$ of 40

(j) $\frac{1}{3}$ of 24 (k) $\frac{1}{4}$ of 32 (l) $\frac{1}{9}$ of 36

2. Find:

(a) $\frac{3}{4}$ of 24 (b) $\frac{4}{5}$ of 20 (c) $\frac{3}{7}$ of 14

(d) $\frac{2}{9}$ of 18 (e) $\frac{5}{6}$ of 30 (f) $\frac{4}{7}$ of 28

(g) $\frac{3}{5}$ of 15 (h) $\frac{7}{9}$ of 45 (i) $\frac{3}{8}$ of 64

(j) $\frac{5}{9}$ of 36 (k) $\frac{3}{5}$ of 45 (l) $\frac{7}{8}$ of 56

3. In a test there are 30 marks. Nasir gets $\frac{3}{5}$ of the marks. How many marks does he get?

4. In a school $\frac{1}{2}$ of the pupils are girls. There are 382 pupils in the school. How many girls are there in the school?

5. In a class there are 32 pupils. Of these, $\frac{3}{8}$ come to school by bus. How many pupils come to school by bus?

6. In a school, $\frac{3}{10}$ of the pupils have pets. There are 510 pupils in the school.

(a) How many of them have pets?

(b) How many of them do not have pets?

7. There are 20 houses in a street and $\frac{3}{4}$ of them have satellite TV. How many of these houses do not have satellite TV?

8. Rachel has 360 stamps in her stamp collection. $\frac{5}{8}$ of these stamps are foreign. How many foreign stamps has she got?

9. Ben and Chris sell their old toys at a car boot sale. They get £45. They agree that Ben will have $\frac{2}{5}$ of the money and Chris the rest. How much do they each get?

10. In a school there are 550 pupils. If $\frac{3}{50}$ of the pupils are left-handed, how many left-handed pupils are there in the school?

10.4 Mixed Numbers and Vulgar (Improper) Fractions

So far we have worked with fractions of the form $\frac{a}{b}$ where $a < b$,

e.g. $\frac{3}{4}, \frac{2}{7}, \frac{5}{6}, \ldots$

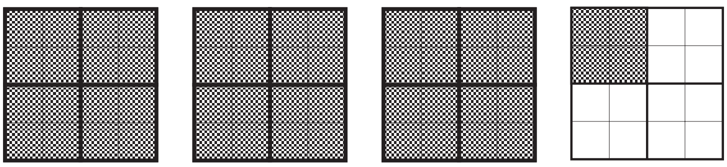
We also need to work with what are sometimes called *vulgar* or *improper* fractions, e.g. $\frac{5}{4}, \frac{7}{2}$, which are of the form $\frac{a}{b}$ when a and b are whole numbers and $a > b$.

Example

Show that the numbers $\frac{13}{4}$ and $3\frac{1}{4}$ are equivalent.

Solution

You can illustrate $3\frac{1}{4}$ as opposite.



If you count the quarters, you can see that there are $4 + 4 + 4 + 1 = 13$,

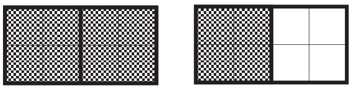
i.e. the number is $\frac{13}{4}$.

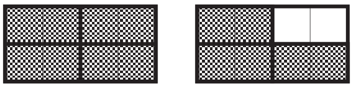
(You can see that $3\frac{1}{4} = \frac{(3 \times 4) + 1}{4} = \frac{13}{4}$.)

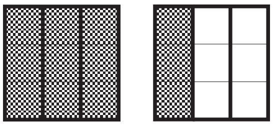
Note that $3\frac{1}{4}$ is called a *mixed* number.

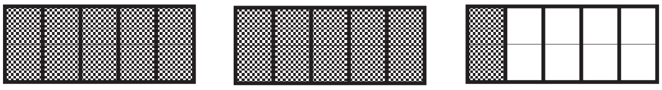
Exercises

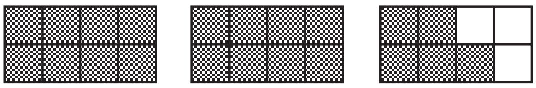
1. Write each of the numbers represented by these diagrams, in two different ways.

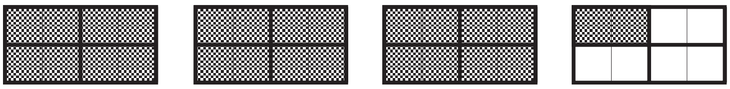
(a) 

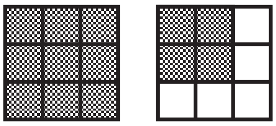
(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

2. Draw diagrams for these mixed numbers.

(a) $1\frac{1}{5}$ (b) $2\frac{1}{4}$ (c) $2\frac{2}{3}$

Write each number as an improper fraction.

3. Draw diagrams to show these improper fractions.

(a) $\frac{7}{2}$ (b) $\frac{8}{3}$ (c) $\frac{18}{5}$

Write each improper fraction as a mixed number.

4. Convert these improper fractions to mixed numbers.

(a) $\frac{9}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$

(d) $\frac{12}{5}$ (e) $\frac{18}{5}$ (f) $\frac{9}{7}$

(g) $\frac{11}{9}$ (h) $\frac{9}{4}$ (i) $\frac{8}{5}$

(j) $\frac{22}{9}$ (k) $\frac{6}{5}$ (l) $\frac{14}{5}$

(m) $\frac{13}{7}$ (n) $\frac{19}{7}$ (o) $\frac{20}{9}$

5. Convert these mixed numbers to improper fractions.

(a) $1\frac{3}{5}$ (b) $4\frac{1}{2}$ (c) $2\frac{1}{4}$

(d) $6\frac{1}{2}$ (e) $7\frac{1}{3}$ (f) $5\frac{2}{3}$

(g) $8\frac{3}{7}$ (h) $4\frac{2}{5}$ (i) $7\frac{1}{5}$

(j) $3\frac{5}{9}$ (k) $4\frac{3}{7}$ (l) $3\frac{4}{5}$

(m) $6\frac{1}{9}$ (n) $7\frac{2}{3}$ (o) $4\frac{7}{8}$

6. Write these fractions in order of increasing size.

$$6\frac{1}{2}, \quad \frac{18}{5}, \quad 3\frac{1}{4}, \quad 5\frac{1}{3}, \quad \frac{17}{3}$$

7. Write out each of these pairs of fractions, inserting either $<$, $>$ or $=$ into each one to make it correct.

(a) $2\frac{3}{5}$ $\frac{13}{5}$ (b) $3\frac{4}{7}$ $\frac{26}{7}$ (c) $3\frac{7}{8}$ 4

(d) $6\frac{1}{2}$ $\frac{22}{3}$ (e) $4\frac{1}{4}$ $\frac{19}{4}$ (f) $\frac{3}{2}$ $\frac{2}{3}$

(g) $\frac{2}{5}$ $\frac{1}{2}$ (h) $\frac{7}{10}$ $\frac{7}{8}$ (i) $\frac{5}{7}$ $\frac{3}{5}$

(j) $\frac{5}{6}$ $\frac{5}{7}$ (k) $\frac{2}{3}$ $\frac{5}{6}$ (l) $\frac{4}{5}$ $\frac{5}{6}$

8. Explain why $3\frac{5}{8} = \frac{58}{16}$.

9. In an office there are $2\frac{1}{2}$ packets of paper. There are 500 sheets of paper in each full packet. How many sheets of paper are there in the office?

10. A young child is 44 months old. Find the age of the baby in years as a mixed number in the simplest form.

11 Data Collection and Presentation

This unit deals with data - how we collect, organise and display it.

11.1 Types of Data

Qualitative data is data that is not given numerically;

e.g. favourite colour, place of birth, favourite food, type of car.

Quantitative data is numerical. There are two types of quantitative data.

Discrete data can only take specific numeric values;

e.g. shoe size, number of brothers, number of cars in a car park.

Continuous data can take any numerical value;

e.g. height, mass, length.

Example

The chart below gives information about the two finalists in the men's Wimbledon championship 1998.

Read through the information and answer these questions.

(a) Choose which of these terms

Qualitative data
Continuous Quantitative Data
Discrete Quantitative Data

best describes the following information.

(i) Age

(ii) Birthplace

(iii) Height

(iv) World Ranking

(v) Aces

(vi) First Serve Max Speed

(vii) Love Life

(b) Find another attribute that can be described as

(i) Qualitative data
(ii) Continuous Quantitative Data
(iii) Discrete Quantitative Data

Pete Sampras		**Goran Ivanisevic**
26	**Age**	26
Washington DC	**Birthplace**	Split, Croatia
Orlando, Florida	**Residence**	Monte Carlo
6 ft 1 in	**Height**	6 ft 4 in
170 lb	**Weight**	180 lb
$ 32,422,649	**Career Winnings**	$16,536,936
1	**World Ranking**	25
10	**Grand Slam Titles**	0
10	**Head to Head**	6
	Wimbledon 1998	
6	**Matches**	6
105	**Aces**	161
41	**Double Faults**	78
55%	**First Serve Percentage**	55%
89%	**First Serve Points Won**	87%
60%	**Second Serve Points Won**	52%
136 mph	**First Serve Max Speed**	128 mph
123 mph	**First Serve Average**	118 mph
126 mph	**Second Serve Max Speed**	116 mph
109 mph	**Second Serve Average**	104 mph

Lifestyle

Pete Sampras	Goran Ivanisevic
Car: A black Porsche Turbo S	**Car:** Does not drive in Monte Carlo.
Love life: His girlfriend is Kimberly Williams, a 26 year old actress who starred in Father of the Bride.	**Love life:** On the rocks. Has split up with girlfriend of five years.
Likes: Italian food, playing golf and flying in his private jet.	**Likes:** Italian food, playing golf and competitive football.
Coach: Former player Paul Annacone.	**Coach:** Has split with his long term coach Bob Brett and now travels with his good friend Vedran Martic.

Solution

(a) (i) Discrete quantitative, becauses it is given as a whole number.

(ii) Qualitative.

(iii) Continuous quantitative - it can take any value, although it is given here to the nearest inch.

(iv) Discrete quantitative - it can only take positive whole numbers.

(v) Discrete quantitative.

(vi) Continuous quantitative - although it should be noted that it is given here as a whole number.

(vii) Qualitative - definitely!

(b) (i) Coach

(ii) Weight

(iii) Grand Slam Titles

Exercises

1. Mr. Jenkin starts to make a database for his tutor group.

Name	*Age*	*Primary School*	*Transport to School*	*Height*	*Glasses*
Alice Ascott	11	St. Johns	Bus	145 cm	Yes
Ben Bray	12	At. Andrews	Walk	160 cm	No
Carol Cotton	12	Prince Hill	Car	161 cm	No
David Darby	12	Prince Hill		152 cm	No
Eddie English	11	St. Andrews	Walk	158 cm	Yes
Frederick Franks		St. Andrews	Bike	164 cm	No
Graham Gee	12	St. Johns	Bus	166 cm	Yes

(a) What is missing from Mr. Jenkin's data base?

(b) Which columns in the database contain quantitative data?

(c) Which columns in the database contain qualitative data?

(d) Write down what Mr. Jenkin would put in his database if you joined his class.

2. Which of the following would give:

(a) qualitative data

(b) discrete quantitative data

(c) continuous quantitative data?

(i) Mass

(ii) Number of cars

(iii) Favourite football team

(iv) Colour of car

(v) Price of chocolate bars

(vi) Amount of pocket money

(vii) Distance from home to school (viii) Number of pets

(ix) Number of sweets in a jar (x) Mass of crisps in a packet

3. A traffic survey records information about cars passing a check point. Some data is given in the table below.

Registration Year letter	*Colour*	*Speed*	*Number of Passengers*	*Trailer / Caravan*
K	Red	26 mph	1	No
L	Blue	47 mph	0	No
C	White	36 mph	4	No
D	Red	31 mph	3	No
J	Silver	33 mph	2	Yes
M	Green	29 mph	0	No
R	White	30 mph	1	Yes
P	Red	31 mph	3	No
N	Blue	42 mph	2	No
G	Grey	28 mph	2	No

(a) Explain why the *Number of Passengers* is discrete data.

(b) Explain why *Speed* is continuous data.

(c) Which columns contain qualitative data?

(d) How fast was the silver car travelling?

(e) How many cars were towing a trailer or caravan?

(f) What colour was the slowest car?

(g) How fast was the car with the most passengers travelling?

(h) What was the registration letter of the car with the highest speed?

4. The table below shows a database that has no entries.

Name	*Age*	*Favourite food*	*Favourite T.V show*	*Favourite pop group*	*Time spent watching T.V yesterday*		

(a) You can add headings to the last two columns.

(b) Collect data from 10 people to complete the database.

(c) State whether each column contains

(i) qualitative data;

(ii) continuous quantitative data;

or (iii) discrete quantitative data.

(d) Answer the following questions

(i) What is the most popular T.V show?

(ii) Who is the oldest?

(iii) What is the favourite pop group for the youngest person?

(e) Write 3 more questions you could answer using your database and write the answers to them.

11.2 Collecting Data

In this section, we will see how data is collected and organised, using a tally chart and then displayed, using

- pictograms
- bar charts
- pie charts

Note

An *hypothesis* is an idea that you want to investigate to see if it is true or false. For example, you might think that most people in your school get there by bus. You could investigate this using a survey. A tally chart can be used to record your data.

Example

The pupils in a class were asked how they got to school.

Method of Travel	*Tally*	*Frequency*
Walk	~~||||~~ ||||	9
Bike	|||	3
Car	~~||||~~ |	6
Bus	~~||||~~ ~~||||~~ ||	12
	TOTAL	30

Illustrate this data, using:

(a) a pictogram

(b) a bar chart

(c) a pie chart

What are the main conclusions that you can deduce from the data?

Solution

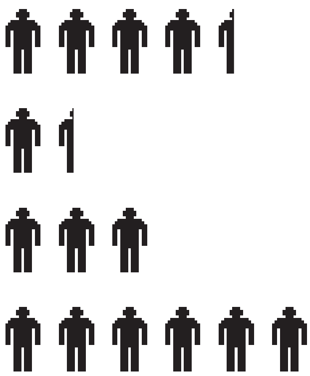
(a) If  is taken to represent 2 people, then the pictogram looks like

Walk

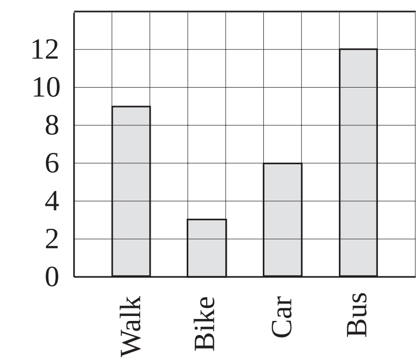
Bike

Car

Bus



(b) A bar chart for the data is illustrated below.



(Note the gaps between the bars. There should be gaps for qualitative data and discrete quantitative data.)

(c) To illustrate the data with a pie chart, you need to find out what angle is equivalent to one pupil. Since there are $360°$ in a circle and 30 pupils,

$$\text{angle per pupil} = \frac{360}{30} = 12°$$

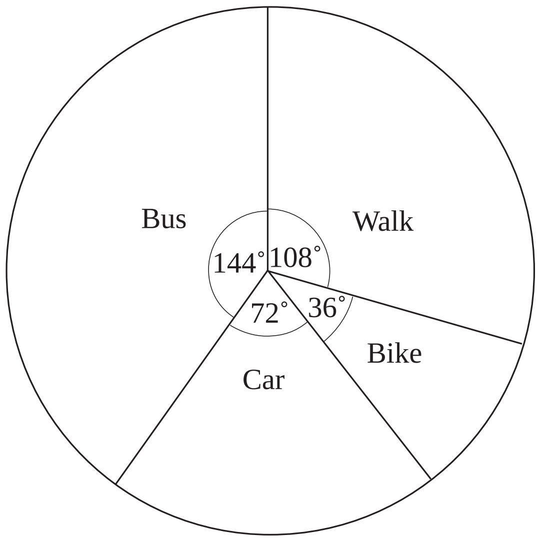
To find the angle for walk, when there are 9 pupils, it is simply

$$9 \times 12 = 108°.$$

The complete calculations are shown below:

Method of Travel	*Frequency*	*Calculation*	*Angle*
Walk	9	$9 \times \dfrac{360}{30} =$	108°
Bike	3	$3 \times \dfrac{360}{30} =$	36°
Car	6	$6 \times \dfrac{360}{30} =$	72°
Bus	12	$12 \times \dfrac{360}{30} =$	144°
		TOTAL	360°

The corresponding pie chart is shown below:



From the data we can see that

- the most common way of getting to school is by bus. (this is called the *modal class* or the *mode*)
- the least popular way of getting to school is by bike.

Exercises

1. The children in a class were asked to state their favourite crisps.

   The results are given in the tally chart below:

Flavour	*Tally*	*Frequency*
Ready Salted	𝍸	
Salt and Vinegar	𝍸 𝍷𝍷𝍷𝍷	
Cheese and Onion	𝍸 𝍷𝍷	
Prawn Cocktail	𝍷𝍷𝍷	
Smokey Bacon	𝍸 𝍷	
	TOTAL	

(a) Copy and complete the table.

(b) Represent the data on a bar chart.

(c) Draw a pictogram for this data.

(d) Copy and complete the following table and draw a pie chart.

Flavour	*Frequency*	*Calculation*	*Angle*
Ready Salted	5	$\frac{5}{30} \times 360° =$	60°
		TOTAL	

(e) What flavour is the mode?

2. (a) Do you think salt and vinegar crisps will be the most popular crisps in your class?

(b) Carry out a favourite crisps survey for your class. Present the results in a bar chart and state which flavour is the mode.

(c) Was your hypothesis in (a) correct?

3. *"More children in my class travel to school by bus than by any other method."*

   (a) Collect data to test this hypothesis.

   (b) Present your data in a suitable diagram.

   (c) Was the original hypothesis correct?

4. Is the pop group that is most popular with the boys in your class the same as the pop group that is most popular with the girls?

   (a) Write down a hypothesis that will enable you to answer this question.

   (b) Collect suitable data from your class.

   (c) Present your data using a suitable diagram.

   (d) Was the hypothesis correct?

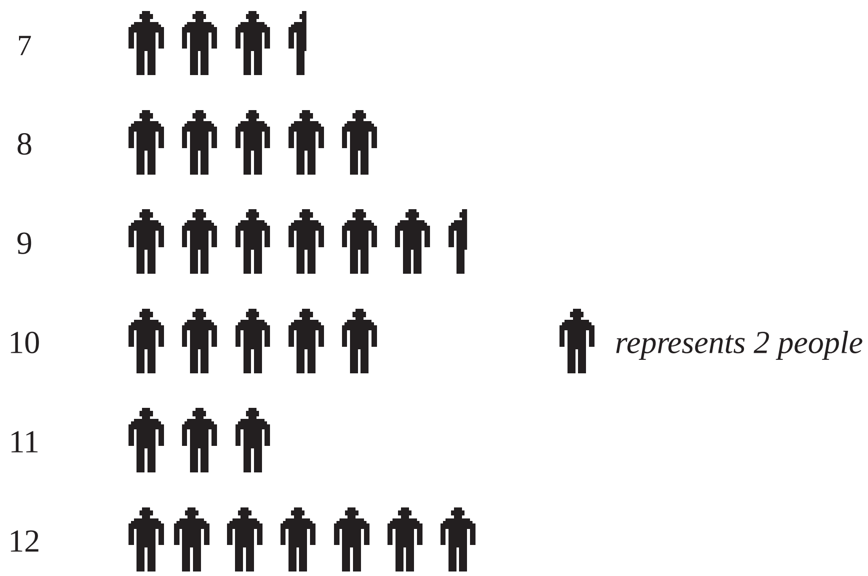
5. (a) State a hypothesis about one of the following for your class.

   Favourite football team
   Favourite pop group
   Favourite T.V soap opera
   Favourite cartoon character

   (b) Collect data for your class and display it using suitable diagrams.

   (c) Was your hypothesis correct?

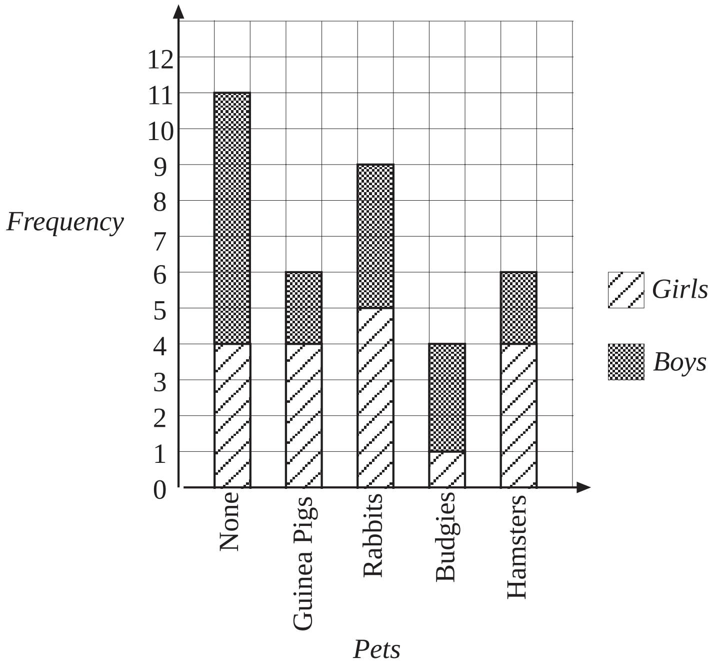
6. The ages of the children that belong to a junior tennis club are illustrated in the pictogram.



(a) What is the modal age?

(b) Draw a pie chart to illustrate this information.

7. The bar chart gives information about the pets owned by the children that live in one road.



Answer the following questions:

(a) How many girls do not have a pet?

(b) How many children own hamsters?

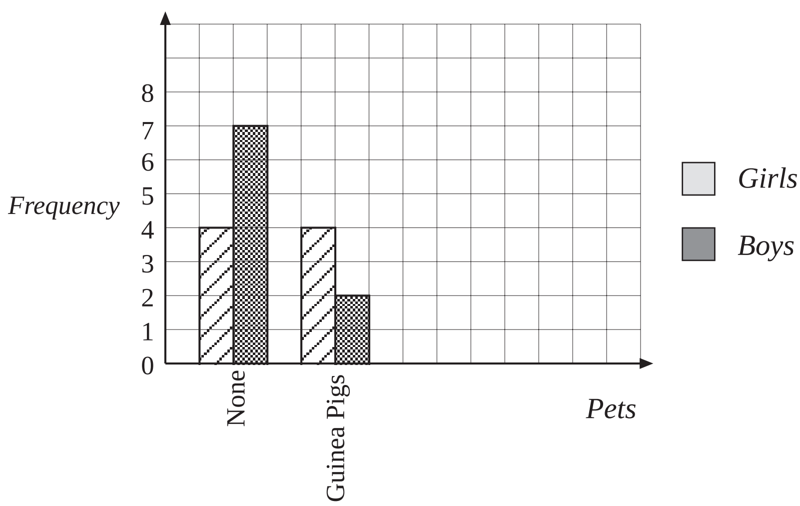
(c) Are the hamsters more popular with girls or boys?

(d) How many girls have rabbits?

(e) What is the most popular pet with the boys?

(f) What is the most popular pet with the girls?

Another way of drawing the same bar chart has been started below. Copy and complete this chart.

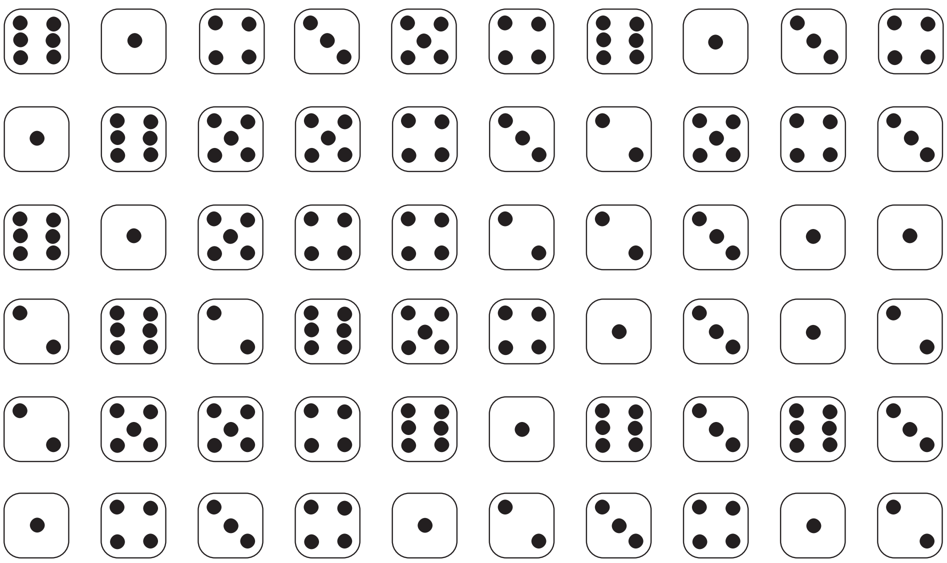


8. Draw a bar chart to illustrate the following data on the favourite colours of a group of children.

	Girls	*Boys*
Yellow	𝍸 ||	𝍸 ||
Red	𝍸	𝍸 |||
Black	|	||||
Purple	||||	||
Green	|||	𝍸 ||
Blue	||||	𝍸 |
Pink	𝍸 |	

9. Malcolm thinks that the dice in his Monopoly set is unfair because he never gets a 6 when he wants one. He decides to test the dice and rolls it 60 times.

The diagram shows what happened.



(a) Show his results on a diagram.

(b) Do you think his dice is fair?

10. Carry out your own experiment with a dice like Malcolm did in question 9. Do you think that your dice is fair?

12 Arithmetic: Revision

In this Arithmetic Unit, you will gain confidence with numerical calculations, particularly for calculations with whole numbers and with decimals. You will revise how to use arithmetic in everyday contexts, particularly with money calculations.

12.1 Arithmetic with Whole Numbers and Decimals

First we revise strategies for calculating answers to numerical problems.

Example

Calculate:

(a) $3.4 + 4.75$

(b) 49×10

(c) 47.3×100

(d) $52 \div 10$

(e) $7.41 \div 100$

(f) 3.6×4

(g) $909 \div 3$

(h) $10.4 \div 1.3$

Solutions

(a) You can write this as
$$\begin{array}{r} 3.40 \\ +\ 4.75 \\ \hline 8.15 \\ \hline \end{array}$$
(remember to keep the decimal points lined up)

(b) $49 \times 10 = 490$

(c) $47.3 \times 100 = 4730$ (since $47.3 \times 10 = 473$, etc.)

(d) $52 \div 10 = 5.2$ (since $5.2 \times 10 = 52$)

(e) $7.41 \div 100 = 0.0741$ (since $0.0741 \times 100 = 7.41$)

(f) 3.6×4 can be written as
$$\begin{array}{r} 3.6 \\ \times\ \ 4 \\ \hline 14.4 \\ \hline \end{array}$$
i.e. $3.6 \times 4 = 14.4$

(g) $909 \div 3 = 303$

(h) $10.4 \div 1.3 = \dfrac{10.4}{1.3}$ (multiplying top and bottom by 10)

$= \dfrac{104}{13}$

$= 8$

Exercises

1. Find the solution to each of these calculations:

(a) $\begin{array}{r} 124 \\ +\ \ 32 \\ \hline \\ \hline \end{array}$ (b) $\begin{array}{r} 1047 \\ +\ 189 \\ \hline \\ \hline \end{array}$ (c) $\begin{array}{r} 3.24 \\ +\ 5.63 \\ \hline \\ \hline \end{array}$

(d) $\begin{array}{r} 18.7 \\ -\ \ 2.6 \\ \hline \\ \hline \end{array}$ (e) $\begin{array}{r} 1627 \\ -\ \ 315 \\ \hline \\ \hline \end{array}$ (f) $\begin{array}{r} 1742 \\ -\ \ 351 \\ \hline \\ \hline \end{array}$

(g)	$37 + 120$	(h)	$157 + 36$	(i)	$4.72 + 3.6$
(j)	$6.4 + 8.21$	(k)	$3.56 + 8.24$	(l)	$6.3 + 8.71$
(m)	$16.4 + 3.2$	(n)	$18.8 - 7.3$	(o)	$17.4 - 8.25$

2. Find the solution to each of the following;

(a)	37×10	(b)	4.71×10	(c)	8.62×10
(d)	57×100	(e)	8.71×100	(f)	8.2×1000
(g)	$117 \div 10$	(h)	$84 \div 10$	(i)	$18.92 \div 10$
(j)	$84 \div 100$	(k)	$8.72 \div 1000$	(l)	$0.421 \div 10$
(m)	8.201×1000	(n)	$52.3 \div 1000$	(o)	$18.62 \div 10$

3. Find the solution to each of the following:

(a)	82×4	(b)	8.7×3	(c)	5.2×2
(d)	64.7×7	(e)	3.8×5	(f)	19.2×5
(g)	16.4×8	(h)	3.21×7	(i)	8.47×5
(j)	3.61×0.4	(k)	5.7×0.8	(l)	4.2×0.9
(m)	6.3×0.02	(n)	8.42×0.3	(o)	9.71×0.02

4. Find the solution to each of the following:

(a) 88×12	(b) 42×17	(c) 56×14
(d) 42×21	(e) 37×31	(f) 84×23
(g) 4.2×8	(h) 32×1.7	(i) 84×1.4
(j) 3.2×2.4	(k) 8.7×0.5	(l) 5.4×9.2
(m) 1.26×3.2	(n) 142×0.51	(o) 3.21×4.2

5. Find the solution to each of the following:

(a) $18 \div 3$	(b) $24 \div 2$	(c) $369 \div 3$
(d) $848 \div 4$	(e) $738 \div 6$	(f) $924 \div 4$
(g) $1332 \div 12$	(h) $1107 \div 9$	(i) $4344 \div 8$
(j) $4860 \div 15$	(k) $5304 \div 17$	(l) $11\,277 \div 21$
(m) $924 \div 11$	(n) $10.44 \div 12$	(o) $63.14 \div 14$
(p) $17.28 \div 1.2$	(q) $25.2 \div 2.1$	(r) $9.63 \div 4.5$

12.2 Problems with Arithmetic

Here numerical calculations are put into practical contexts, particularly involving money.

Example 1

Sarah buys 8 ice creams costing 95p each. How much does she spend?

Solution

The answer is 8×95p

$= 760$p

$= £7.60$

Example 2

It takes Paul 22 minutes to wash a car. How many cars can he wash in 1 hour and 50 minutes?

Solution

The answer is given by the number of times that 22 minutes goes into 1 hour and 50 minutes (or 110 minutes).

i.e. $110 \div 22 = \dfrac{110}{22}$ (divide top and bottom by 11)

$= \dfrac{10}{2}$

$= 5$

Exercises

1 Sally is given £40 for her birthday. She spends £28.95 on a pair of roller-blades. How much money does she have left?

2. Clive joins a music club. He has to buy 4 CDs costing £13.99 each. How much will they cost altogether?

3. A shopkeeper buys 40 cyber-pets at £5.20 each and sells them for £8.00 each. How much profit does she make altogether?

4. Chocolate bars cost 35p each from a machine. At the end of the day there is £8.05 in the machine. How many chocolate bars have been sold?

5. Charlie loves 'Chocnut Bars'. They cost 31p each. She buys 7 bars. How much do they cost in total? How much change does she get from a £5 note?

6. Ben and his family are going camping. The camp site fee for their caravan is £18.50 per night. How much are the fees for the caravan for 14 nights?

7. Elizabeth needs 3.4 metres of material to make her costume for the school play. The material costs £3.20 per metre. How much does Elizabeth have to pay for her material?

8. Tickets for a school talent show cost £1.20 each. The total amount paid for tickets is £241.20. How many tickets were sold?

9. Tariq raised £26.70 on a 15-mile sponsored walk. How much was he sponsored for each mile?

10. Jenny makes some ginger beer. She fills 21 bottles each with 550 ml of ginger beer, and has 360 ml left over. How much ginger beer did she make altogether? (Try writing your answer in litres as well as millilitres.)

13 Searching for Pattern

13.1 Pictorial Logic

In this section we will see how to continue patterns involving simple shapes.

Example

Continue these patterns by drawing the next 5 shapes in each case:

(a) 

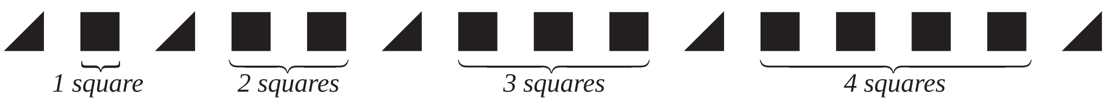
(b) 

Solution

(a) This pattern consists of four shapes that repeat in the same order. The repeating pattern is:

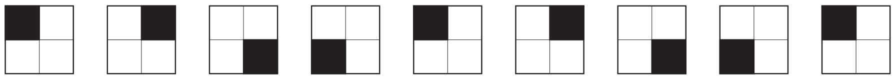
The pattern can now be extended:

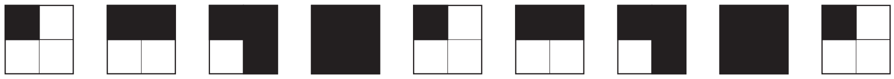
(b) This pattern consists of increasing numbers of squares separated by triangles. The pattern can be extended by adding 4 squares and another triangle:

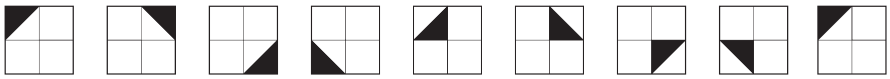


Exercises

1. Add the next 5 shapes to each of the repeating patterns below:

(a) 

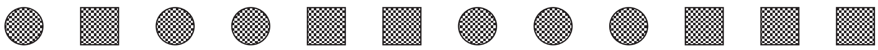
(b) 

(c) 

(d) 

2. Add the next 5 shapes to each of these patterns:

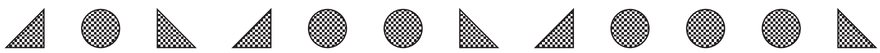
(a) 

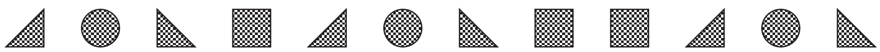
(b) 

(c) 

(d) 

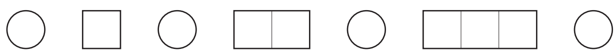
(e) 

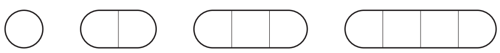
(f) 

(g) 

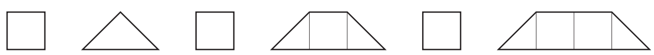
(h) 

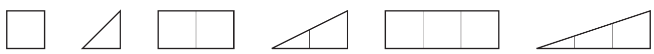
3. Extend each pattern until you obtain a shape that is 5 squares long:

(a) 

(b) 

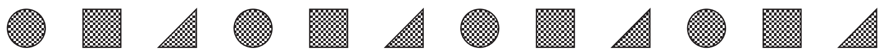
(c) 

(d) 

(e) 

(f) 

4. Consider the pattern of shapes shown below:



(a) What is the 3rd shape in the pattern?

(b) What is the 15th shape in the pattern?

(c) What is the 30th shape in the pattern?

(d) What is the 31st shape in the pattern?

5. Consider this pattern of shapes:

(a) Draw the 8th shape.

(b) Draw the 20th shape.

(c) Draw the 21st shape.

(d) Draw the 19th shape.

6. Consider this pattern of shapes:

(a) What is the 11th shape?

(b) What is the 21st shape?

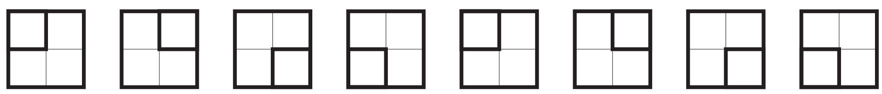
(c) What is the 41st shape?

(d) How long is the 4th shape?

(e) How long is the 6th shape?

(f) How long is the 20th shape?

7. Look at this pattern:



(a) Draw the 4th and 8th shapes.

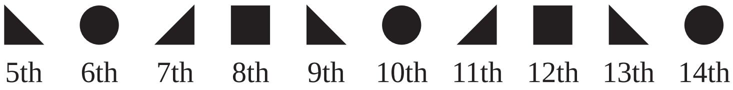
(b) Draw the 16th shape.

(c) Draw the 17th shape.

(d) Draw the 40th shape.

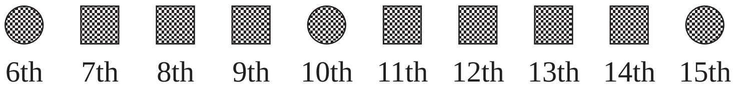
(e) Draw the 38th shape.

8. The diagram shows the 5th to 14th shapes in a pattern:



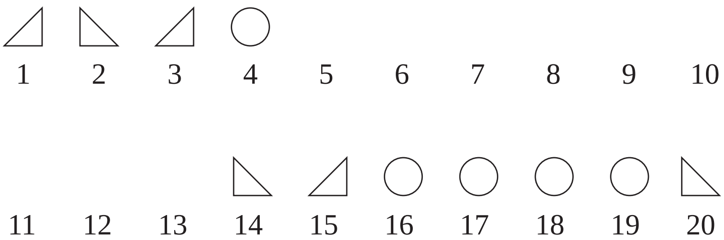
Draw and label the first 4 shapes in the correct order.

9. The diagram shows the 6th to 15th shapes of a pattern:



Draw and label the first 5 shapes of the pattern.

10. Fill in the missing shapes in this pattern. There should be one shape for each number.

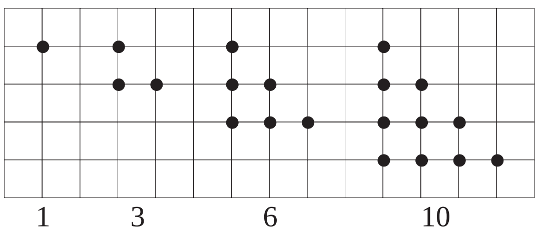


13.2 Extending Number Sequences

You will have studied some sequences in Unit 7. This section takes these ideas further and introduces some other types of sequences.

Example

The first 4 triangular numbers are represented by the diagrams below:

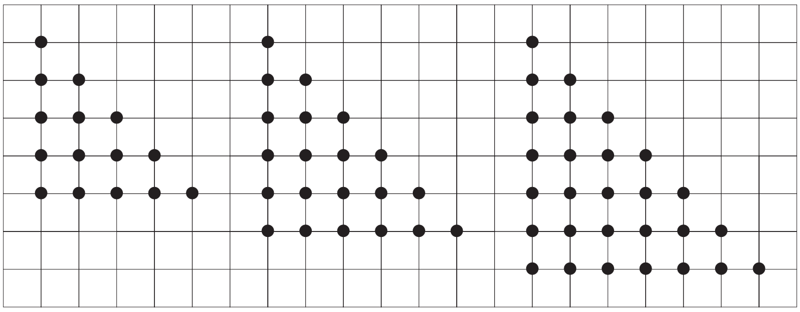


(a) Draw the next 3 triangular numbers.

(b) Describe how to find the 8th, 9th and 10th triangular numbers without drawing the diagrams.

Solution

(a) Note that an extra row of dots is added to each triangle and that the extra row has one more dot than the previous row. The next 3 triangular numbers are shown below:



15 21 28

(b) To extend the sequence of triangular numbers, look at the difference between the terms:

1 3 6 10 15 21 28

2 3 4 5 6 7

Note that the difference between each term increases by 1 as you move along the sequence.

So,

8th term $= 28 + 8$
$= 36$

9th term $= 36 + 9$
$= 45$

10th term $= 45 + 10$
$= 55$

Example

Write down the next 3 terms of the sequence:

3, 7, 10, 17, 27, 44, 71, . . .

Solution

Look at the differences between each term:

3 7 10 17 27 44 71

4 3 7 10 17 27

The first difference is not very helpful, but then note how the sequence of differences is the same as the original sequence.

For example, $10 + 7 = 17$

To find the next 5 terms:

$$\begin{aligned} \text{8th term} &= 71 + 44 \\ &= 115 \\ \text{9th term} &= 115 + 71 \\ &= 186 \\ \text{10th term} &= 186 + 115 \\ &= 301 \end{aligned}$$

In this type of sequence, called a *Fibonacci* sequence, each term is the sum of the two previous terms. For example, this sequence begins:

3, 7, 10 where $3 + 7 = 10$

and the next term is $10 + 7 = 17$.

Triangular Numbers	1, 3, 6, 10, 15, 21, 28, . . .
Square Numbers	1, 4, 9, 16, 25, 36, 49, . . .
Cubic Numbers	1, 8, 27, 64, 125, . . .
Fibonacci Sequence	1, 1, 2, 3, 5, 8, 13, . . .
(formed by adding the two previous terms to get the next one)	

Exercises

1. Write down the next 4 terms of each of these sequences:

   (a) 4, 7, 10, 13, 16, 19, . . .

   (b) 5, 11, 17, 23, 29, 35, . . .

   (c) 6, 8, 11, 15, 20, 26, . . .

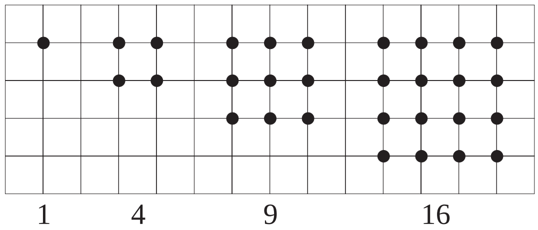
   (d) 8, 10, 14, 20, 28, 38, . . .

   (e) 24, 23, 21, 18, 14, 9, . . .

   (f) 2, 12, 21, 29, 36, 42, . . .

   (g) 1, 1, 2, 4, 7, 11, . . .

2. The diagram shows the first 4 square numbers:



(a) Draw the next 2 square numbers, and write their actual value underneath.

(b) What is the 10th square number?

(c) What is the 20th square number?

(d) Find the differences between each of the first 6 square numbers in turn. What would be the difference between the 6th and 7th square numbers? Check that your answer is correct by drawing the 7th square number.

3. (a) Write down the next 3 terms in each of these sequences:

(i) 0, 3, 8, 15, 24, . . .

(ii) 2, 5, 10, 17, 26, . . .

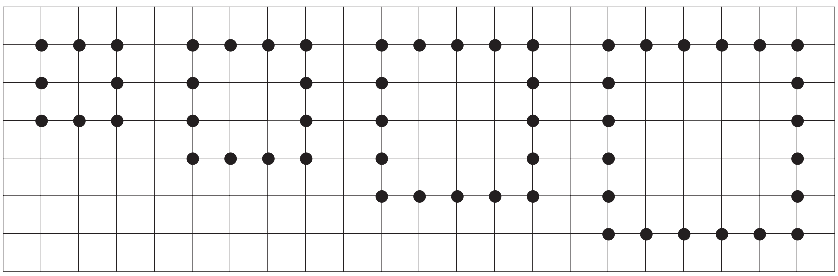
(iii) 11, 14, 19, 26, 35, . . .

(iv) 6, 9, 14, 21, 30, . . .

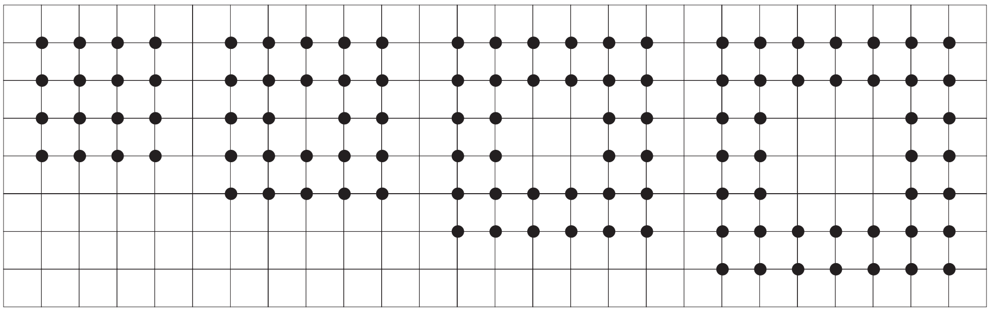
(b) In each case above, explain how the sequence is related to the sequence of square numbers 1, 4, 9, 16, 25, . . . , . . .

4. For each sequence below, draw the next two diagrams and write down the number of dots in each of the first 10 diagrams:

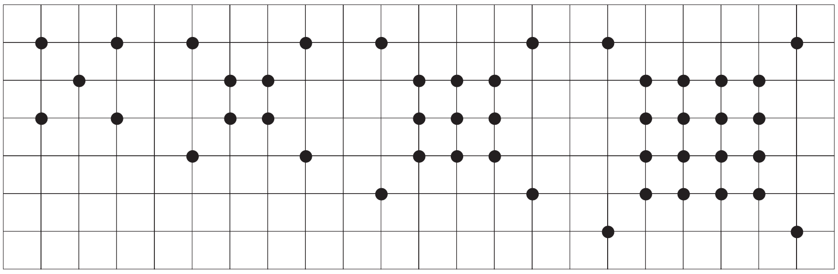
(a)



(b)

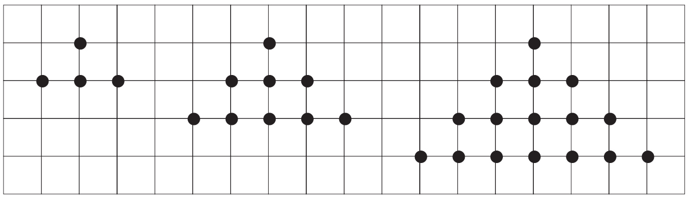


(c)

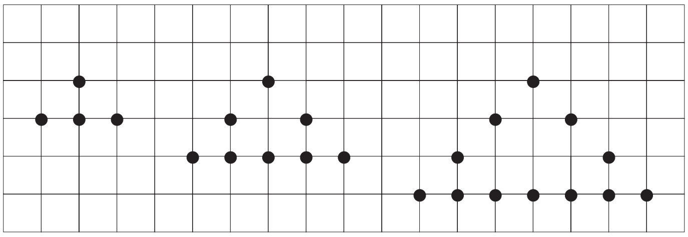


5. For each sequence below, draw the next 3 diagrams and write down the number represented by each of the first 8 diagrams:

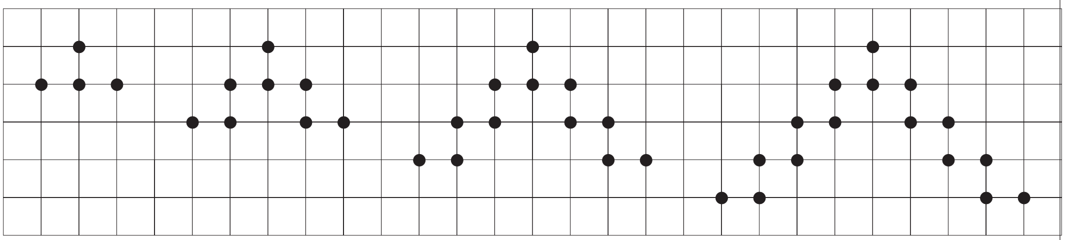
(a)



(b)

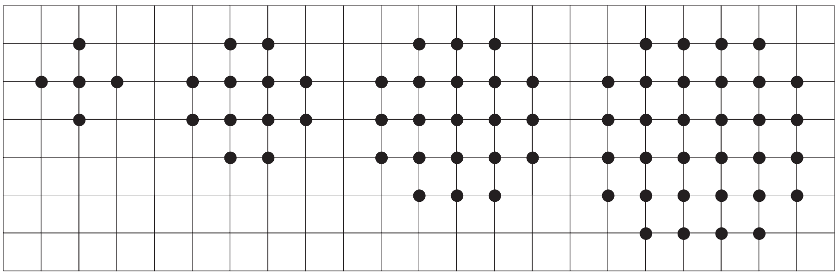


(c)

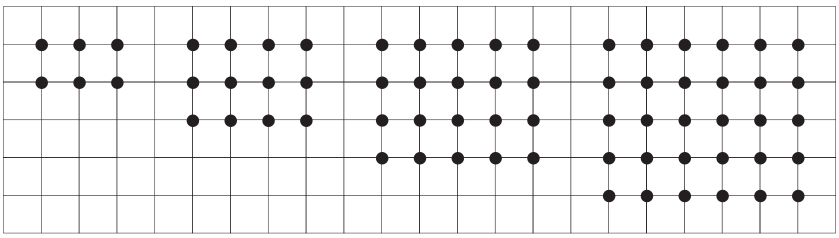


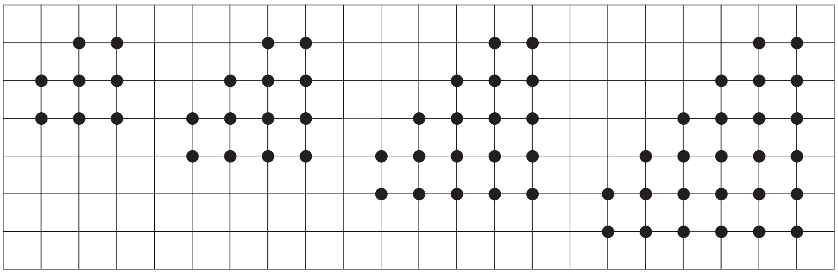
6. What number is represented by the 10th diagram in each of the sequences illustrated in the following diagrams:

(a)



(b)



(c) 

7. The Fibonacci sequence begins:

1, 1, 2, 3, 5, 8

Calculate the 10th and 20th terms in this sequence.

8. Write down the next 5 terms in each of these sequences:

(a) 2, 2, 4, 6, 10, . . .

(b) 1, 3, 4, 7, 11, . . .

(c) 2, 5, 7, 12, 19, . . .

(d) 1, 9, 10, 19, 29, . . .

9. Write down the missing terms in each sequence:

(a) ☐, ☐, 5, 9, 14, 23, 37, ☐, ☐, . . .

(b) ☐, ☐, ☐, ☐, 20, 33, 53, 86, 139, . . .

(c) ☐, ☐, ☐, ☐, 7, 11, 18, 29, 47, . . .

10. A sequence begins:

1, 2, 3, 6, 11, 20, 37, 68, . . .

(a) What do you get if you add: (i) the first three terms,

(ii) the 2nd, 3rd and 4th terms,

(iii) the 3rd, 4th and 5th terms?

(b) What are the next 3 terms in the sequence?

(c) A similar sequence is given below. Write down the missing terms.

☐, ☐, ☐, 14, 26, 48, 88, 162, . . .

(d) A sequence begins:

1, 1, 3, 5, 9, 17, 31, . . .

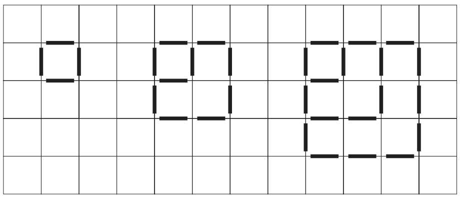
Write down the next 3 terms in the sequence.

13.3 Patterns and Matchsticks

In this section we look at forming patterns with matches, to generate sequences. We then look at how to describe these sequences.

Example 1

(a) Draw the next three shapes in this sequence:

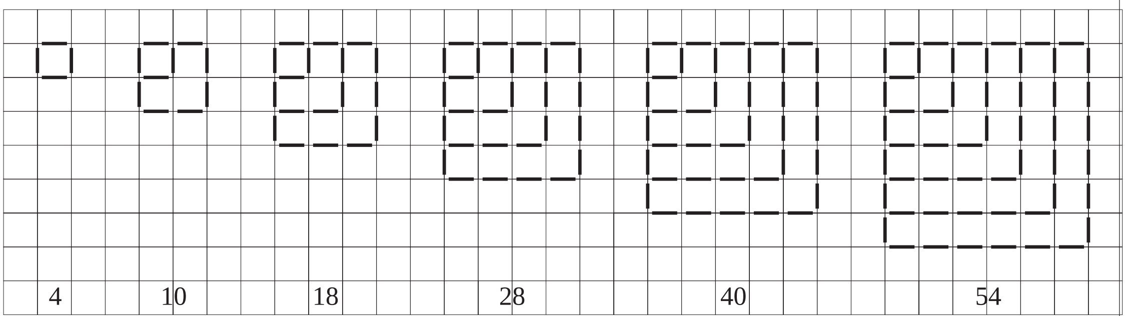


(b) How many matches are used in each shape?

(c) How many matches are used in the 10th shape?

Solution

(a) Here is the sequence with the next 3 shapes:



(b) The number of matches is written under each shape.

(c) The sequence is listed here with the differences between terms:

4 10 18 28 40 54

6 8 10 12 14

Note how the differences increase by 2 as the sequence continues.

The 6th term is 54.

The 7th term is $54 + 16 = 60$.

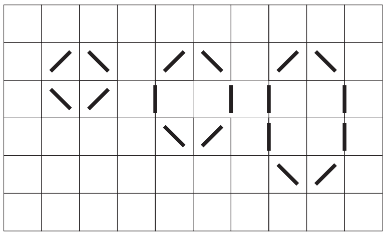
The 8th term is $60 + 18 = 78$.

The 9th term is $78 + 20 = 98$.

The 10th term is $98 + 22 = 120$.

Example 2

The diagram shows the first 3 shapes in a pattern made from matches:



(a) Draw the next 3 shapes and state how many matches are used to make each shape.

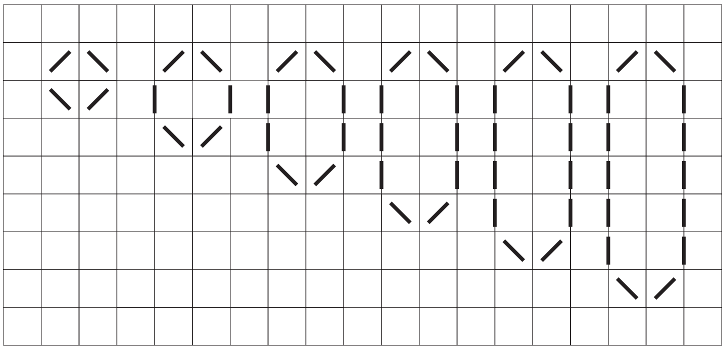
(b) Write down the 10th and 20th terms in this sequence.

(c) What is the *n*th term in this sequence?

(d) One shape needs 20 matches. Which one is it?

Solution

(a) The diagram shows the next 3 shapes:



The number of matches in each shape is listed below:

4 6 8 10 12 14 . . .

2 2 2 2 2

Notice that the difference between each term is 2.

(b) Note that:

1st term	4	=	$2 + 2 \times 1$
2nd term	6	=	$2 + 2 \times 2$
3rd term	8	=	$2 + 2 \times 3$
4th term	10	=	$2 + 2 \times 4$

So to find the 10th term,

$$2 + 2 \times 10 = 22$$

and the 20th term,

$$2 + 2 \times 20 = 42$$

(c) The *n*th term is $2 + 2 \times n = 2 + 2n$.

(d) For the shape that needs 20 matches, we need to find the missing number in the calculation:

$$2 + 2 \times \square = 20$$

The missing number is 9.

We can write this in steps:

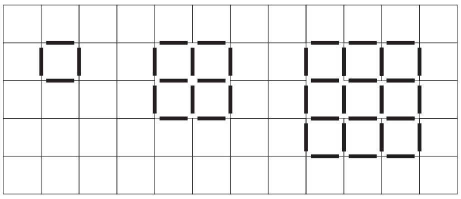
$$2 + 2 \times \square = 20$$

$$2 \times \square = 18$$

$$\square = 9$$

Exercises

1. Here is a pattern formed with matches:

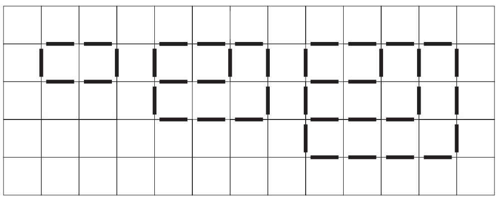


   (a) Draw the next 3 shapes.

   (b) How many matches are used in each of the first 6 shapes?

   (c) How many matches are needed for each of the 7th and 8th shapes?

2. Here is a pattern of shapes made with matches:

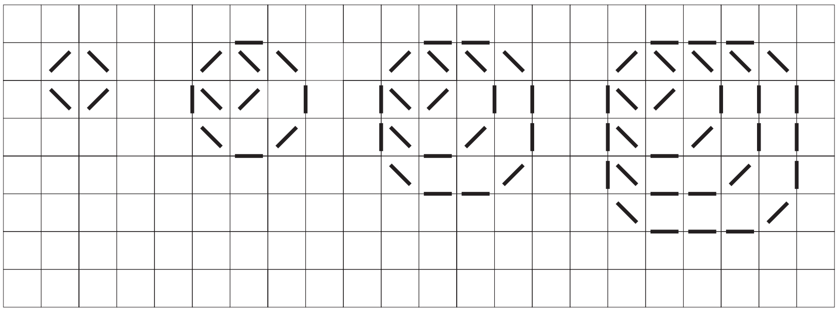


   (a) Draw the next 3 shapes.

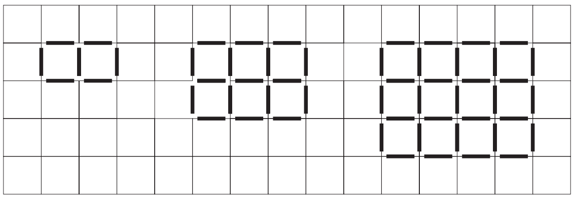
   (b) How many matches are needed for the 10th shape?

   (c) Which shape needs 97 matches?

3. How many matches are needed to make the 8th shape in this pattern?



4. A pattern of rectangles is made using matches:

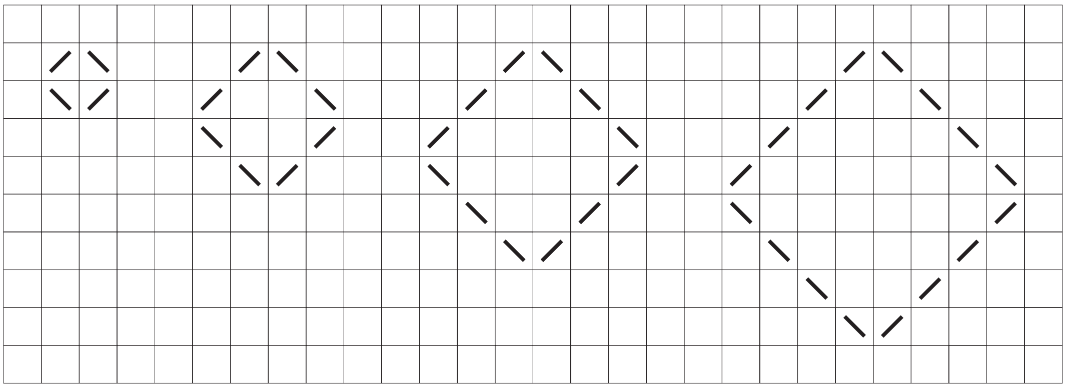


(a) Draw the next two rectangles.

(b) How many matches would be needed for the 7th rectangle?

(c) Which rectangle requires 199 matches?

5. The shapes below are made using matches:

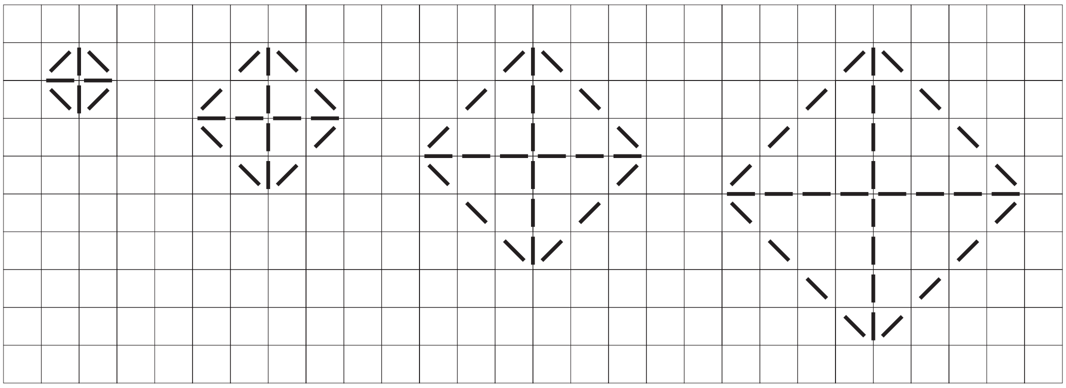


(a) How many matches would be needed for each of the 5th and 6th shapes?

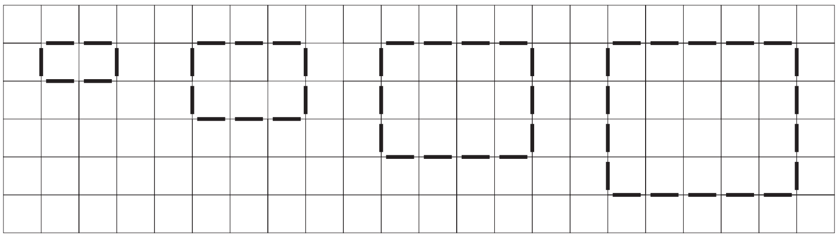
(b) How many matches would be needed for the nth shape?

(c) Which shape contains 88 matches?

6. How many matches would the *n*th shape in the pattern below contain?



7. A pattern of rectangles is made from matches:

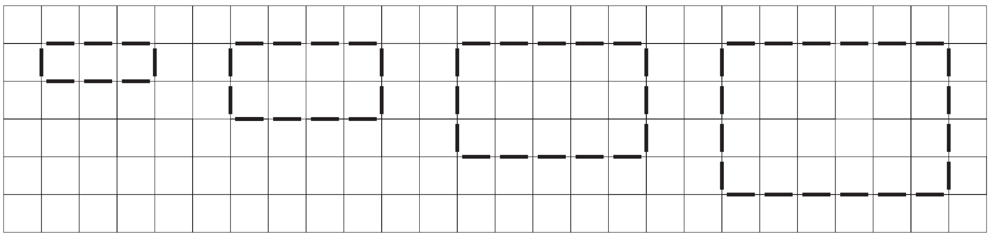


(a) How many matches are needed for the 10th rectangle?

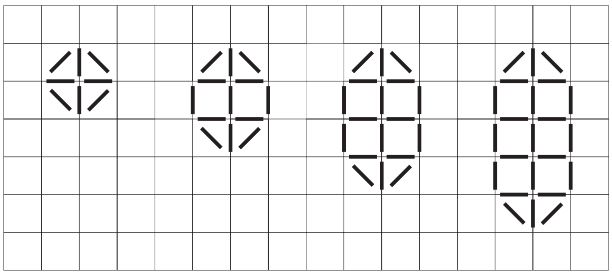
(b) How many matches are needed for the *n*th rectangle?

(c) Which rectangle requires 50 matches?

8. How many matches are needed to make the *n*th shape in the pattern of rectangles below?



9. A pattern of shapes is made from matches:



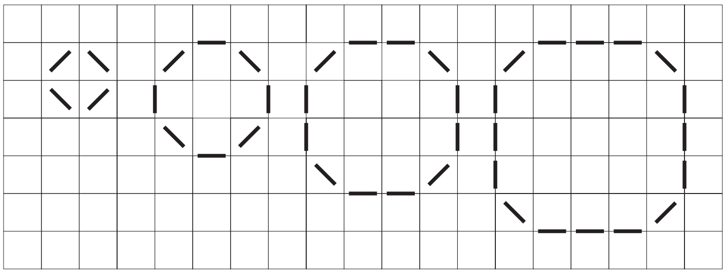
Write down:

(a) the number of matches in each of the 5th and 10th shapes,

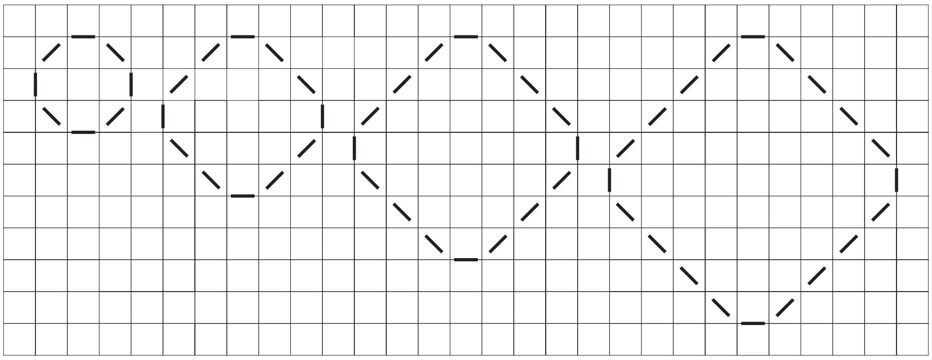
(b) the number of matches in the nth shape.

10. How many matches are needed to make the nth shape of each of these patterns?

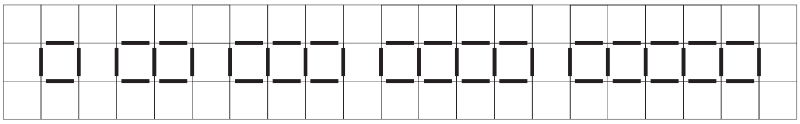
(a)



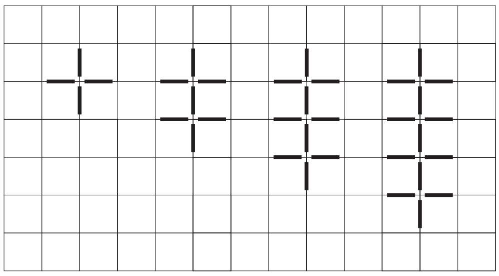
(b)



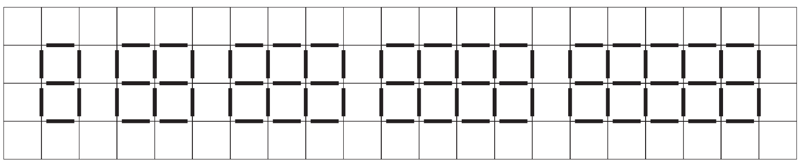
(c)



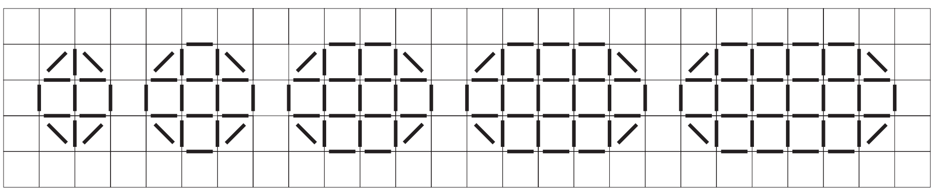
(d)



(e)



(f)



13.4 Two-Dimensional Number Patterns

This section explores 2-dimensional number patterns. One of the most famous of these is *Pascal's triangle*.

Example 1

Here are the first 4 rows of Pascal's triangle.

1

1 1

1 2 1

1 3 3 1

Write down the next 3 rows of the triangle.

Solution

Note that each row starts and ends with a 1.

1

1 1

1 2 1

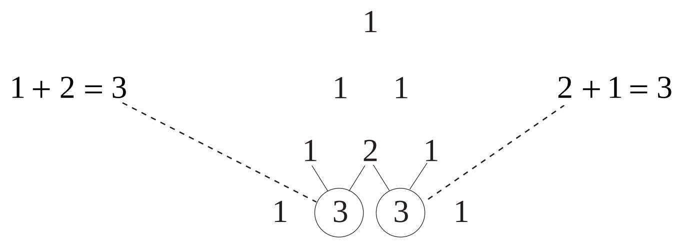
1 3 3 1

1 1

1 1

1 1

The other numbers are found by adding together the two numbers that are diagonally above them in the previous row.



Using this rule the triangle can be completed:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Example 2

What are the next 2 diagrams in the sequence:

1	,	1 1	,	1 1 1	,	. . .
		2		2 2		
				4		

Solution

Note how the numbers in the next row are obtained by adding together two of the numbers in the row above.

For example, in the 2nd diagram, $1 + 1 = 2$

and in the 3rd diagram, $2 + 2 = 4$.

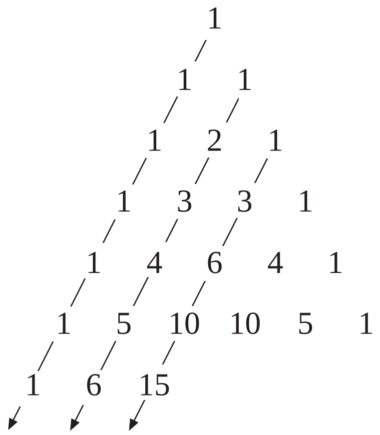
Using this rule, the sequence can be extended:

1	,	1 1	,	1 1 1	,	1 1 1 1	,	1 1 1 1 1	,	. . .
		2		2 2		2 2 2		2 2 2 2		
				4		4 4		4 4 4		
						8		8 8		
								16		

Exercises

1. In example 1, the first 7 rows of Pascal's triangle are listed. By adding the next 3 rows, write down the first 10 rows of the triangle.

2. Patterns can be found in the diagonals of Pascal's triangle. Copy the part of the triangle shown here and add the next 4 terms to the three diagonals shown.



3. Write down the next two diagrams in the sequence:

```
2  ,  2   2  ,  2   2   2  ,  . . .
        4         4   4
                    8
```

4. Write down the next three diagrams in this sequence:

```
1  ,  1   2  ,  1   2   3  ,  . . .
        3         3   5
                    8
```

5. Write down the next three diagrams in the sequence:

```
1  ,  1   2  ,  1   2   1  ,  1   2   1   2  ,  . . .
        3         3   3         3   3   3
                    6             6   6
                                    12
```

6. Write down the next three diagrams in the sequence:

```
1 , 1   3 , 1   3   5 ,  . . .
      4       4   8
               12
```

7. Here are the first 5 rows of a triangle of numbers:

```
                2
            2       2
        2       4       2
    2       6       6       2
2       8      12       8       2
```

(a) Write down the next 3 rows of the triangle.

(b) Explain why the triangle contains no odd numbers.

8. Here are the first three diagrams in a sequence:

```
3   ,   3   3   ,   3   3   3   ,   . . .
          6           6   6
                       12
```

(a) What is the next diagram in the sequence?

(b) What is the largest number in the 5th diagram in the sequence?

(c) What is the largest number in the 10th diagram in the sequence?

(d) For which diagram is the largest number 384?

9. Here is a sequence of number diagrams:

```
1   ,   1   2   ,   1   2   3   ,   . . .
          2           2   6
                       12
```

Write down the next 2 diagrams in this sequence.

10. Write down the next 3 diagrams in this sequence:

```
1 , 1   2 , 1   2   3 , 1   2   3   4 , 1   2   3   4   5 ,
      3       3   5       3   5   7       3   5   7   9
                8           8   2           8   2   6
                              0               0   8
                                                8
```

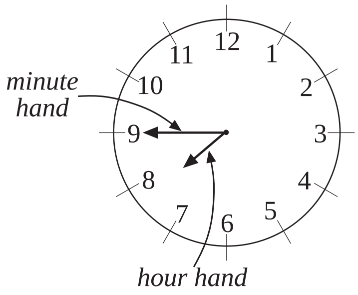
What is the largest number that will appear in any of the diagrams of this sequence?

14 Time and Timetables

14.1 Telling the Time

In this section we look at different ways of writing times; for example, '7:45' is the same time as 'quarter to eight'.

On a clockface, this can be represented as shown here:



Also remember that

one hour $=$ 60 minutes

so that

half an hour $=$ 30 minutes
quarter of an hour $=$ 15 minutes
three quarters of an hour $=$ 45 minutes

Example 1

Write each time using digits and show the position of the hands on a clockface:

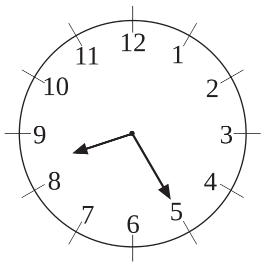
(a) twenty five past eight,

(b) quarter to ten.

Solution

(a) Twenty five past eight using digits is

8:25



(a) Quarter to ten can be thought of as:

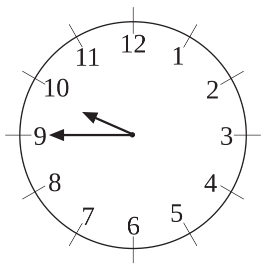
15 minutes to 10 o'clock

or

45 minutes past 9 o'clock

so, using digits, quarter to ten is

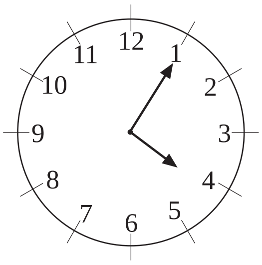
9:45

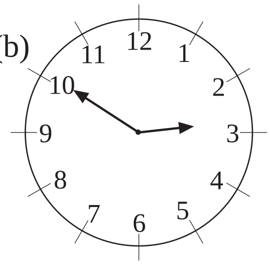


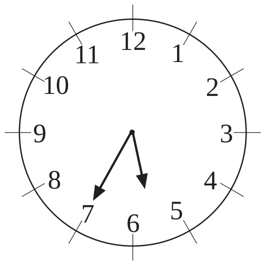
Example 2

Write each of the times shown on these clocks:

(i) in words, and (ii) using digits.

(a) 

(b) 

(c) 

Solution

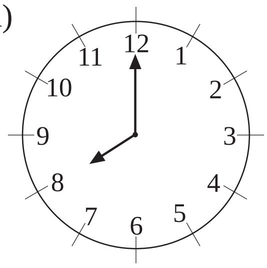
(a) (i) Five past four (ii) 4:05

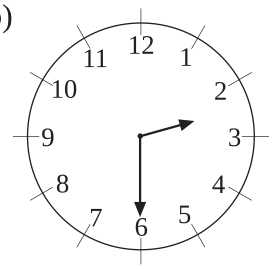
(b) (i) Ten to three (ii) 2:50

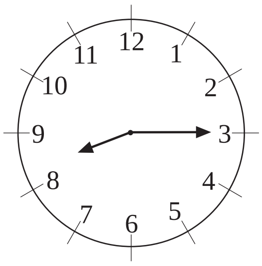
(c) (i) Twenty five to six (ii) 5:35

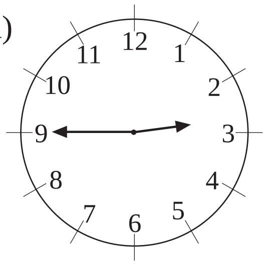
Exercises

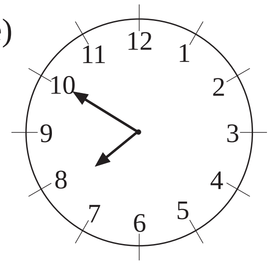
1. Write the times shown on each of these clocks in words and digits:

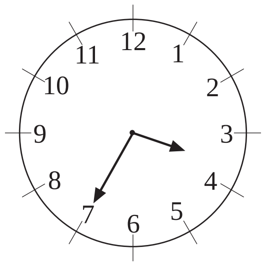
(a) 

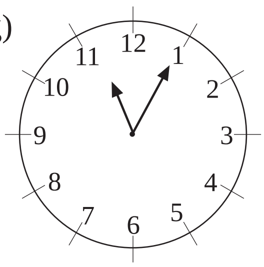
(b) 

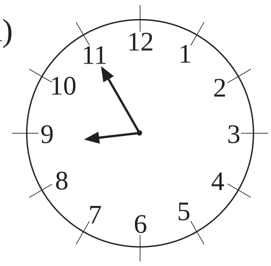
(c) 

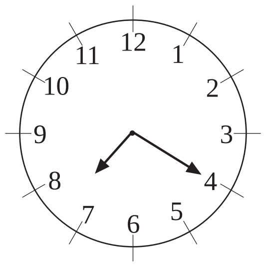
(d) 

(e) 

(f) 

(g) 

(h) 

(i) 

2. Draw these times on clock faces:

(a)	ten past five	(b)	ten to nine	(c)	quarter to seven
(d)	quarter past twelve	(e)	half past ten	(f)	twenty to nine
(g)	ten to two	(h)	twenty five to six	(i)	twenty past four

3. Draw these times on clock faces:

(a)	4:00	(b)	5:30	(c)	7:15
(d)	8:20	(e)	2:45	(f)	3:50
(g)	1:55	(h)	6:05	(i)	11:35

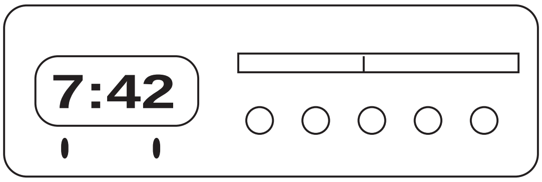
4. Write these times in words:

(a)	9:30	(b)	4:00	(c)	4:25
(d)	8:45	(e)	7:35	(f)	9:05

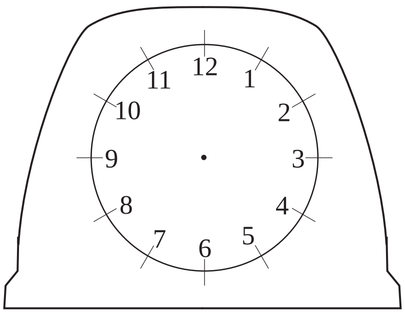
5. Write these times using digits:

(a)	eight o'clock	(b)	quarter to seven
(c)	ten past five	(d)	half past six
(e)	ten to three	(f)	five to four
(g)	twenty five to nine	(h)	twenty to three

6. This picture shows the time on Vicki's radio clock:



(a) Draw a picture to show where the hands would be on this clock:

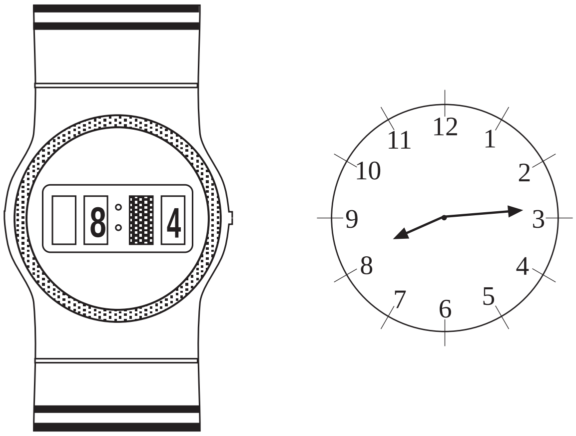


(b) Write the time in words.

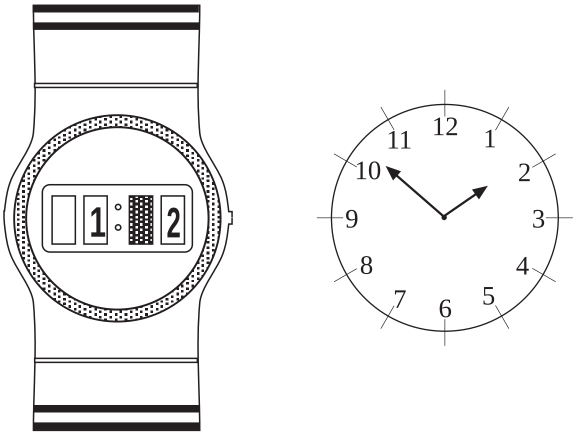
7. Daniel's digital watch has gone wrong and shows a grey blob instead of the first digit of the minutes.

Use the ordinary clock to tell Daniel what the missing digit is for each of the times shown below:

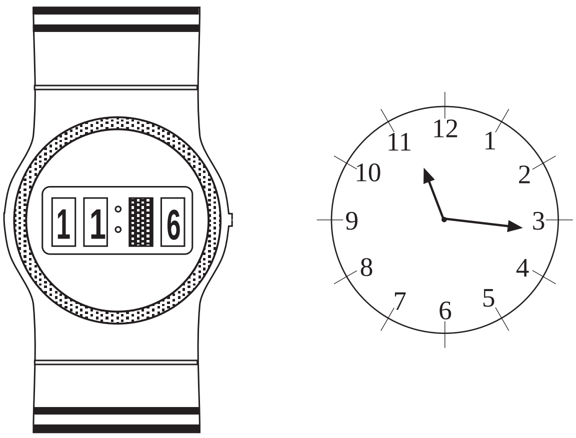
(a)



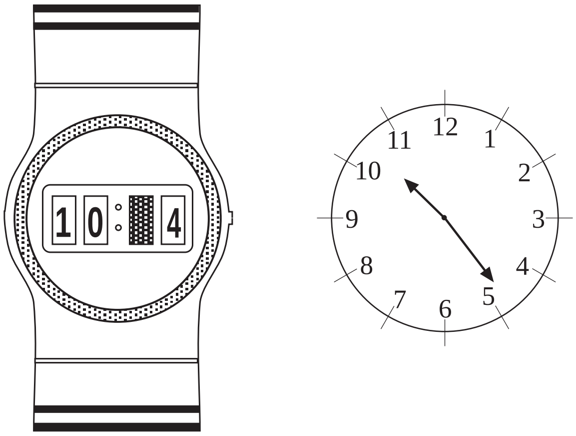
(b)



(c)



(d)



8. Halim looks at his watch and sees that the time is 7:46.

   (a) Write this time in words.

   (b) What will be the time 10 minutes later?

9. A bus leaves school at five minutes past four and Tony gets off 20 minutes later. What is the time when Tony gets off the bus? Write your answers in words and in figures.

14.2 12- and 24-hour Clocks

The 24-hour clock system can be used to tell if times are in the morning or the afternoon. Alternatively, times can be given as 'a.m.' or 'p.m.'

Example 1

Write these times in 24-hour clock time:

(a) 3.06 a.m. (b) 8:14 p.m.

Solution

(a) As this is an 'a.m.' time it remains the same but with a zero in front of the 3,

0306

(b) As this is a 'p.m.' time, add 12 to the hours to give

2014

Example 2

Write these times using 'a.m.' or 'p.m':

(a) 1428 (b) 0742

Solution

(a) As the hours figure, 14, is greater than 12, subtract 12 and write as a 'p.m.' time:

2:28 p.m.

(b) As the hours figure, 07, is less than 12, simply remove the zero and write as an 'a.m.' time:

7:42 a.m.

Exercises

1. Convert these times to 24-hour clock times:

(a) 9:42 a.m.	(b) 11:28 p.m.	(c) 11:14 a.m.
(d) 7:13 p.m.	(e) 9:21 p.m.	(f) 4:18 p.m.
(g) 10:05 a.m.	(h) 12:15 a.m.	(i) 12:15 p.m.

2. Write these times in 24-hour clock time:

(a) quarter to eight in the morning,	(b) ten minutes to midnight,
(c) ten past nine in the evening,	(d) five to seven in the morning,
(e) quarter past five in the afternoon,	(f) half past two in the afternoon.

3. Write these 24-hour clock times in 12-hour clock times, using 'a.m.' or 'p.m.':

(a) 1204	(b) 0822	(c) 1842
(d) 1330	(e) 1440	(f) 2305
(g) 1735	(h) 1605	(i) 0342

4. Write these 24-hour clock times in words, as in question 2:

(a) 1430	(b) 1555	(c) 0745

5. Insert 'a.m.' or 'p.m.' into each sentence so that it makes sense:

   (a) Joshua woke up at 6:45.

   (b) Miles came home from school at 3:55.

   (c) Andy started his night shift at 10:15.

   (d) Grace ate her lunch at 11:45.

   (e) James went to bed at 8:55.

   (f) Heidi cooked a meal at 5:15.

6. Here are the timings for a school day.

   Convert them to 24-hour clock times.

Registration	8:55 a.m.
Lesson 1	9:05 a.m.
Lesson 2	10:05 a.m.
Lesson 3	11:05 a.m.
Lunch	12:05 p.m.
Lesson 4	1:05 p.m.
Lesson 5	2:05 p.m.
Day ends	3:05 p.m.

7. A train leaves at 0020. Write this time in words.

8. Shamil leaves home at 0900 and returns 7 hours later. Write the time that Shamil gets home in 24-hour clock time, and in 12-hour clock time using 'a.m.' or 'p.m.'.

14.3 Units of Time

In this section we explore the different units of time.

1 minute	=	60 seconds
1 hour	=	60 minutes
1 day	=	24 hours
1 week	=	7 days
1 year	=	365 days (366 days in a leap year)

January	has	31 days
February	has	28 days (29 days in a leap year)
March	has	31 days
April	has	30 days
May	has	31 days
June	has	30 days
July	has	31 days
August	has	31 days
September	has	30 days
October	has	31 days
November	has	30 days
December	has	31 days

Example 1

How many hours are there in May?

Solution

Number of hours in May $= 31 \times 24$

$= 744$ hours

Example 2

25 February is a Friday. What will be the date on the next Friday:

(a) if it is *not* a leap year,

(b) if it *is* a leap year?

Solution

(a) You could write out the next 7 days like this:

Friday	25
Saturday	26
Sunday	27
Monday	28
Tuesday	1
Wednesday	2
Thursday	3
Friday	4

or

$25 + 7 = 32$

$32 - 28 = 4$

So the next Friday will be 4 March.

(b) Using the addition method:

$25 + 7 = 32$

$32 - 29 = 3$

So, in a leap year, the next Friday will be 3 March.

Exercises

1. How many hours are there in a week?

2. How many hours are there in:

   (a) September,

   (b) February (2 answers needed),

   (c) one year (2 answers needed)?

3. How many minutes are there in:

   (a) one day, (b) one week?

4. How many seconds are there in:

   (a) one hour, (b) one day?

5. If 25 March is a Friday, what will be the date on the following Friday?

6. Sasha goes on holiday on Monday 20 June. She returns 14 days later. On what date does she return from her holiday?

7. If 3 October is a Monday:

   (a) what day of the week will 1 November be,

   (b) what will be the date of the first Monday in November?

8. Hannah goes to the bank every Tuesday. The last time she went was on Tuesday 20 October.

   (a) What will be the dates of her next 2 visits to the bank?

   (b) On the second Tuesday in November she is ill and goes to the bank on Wednesday instead. What is the date of that Wednesday?

9. This year Mike's birthday is on a Saturday in June. What day will his birthday be on next year if:

   (a) next year is a leap year,

   (b) next year is *not* a leap year?

10. In 1998, Christmas Day was on a Friday. Name the day of the week for Christmas Day in:

    (a) 1997,

    (b) 2000.

11. (a) For how many days have you been alive?

(b) How many *hours* is this?

(c) How many *minutes* is this?

(d) How many *seconds* is this?

(e) David has been alive for approximately 1 892 160 000 seconds. In which year do you think he was born?

14.4 Timetables

In this section we consider how to extract information from timetables.

Example 1

London		Bristol Parkway - Cardiff - Swansea Cheltenham - Bath - Bristol								
Sundays from 26 July										
London Paddington	d	1903	1915	1930	2000	2030	2100	2130	2200	2215
Reading	d	1935	1945	2003	2030	2100	2130	2200	2230	2245
Didcot Parkway	d	1949	–	2017	2044	2114	2144	2214	2244	2259
Swindon	d	2009	2015	2036	2103	2133	2203	2233	2303	2319
Kemble	a	2022	–	–	–	–	–	2250	–	–
Stroud	a	2037	–	–	–	–	–	2305	–	–
Stonehouse	a	2042	–	–	–	–	–	2310	–	–
Gloucester	a	2053	–	–	–	–	–	2323	–	–
Cheltenham Spa	a	2108	–	–	-	–	–	2335	–	–
Chippenham	d	–	–	2049	–	2145	–	2245	–	2332
Bath Spa	a	–	2035	2105	–	2157	–	2257	–	2343
Bristol Parkway	d	–	–	–	2129	–	2229	–	2329	–
Bristol Temple Meads	a	–	2051	2117	–	2211	–	2311	–	2357
Weston-super-Mare	a	–	2110	–	–	–	–	–	–	0018
Newport	a	–	–	–	2149	–	2249	–	2353	–
Hereford	a	–	–	–	–	–	–	–	–	–
Cardiff Central	a	–	–	–	2205	–	2305	–	0014	–
Bridgend	a	–	–	–	2225	–	2325	–	0034	–
Port Talbot Parkway	a	–	–	–	2237	–	2337	–	0046	–
Neath	a	–	–	–	2244	–	2344	–	0053	–
Swansea	a	–	–	–	2257	–	2357	–	0106	–

a: arrives d: departs

Use the train timetable above to answer these questions:

(a) If you catch the 1915 from London Paddington, at what time would you arrive in Weston-super-Mare?

(b) Ted arrives in Chippenham at 2245. At what time did he leave London Paddington?

(c) Tina catches the 2205 at Cardiff Central. At what time does she arrive in Neath?

Solution

(a) The 1915 from London Paddington arrives at Weston-super-Mare at 2110.

Sundays from 26 July			
London Paddington	d	1903	(1915)
Reading	d	1935	1945
Didcot Parkway	d	1949	–
Swindon	d	2009	2015
Kemble	a	2022	–
Stroud	a	2037	–
Stonehouse	a	2042	–
Gloucester	a	2053	–
Cheltenham Spa	a	2108	–
Chippenham	d	–	–
Bath Spa	a	–	2035
Bristol Parkway	d	–	–
Bristol Temple Meads	a	–	2051
Weston-super-Mare	a	–	(2110)
Newport	a	–	–

(b) To arrive in Chippenham at 2245 Ted must have left London Paddington at 2130.

Sundays from 26 July								
London Paddington	d	1903	1915	1930	2000	2030	2100	(2130)
Reading	d	1935	1945	2003	2030	2100	2130	2200
Didcot Parkway	d	1949	–	2017	2044	2114	2144	2214
Swindon	d	2009	2015	2036	2103	2133	2203	2233
Kemble	a	2022	–	–	–	–	–	2250
Stroud	a	2037	–	–	–	–	–	2305
Stonehouse	a	2042	–	–	–	–	–	2310
Gloucester	a	2053	–	–	–	–	–	2323
Cheltenham Spa	a	2108	–	–	-	–	–	2335
Chippenham	d	–	–	2049	–	2145	–	(2245)
Bath Spa	a	–	2035	2105	–	2157	–	2257
Bristol Parkway	d	–	–	–	2129	–	2229	–

(c) The 2205 from Cardiff Central arrives in Neath at 2244.

Sundays from 26 July					
London Paddington	d	1903	1915	1930	2000
Reading	d	1935	1945	2003	2030
Didcot Parkway	d	1949	–	2017	2044
Swindon	d	2009	2015	2036	2103
Kemble	a	2022	–	–	–
Stroud	a	2037	–	–	–
Stonehouse	a	2042	–	–	–
Gloucester	a	2053	–	–	–
Cheltenham Spa	a	2108	–	–	-
Chippenham	d	–	–	2049	–
Bath Spa	a	–	2035	2105	–
Bristol Parkway	d	–	–	–	2129
Bristol Temple Meads	a	–	2051	2117	–
Weston-super-Mare	a	–	2110	–	–
Newport	a	–	–	–	2149
Hereford	a	–	–	–	–
Cardiff Central	a	–	–	–	(2205)
Bridgend	a	–	–	–	2225
Port Talbot Parkway	a	–	–	–	2237
Neath	a	–	–	–	(2244)
Swansea	a	–	–	–	2257

Exercises

1. The table below gives the timetable for a ski train that runs from London Waterloo International:

London Waterloo International	depart	0857
Ashford International	depart	1006
Moûtiers	arrive	1657
Aime la Plagne	arrive	1723
Bourg-St-Maurice	arrive	1742

(a) At what time does the train leave Ashford International?

(b) At what time does the train arrive at Bourg-St-Maurice?

(c) Where does the train arrive at 1657?

(d) John arrives at London Waterloo International at five past nine. Can he catch the train?

2. The timetable below gives the times of early morning trains from Norwich to London Liverpool Street:

NORWICH	Dep	0500	0520	0530	0600	0630	0655	0710	0755	0805	0835	0905
Diss	Dep	0518		0547	0618	0647	0713	0728	....	0823	0852	0922
Stowmarket	Dep	0531	....	0558	0630	0658	0725	0740	....	0835	0903	0933
IPSWICH	Arr	0541	0553	0610	0641	0709	0736	0751	0828	0846	0913	0944
	Dep	0543	0553	0612	0642	0710	0737	0752	0830	0847	0915	0945
Manningtree	Dep	0553	....	0620	0652	0721	......	0802	....	....	0925	....
COLCHESTER	Dep	0604	0610	0632	0704	0732	....	0812	....	0906	0935	1003
Chelmsford	Dep	....	....	....	....	....	....	....	....	....	....	1020
LIVERPOOL STREET	Arr	0653	070	0721	0756	0826	0848	0903	0933	0955	1025	1054

(a) At what time does the 0630 from Norwich arrive at Liverpool Street?

(b) Jason arrives by train at Liverpool street at 0903. At what time did he catch the train at Diss?

(c) If you catch the 0655 from Norwich, at what time do you arrive at Ipswich?

(d) Alex arrives at Ipswich station at 0900 and catches the next train to Liverpool Street. At what time does he arrive at Liverpool Street?

(e) Scott catches the 0612 at Ipswich. At what time does he arrive at Manningtree?

3. The timetable below gives information about trains from London Waterloo to Paris:

Waterloo International	0508	0619	0723	0753	0823	0853	0953	1023	1053	1157
Ashford International	0616	0719	0823	....	0923	....	1053	....	....	....
Calais-Fréthun	....	0856	....	....	....	....	....	....	....	1429
Lille Europe	....	....	....	....	....	1150	....	....	....	....
Paris Nord	0923	1023	1123	1147	1223	1253	1353	1417	1453	1556

(a) Scott arrives in Paris at 1123. At what time did he leave Waterloo?

(b) Chelsea arrives in Paris at 1223. At what time did she leave Ashford?

(c) Jai wants to go from Waterloo to Lille Europe. Which train should he catch?

(d) Halim wants to arrive in Calais-Fréthun before 9:00 a.m. Which train should he catch?

4. Mike is in Brussels and wants to return to Ashford. He looks at this train timetable:

Brussels to Waterloo

Brussels Midi	0856	1102	1302	1456	1702	1756	1856	2102
Lille Europe	0937	1142	1342	1536	1742	1836	1936	2142
Ashford International	0938	1141	1341	1536	1741	1837	1938	....
Waterloo International	1047	1247	1447	1639	1843	1939	2039	2239

(a) At what time should he catch a train if he wants to arrive in Ashford at 1741?

(b) Which train should he *avoid* if he wants to go to Ashford?

(c) If he catches the 1456, at what time does he arrive in Ashford?

(d) He catches the 1456, but falls asleep and does not get off at Ashford. At what time does he get to Waterloo?

5. Use the following timetable to answer these questions:

(a) Rachel catches the 1600 at Reading. At what time does she arrive in Bristol?

(b) Emma catches the 1330 at London Paddington. At what time does she arrive in Bristol?

(c) Hannah arrives in Newport at 1545. At what time did she leave Reading?

(d) Ben arrives at Neath at 1451. At what time did he leave Swindon?

London	Bristol Parkway - Cardiff - Swansea Cheltenham - Bath - Bristol									
Sundays from 26 July										
London Paddington	d	1152	1200	1230	1300	1330	1400	1430	1500	1530
Reading	d	1222	1230	1301	1330	1403	1430	1500	1530	1600
Didcot Parkway	d	–	1244	1314	–	1417	–	1514	1544	1614
Swindon	d	1252	1303	1333	1358	1437	1458	1533	1603	1633
Kemble	a	–	1320	–	–	–	–	1559	–	–
Stroud	a	–	1335	–	–	–	–	1614	–	–
Stonehouse	a	–	1340	–	–	–	–	1619	–	–
Gloucester	a	–	1353	–	–	–	–	1632	–	–
Cheltenham Spa	a	–	1409	–	–	–	–	1644	–	–
Chippenham	d	–	–	1346	–	1449	–	1545	–	1645
Bath Spa	a	–	–	1358	–	1501	–	1557	–	1701
Bristol Parkway	d	–	1329	–	1424	–	1524	–	1629	–
Bristol Temple Meads	a	1324	–	1412	–	1515	–	1611	–	1713
Weston-super-Mare	a	–	–	1432	–	1543	–	1651	–	1751
Newport	a	–	1349	–	1445	–	1545	–	1649	–
Hereford	a	–	–	–	1605	–	1720	–	1820	–
Cardiff Central	a	–	1412	–	1508	–	1608	–	1705	–
Bridgend	a	–	1432	–	1528	–	1628	–	1725	–
Port Talbot Parkway	a	–	1444	–	1540	–	1640	–	1737	–
Neath	a	–	1451	–	1547	–	1647	–	1744	–
Swansea	a	–	1504	–	1559	–	1659	–	1757	–

6.

Penzance - Plymouth - Torbay - Exeter - Taunton - Reading				London						
Sundays										
Penzance	d	–	–	–	–	–	0832	–	0926	0926
St Erth	d	–	–	–	–	–	–	–	0937	0937
Camborne	d	–	–	–	–	–	–	–	0948	0948
Redruth	d	–	–	–	–	–	0855	–	0955	0955
Truro	d	–	–	–	–	–	0907	–	1007	1007
St Austell	d	–	–	–	–	–	0924	–	1024	1024
Par	d	–	–	–	–	–	0932	–	1032	1032
Bodmin Parkway	d	–	–	–	–	–	0943	–	1043	1043
Liskeard	d	–	–	–	–	–	0955	–	1055	1055
Plymouth	d	–	–	0830	0844	0844	1020	1042	1123	1123
Totnes	d	–	–	0857	0910	0910	1050	1108	1150	1150
Paignton	d	–	–	–	0940	0940	1015	1059	1140	1140
Torquay	d	–	–	–	0946	0946	1021	1105	1145	1145
Newton Abbot	d	–	–	0910	0958	0958	1103	1121	1203	1203
Teignmouth	d	–	–	0916	1004	1004	1041	–	–	–
Dawlish	d	–	–	0921	1009	1009	1046	–	–	–
Exeter St Davids	d	0800	0815	0939	1022	1022	1124	1140	1224	1224
Tiverton Parkway	d	0814	0829	0953	0958	0958	1043	–	1238	1238
Taunton	d	0828	0843	1006	1045	1045	1148	1230	1252	1252
Bristol Temple Meads	a	0902	–	1040	–	–	–	1312	–	1325
Castle Cary	d	–	–	–	–	–	–	–	–	–
Westbury	d	–	0929	–	1132	–	–	–	1338	–
Pewsey	d	–	0945	–	–	–	–	–	–	–
Newbury	a	–	1004	–	–	–	–	–	–	–
Reading	a	1000	1034	1135	1226	1227	1316	1424	1432	1430
Gatwick Airport	a	1131	1231	1331	1431	1431	1511	–	1631	1631
Heathrow Terminal 1	a	1100	1200	1300	1330	1330	1430	–	1600	1530
London Paddington	a	1043	1115	1213	1305	1305	1359	1500	1514	1514

Use the timetable to answer these questions:

(a) David catches the 0939 at Exeter St Davids. At what time does he arrive in London?

(b) Stewart arrives in London at 1514. At what time did he leave St Erth?

(c) Helen wants to travel from Camborne to Plymouth. What is the earliest time she can get there by train?

(d) Misha arrives in Reading at 1432. At what time did she leave Truro?

7. Use the following bus timetable to answer these questions:

(a) What bus should you catch from Glasgow to be at Clydebank by 1:00 p.m?

(b) You arrive at the bus stop in Tarbet at 1300. At what time could you arrive in Connel?

(c) If you leave Glasgow at 1000, at what time will you arrive in Oban?

(d) If you catch the 1233 bus from Anniesland, where will you get off the bus in Dalmally?

GLASGOW	Buchanan Bus Stn	0815	1000	–	1215	1800
Hillhead	Gt Western Rd at Kersland St	0825	1010	–	1225	1810
Gartnavel	Gt Western Rd Bus Stop	0829	1015	–	1229	1814
Anniesland	Cross	0833	1019	–	1233	1818
Drumchapel	Drumry Roundabout	0841	1026	–	1241	1826
Clydebank	Gt Western Rd at Kilbowie Rd	0844	1029	–	1244	1829
Dumbarton	Barloan Toll	0855	1043	–	1255	1840
Balloch	Layby near Roundabout	0900	1048	–	1300	1845
Luss	Bypass	0911	1100	–	1311	1856
Tarbet	Hotel	0921	1111	–	1321	1906
Arrochar	Braeside Stores	0925	–	–	1325	1910
Cairndow	War Memorial	0944	–	–	1344	1928
INVERARAY *arr*	Front Street	1000	–	–	1405	1945
INVERARAY *dep*	Front Street	1010	–	–	1415	1955
TYNDRUM *arr*	Little Chef	–	1200	–	–	–
TYNDRUM *dep*	Little Chef	–	–	1215	–	–
Dalmally	opp Glen Orchy Hotel	1035	–	–	1440	2020
Dalmally	opp Police Station	–	–	1233	–	–
Taynuilt	Hotel	1054	–	1252	1459	2039
Connel	Bypass at Bridge	1104	–	1304	1509	2049
OBAN	Railway Station	1115	–	1315	1520	2100

8. This timetable gives the times of departures of the ferry between Penzance and St Mary's on the Isles of Scilly, for 1998.

Mondays to Fridays, daily	
6 April to 2 October	Depart Penzance 0915 Depart St Mary's 1630
Saturdays only	
4 April to 2 May **11 July**	Depart Penzance 0915 Depart St Mary's 1630
9 May & 16 May **6 June to 4 July** **5 Sept to 3 October**	Depart Penzance 1100 Depart St Mary's 1500
23 May & 30 May **18 July to 29 August**	Depart Penzance 0630 and 1345 Depart St Mary's 0945 and 1700
Sundays only	
30 August	Depart Penzance 0915 Depart St Mary's 1630
Monday, Wednesday, Friday, Saturday	
5 October to 31 October	Depart Penzance 0915 Depart St Mary's 1630

(a) What was the date of the only Sunday in 1998 on which you could catch the ferry?

(b) At what time did the ferry leave Penzance on Saturday 6 June?

(c) At what time did the ferry leave St Mary's on Wednesday 14 October?

(d) Was there a ferry on Thursday 15 October?

(e) On how many days were there *two* return journeys instead of the usual one?

9. The table gives the flight numbers and times of flights to and from the Isles of Scilly from Newquay, Plymouth and Southampton.

To ISLES OF SCILLY			**From** ISLES OF SCILLY		
Newquay (Flight time 30 mins)					
5Y201	0930	TWTh	5Y200	0830	TWTh
5Y203	1000	M FS	5Y202	0900	M FS
5Y205	1445	M W FS	5Y204	1330	M W FS
5Y207	1745	T Th	5Y206	1630	T Th
Plymouth (Flight time 45 mins)					
5Y301	1335	M W F	5Y300	1220	M W F
Southampton (Flight time 90 mins)					
5Y501	1000	M F	5Y500	0800	M F
M - Mon, T - Tues, W - Wed, Th - Thurs, F - Fri, S - Sat, Su - Sun					

The left hand side of the table gives flights *to* the Isles of Scilly and the right hand side gives flights *from* the Isles of Scilly.

(a) On which days can you fly from Southampton to the Isles of Scilly?

(b) On which day are there no flights from any of these airports to the Isles of Scilly?

(c) If you catch the 1000 from Newquay, at what time do you arrive in the Isles of Scilly?

(d) Khan arrives in the Isles of Scilly at 1130. Where did he fly from?

(e) What is the latest time that you could land in the Isles of Scilly?

10. The following timetable gives details of flights from the Isle of Man to Heathrow:

Isle of Man to London Heathrow

Flight Number	Operation Dates	Days	Routes	Depart - Arrive
JE301	30SEP - 16OCT98	123456-	IOM - LHR	0700 - 0815
JE301	19OCT - 24OCT98	123456-	IOM - LHR	0700 - 0815
JE301	17OCT - 17OCT98	- - - - - 6-	IOM - LHR	0700 - 0815
JE305	30SEP - 23OCT98	12345 - -	IOM - LHR	1440 - 1540
JE307	30SEP - 23OCT98	12345 - 7	IOM - LHR	1755 - 1900
JE309	04OCT - 18OCT98	- - - - - - 7	IOM - LHR	0740 - 0900
JE311	03OCT - 10OCT98	- - - - - 6 -	IOM - LHR	1755 - 1855
JE311	24OCT - 24OCT98	- - - - - 6 -	IOM - LHR	1755 - 1855
JE311	17OCT - 17OCT98	- - - - - 6 -	IOM - LHR	1755 - 1855
JE307	25OCT - 25OCT98	- - - - - - 7	IOM - LHR	1755 - 1900
JE313	25OCT - 25OCT98	- - - - - - 7	IOM - LHR	1040 - 1200

Days
1 = Monday
2 = Tuesday
3 = Wednesday
4 = Thursday
5 = Friday
6 = Saturday
7 = Sunday

(a) What is the flight number of the plane that arrives at Heathrow at 0900, and on which day?

(b) On which day does the 1755 flight have the shortest journey time?

(c) How many flights are there to Heathrow on Sunday 25 October?

(d) What is the *earliest* departure time on Wednesday 21 October?

(e) What is the *latest* arrival time on Saturday 10 October?

14.5 Time Problems in Context

In this section we consider further problems involving time, travel and also rates of pay.

Example 1

The timetable below gives the times of buses from Glasgow to Edinburgh:

GLASGOW	Buchanan Bus Stn	1445	1500	1515	1530	1545
Eurocentral	Gt A8 Interchange	–	–	–	1550	–
Harthill	Service Area	1510	1525	1540	1600	1610
Newbridge East	A8 opp RACAL	1531	1546	1601	1621	1631
Ratho	Sation Road End	1532	1547	1602	1622	1632
Ingliston Showground	West Entrance	1534	1549	1604	1624	1634
Corstorphine	Drum Brae	1538	1553	1608	1628	1638
Corstorphine	Station Road End	1540	1555	1610	1630	1640
Corstorphine	Zoo Park	1542	1557	1612	1632	1642
Murrayfield	Corstorphine Rd, Ice Rink	1545	1600	1615	1635	1645
Edinburgh	Haymarket	1548	1603	1618	1638	1648
Edinburgh	Shandwick Place	1550	1605	1620	1640	1650
EDINBURGH	St Andrew Sq Bus Station	1555	1610	1625	1645	1655

(a) How long does it take for the 1500 from Glasgow to get to Ratho?

(b) How long does it take for the 1545 from Glasgow to get to Edinburgh Haymarket?

(c) How long does it take the 1540 from Harthill to get to Murrayfield?

Solution

(a) The 1500 arrives at Ratho at 1547.

This is 47 minutes later than 1500, so the journey takes 47 minutes.

(b) The journey starts at 1545 and ends at 1648.

From 1545 to 1600 is 15 minutes.

From 1600 to 1648 is 48 minutes.

The total time is $15 + 48 = 63$ minutes or 1 hour 3 minutes.

(c) The journey starts at 1540 and ends at 1615.

From 1540 to 1600 is 20 minutes.

From 1600 to 1615 is 15 minutes.

The total time is $20 + 15 = 35$ minutes.

Example 2

The table gives the time differences between the UK and some other countries:

Austria	1 hour	ahead
Honduras	6 hours	behind
Samoa	11 hours	ahead
Tanzania	3 hours	ahead

(a) What time is it in *Austria* when it is 3.00 p.m. in the UK?

(b) What time is it in *Samoa* when it is 5.00 p.m. in the UK?

(c) When it is 5.00 p.m. in *Tanzania*, what time is it in the UK?

(d) If it is 4.00 p.m. in *Honduras*, what time is it in *Tanzania*?

Solution

(a) In Austria the time is 1 hour ahead of the UK, so it will be 4.00 p.m.

(b) In Samoa the time is 11 hours ahead of the UK, so it will be 4.00 a.m. the next day.

(c) In Tanzania the time is 3 hours ahead of the UK, so it will be 2.00 p.m.

(d) In Honduras the time is 6 hours behind the UK.
In the UK the time will be 10.00 p.m.
In Tanzania the time is 3 hours ahead of the UK, so it will be 1.00 a.m.

Exercises

1. The timetable below gives the times of some buses from Glasgow to Edinburgh:

GLASGOW	Buchanan Bus Stn	1015	1030	1045	1100	1115
Eurocentral	Gt A8 Interchange	–	–	–	–	–
Harthill	Service Area	1040	1055	1110	1125	1140
Newbridge East	A8 opp RACAL	1101	1116	1131	1146	1201
Ratho	Station Road End	1102	1117	1132	1147	1202
Ingliston Showground	West Entrance	1104	1119	1134	1149	1204
Corstorphine	Drum Brae	1108	1123	1138	1153	1208
Corstorphine	Station Road End	1110	1125	1140	1155	1210
Corstorphine	Zoo Park	1112	1127	1142	1157	1212
Murrayfield	Corstorphine Rd, Ice Rink	1115	1130	1145	1200	1215
Edinburgh	Haymarket	1118	1133	1148	1203	1218
Edinburgh	Shandwick Place	1120	1135	1150	1205	1220
EDINBURGH	St Andrew Sq Bus Station	1125	1140	1155	1210	1225

Use the timetable to answer these questions:

(a) You catch the 1030 from Glasgow. How long will it take to get to Harthill?

(b) You catch the 1100 from Glasgow. How long does it take to get to Ratho?

(c) You catch the bus at Harthill and arrive in Edinburgh at 1225. How long did the journey take?

(d) You arrive in Edinburgh at 1210, having travelled from Harthill. How long did the journey take?

(e) Do all the buses take the same time to get from Glasgow to Edinburgh?

2. The timetable below is for the Eurostar train that travels from Disneyland Paris (France) to Waterloo:

Disneyland Paris to Waterloo	
Disneyland Paris	1935
Ashford International	2037
Waterloo International	2139

How long is the journey time from Disneyland to:

(a) Ashford International,

(b) Waterloo International?

(Note that France is 1 hour ahead of UK time.)

3. The timetable below is for the North York Moors Railway.

Grosmont	dep.	0950	1050	1150	1250	1350	1450	1550	1650
Goatland	arr.	1005	1105	1205	1305	1405	1505	1605	1705
Goatland	dep.	1010	1110	1210	1310	1410	1510	1610	1710
Newtondale		1023	1123	1223	1323	1423	1523	1623	1723
Levisham	arr.	1036	1136	1236	1336	1436	1536	1636	1736
Levisham	dep.	1040	1140	1240	1340	1440	1540	1640	1740
Pickering	arr.	1100	1200	1300	1400	1500	1600	1700	1800

(a) Do all the journeys take the same time?

(b) For how long do the trains stay at Goathland?

(c) How long does it take to get from Grosmont to Newtondale?

(d) How long does it take to get from Grosmont to Levisham?

(e) For how long do the trains stay at Levisham?

4. Use the timetable below to answer the questions which follow:

Saturdays								
London Paddington	d	1135	1145	1235	1335	1535	1635	1700
Reading	d	1200	1210	1300	1400	1605	1705	1730
Newbury	d	–	–	–	–	–	1721	–
Pewsey	d	–	–	–	–	–	1740	1855
Westbury	d	–	1256	1346	1445	1650	1802	–
Castle Cary	d	–	1313	–	1503	–	1819	–
Bristol Temple Meads	d	–	–	–	–	–	–	–
Taunton	d	–	1335	1423	1529	1726	1841	–
Tiverton Parkway	d	–	1348	1502	1542	1738	1853	–
Exeter St Davids	d	1337	1404	1450	1600	1754	1909	–
Dawlish	d	1403	1423	1537	1646	1829	1921	–
Teignmouth	d	1409	1429	1543	1651	1834	1926	–
Newton Abbot	d	1415	1437	1512	1620	1814	1933	–
Torquay	a	1426	1448	1616	1711	1835	1955	–
Paignton	a	1433	1500	1623	1717	1841	2000	–
Totnes	d	–	1454	1526	1634	1829	1947	–
Plymouth	d	1436	1523	1558	1705	1902	2020	–
Liskeard	d	1502	1552	1628	1729	1926	–	–
Bodmin Parkway	d	1514	1604	1638	1741	1941	–	–
Par	d	–	1615	1652	1847	1953	–	–
St Austell	d	1530	1623	1658	1757	2001	–	–
Truro	d	1548	1641	1716	1815	2019	–	–
Redruth	d	1600	1653	1731	1827	2031	–	–
Camborne	d	–	1700	1738	1834	2038	–	–
St Erth	d	1614	1711	1751	1846	2051	–	–
Penzance	a	1630	1722	1805	1900	2107	–	–

(a) How long does it take for the 1635 from London Paddington to get to Plymouth?

(b) Adrian catches a train at 2:00 pm at Reading. How long does it take him to get to Truro?

(c) Sam catches the last train from Totnes to Penzance. How long does his journey take?

(d) Clare catches the 1445 from Westbury to Plymouth. At what time does she arrive in Plymouth?

(e) Which train has the shortest journey time from London to Penzance?

5. The timetable below is for the overnight ski train from London to the French Alps:

London Waterloo International	(Friday) depart	1957
Ashford International	(Friday) depart	2107
Moûtiers	(Saturday) arrive	0552
Aime la Plagne	(Saturday) arrive	0625
Bourg-St-Maurice	(Saturday) arrive	0645

How long does it take to travel:

(a) from Waterloo to Moûtiers,

(b) from Ashford to Aime la Plagne,

(c) from Waterloo to Bourg-St-Maurice?

6. Use the information given in Example 2 to answer these questions:

(a) What will be the time in Honduras when it is 7:00 am in the UK?

(b) What will be the time in the UK when it is 7:00 pm in Honduras?

(c) What is the time in Tanzania when it is 10:00 pm in the UK?

(d) What is the time in the UK when it is 10:00 pm in Samoa?

7. The flight from London to Stockholm (Sweden) takes $2\frac{1}{2}$ hours. The time in Sweden is 1 hour ahead of the UK.

(a) If you leave London at 1000, what will be the local time when you land at Stockholm?

(b) If you leave Stockholm at 1745, what will be the local time when you land in London?

8. A flight leaves Kuala Lumpur at 1100 and lands in London at 1710. The time in Kuala Lumpur is 8 hours ahead of the time in London. How long does the flight take?

9. Shainee earns £4 per hour on weekdays, £4.50 per hour on Saturdays and £6 per hour on Sundays.

The table below lists the hours he worked on each day of one week:

Day	*No. hours worked*
Monday	4
Tuesday	2
Wednesday	8
Thursday	10
Friday	3
Saturday	5
Sunday	2

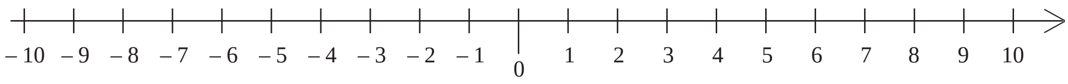
How much money did Shainee earn that week?

10. Ben earns £5 per hour for the first 40 hours he works and £7.50 per hour for any hours of overtime. One week he earned £290. How many hours overtime did he work?

15 Negative Numbers

15.1 Addition and Subtraction

In this section we consider how to add and subtract when working with negative numbers. This is done using a number line.



> To *add a positive number*, move to the *right* on a number line.
>
> To *add a negative number*, move to the *left* on a number line.

Example 1

Calculate:

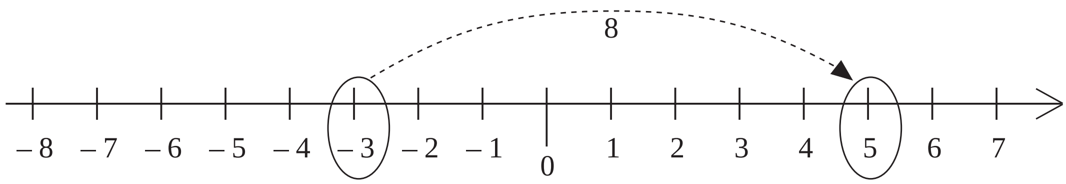
(a) $-3+8$ (b) $4+(-2)$

(c) $-8+5$ (d) $-1+(-3)$

Solution

(a) $-3+8$

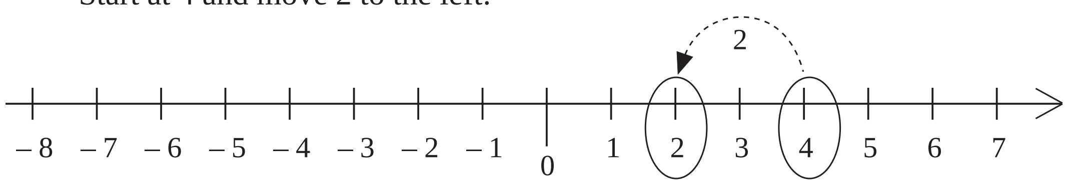
Start at -3 and move 8 to the right:



So, $-3+8=5$.

(b) $4+(-2)$

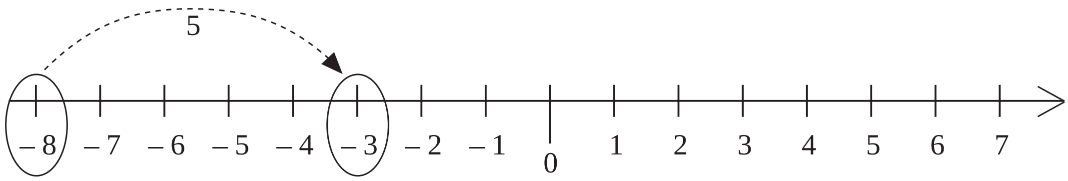
Start at 4 and move 2 to the left:



So, $4+(-2)=2$.

(c) $-8+5$

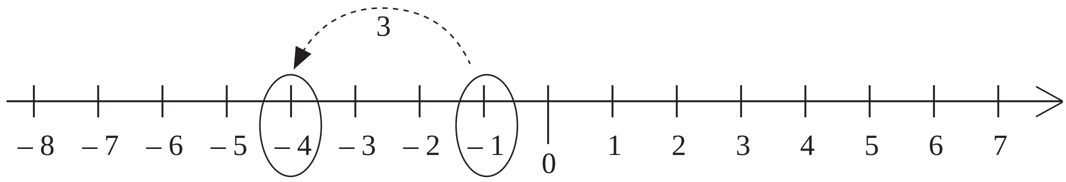
Start at -8 and move 5 to the right:



So, $-8+5=-3$.

(d) $-1+(-3)$

Start at -1 and move 3 to the left:



So, $-1+(-3)=-4$.

Subtraction is the *inverse* (or opposite) of addition, so

> To *subtract a positive number*, move to the *left* on a number line.
>
> To *subtract a negative number*, move to the *right* on a number line.

Example 2

Calculate:

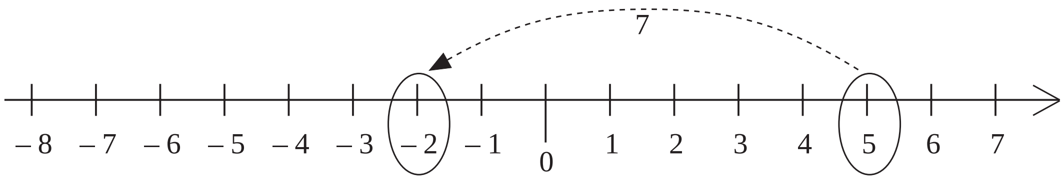
(a) $5-7$ (b) $-2-3$

(c) $4-(-2)$ (d) $-5-(-3)$

Solution

(a) $5-7$

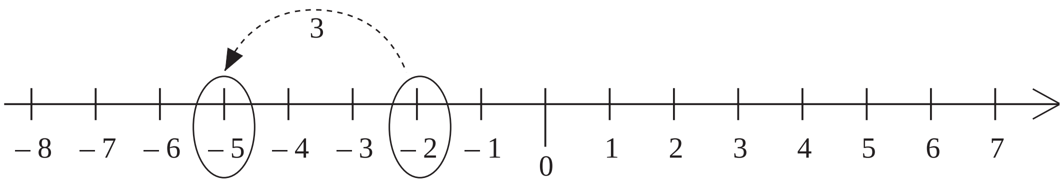
Start at 5 and move 7 to the left:



So, $5-7=-2$.

(b) $-2-3$

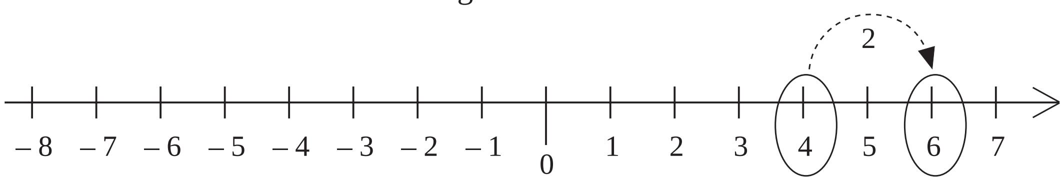
Start at -2 and move 3 to the left:



So, $-2-3=-5$.

(c) $4-(-2)$

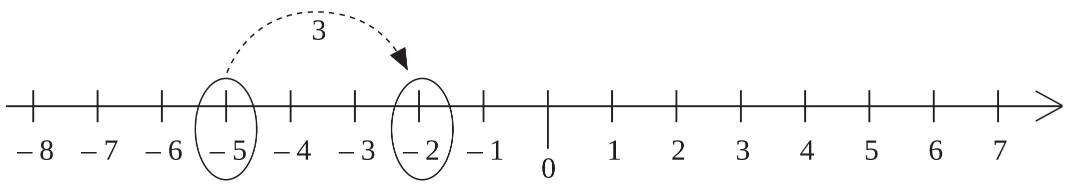
Start at 4 and move 2 to the right:



So, $4-(-2)=6$.

(d) $-5-(-3)$

Start at – 5 and move 3 to the right:



So, $5-(-3)=-2$.

Exercises

1. Use a number line to work out the following calculations:

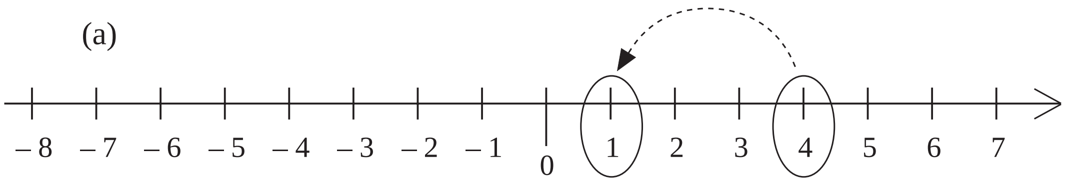
(a)	$-4+6$	(b)	$-5+8$	(c)	$-1+3$
(d)	$-4+7$	(e)	$2+(-3)$	(f)	$-1+(-4)$
(g)	$-2+(-3)$	(h)	$-6+6$	(i)	$-7+4$
(j)	$-6+2$	(k)	$-7+2$	(l)	$5+(-5)$

2. Use a number line to calculate:

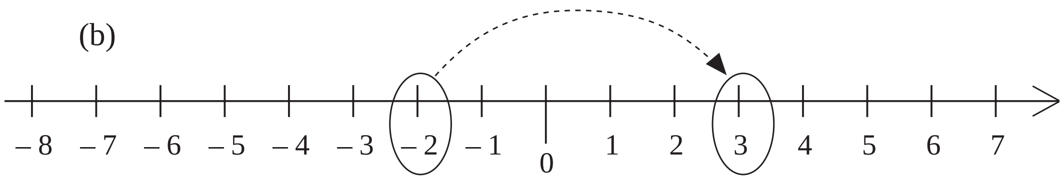
(a)	$4-6$	(b)	$5-7$	(c)	$2-4$
(d)	$-1-1$	(e)	$-3-2$	(f)	$-4-(-1)$
(g)	$3-(-4)$	(h)	$5-(-6)$	(i)	$8-12$
(j)	$-5-(-1)$	(k)	$4-9$	(l)	$-4-(-4)$

3. Write down the two possible sums that could be shown by each number line below:

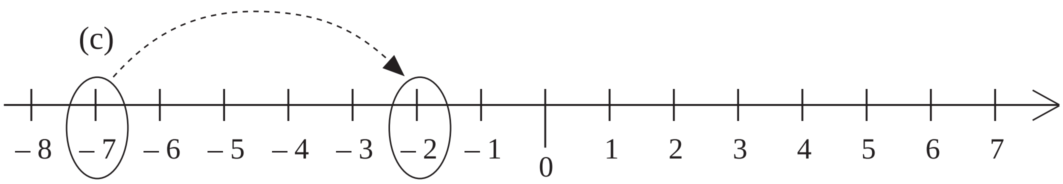
(a)



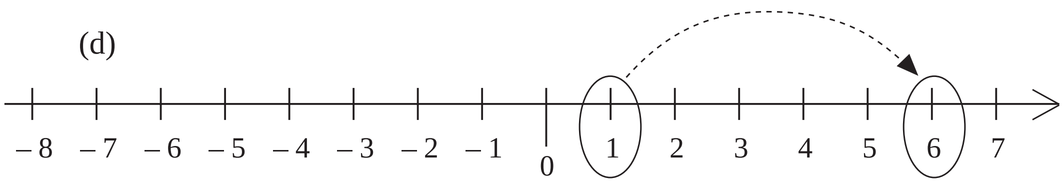
(b)



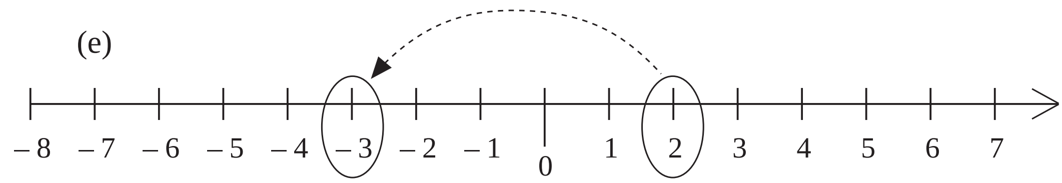
(c)



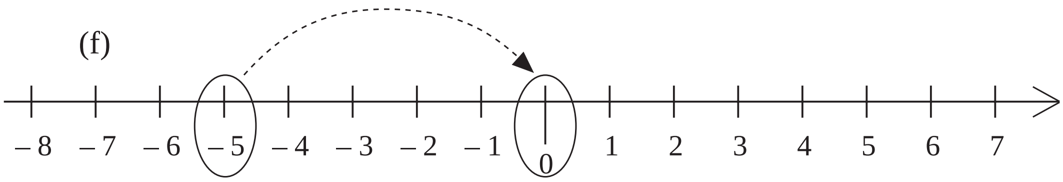
(d)



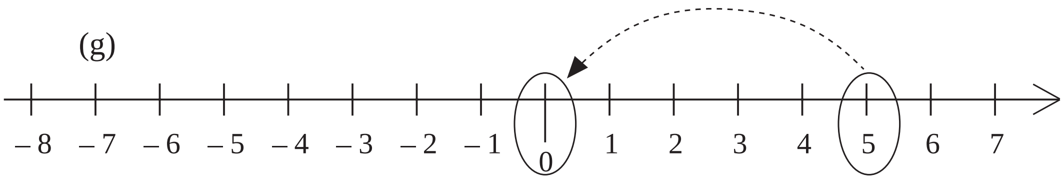
(e)



(f)



(g)



4. Copy and complete this addition table:

+	−4	−2	0	2	4
−3					
−1					
1					
3					

5. Fill in the missing numbers on a copy of this addition table:

+		1		3	
−5	−5				−1
		−3			
				0	
	−2		0		

6. Copy these equations and fill in the missing numbers:

(a) $4 + ... = 1$ (b) $3 - ..... = -6$

(c) $-6 + ... = -7$ (d) $..... + 7 = 2$

(e) $..... - 7 = -8$ (f) $4 - ..... = 7$

7. Write down the next 5 terms in each sequence:

(a) 10, 8, 6, 4, . . .

(b) $-10, \quad -7, \quad -4, \quad -1, \quad . . .$

(c) 19, 14, 9, 4, . . .

(d) $-1, \quad -3, \quad -5, \quad -7, \quad . . .$

(e) $-20, \quad -16, \quad -12, \quad -8, . . .$

(f) $-5, \quad -10, \quad -15, \quad -20 \quad . . .$

(g) 18, 15, 12, 9, . . .

(h) $-16, \quad -12, \quad -8, \quad -4, \quad . . .$

8. Overnight, the temperature dropped from $5\ °C$ to $-14\ °C$. By how many degrees did the temperature fall?

9. One day the level of the water in a river was 30 cm *above* its average level. One week later it was 12 cm *below* its average level. How far did the water level drop in the week?

10. A chest of treasure was hidden in the year 64 BC and found in 284 AD. For how long was the chest hidden?

15.2 Multiplication and Division

In this section we look at how to multiply and divide negative numbers.

Example 1

(a) Calculate

$$(-2) + (-2) + (-2) + (-2) + (-2).$$

(b) Fill in the missing numbers:

$$(-2) + (-2) + (-2) + (-2) + (-2) = ... \times (-2) = ...$$

Solution

(a) $(-2)+(-2)+(-2)+(-2)+(-2)=-10$

(b) $(-2)+(-2)+(-2)+(-2)+(-2)=5\times(-2)=-10$

In Example 1 we see that

a positive number multiplied by
a negative number gives a negative answer.

This table shows what happens to the sign of the answer when positive and negative numbers are *multiplied*:

×	+	−
+	+	−
−	−	+

The same table can be used for *division* of positive and negative numbers.

Example 2

Work out the following:

(a) $5\times(-7)$

(b) $(-8)\times(-10)$

(c) $(-42)\div 6$

(d) $(-88)\div(-8)$

Solution

(a) First calculate $5\times 7=35$.

As a positive number is multiplied by a negative number, the answer will be negative:

$$5\times(-7)=-35$$

(b) First calculate $8\times 10=80$.

Here a negative number is multiplied by a negative number, so the answer will be positive:

$$(-8)\times(-10)=80$$

(c) First calculate $42\div 6=7$.

As a negative number is divided by a positive number, the answer will be negative:

$$(-42)\div 6=-7$$

(c) First calculate $88 \div 8 = 11$.

As a negative number is divided by a negative number, the answer will be positive:

$$(-88) \div (-8) = 11$$

Exercises

1. Calculate:

(a) $(-7) \times 2$ (b) $(-4) \times 8$ (c) $(-2) \times (-5)$

(d) $(-6) \times (-3)$ (e) $(-3) \times 7$ (f) $(-10) \times (-4)$

(g) 8×4 (h) $3 \times (-6)$ (i) $(-7) \times (-2)$

(j) $(-4) \times (-5)$ (k) $(-7) \times 0$ (l) $8 \times (-5)$

2. Calculate:

(a) $(-10) \div (-2)$ (b) $(-15) \div 5$

(c) $18 \div (-3)$ (d) $14 \div (-7)$

(e) $(-21) \div (-3)$ (f) $(-45) \div 9$

(g) $50 \div (-5)$ (h) $(-100) \div (-4)$

(i) $80 \div (-2)$ (j) $26 \div (-13)$

(k) $(-70) \div (-7)$ (l) $(-42) \div 7$

3. Copy and complete these multiplication tables:

(a)

×	1	2	3	4
−1				
−2				
−3				
−4				

(b)

×	1	0	−1	−2	−3
−4					
−2					
0					
1					

4. Copy and complete these multiplication tables:

(a)

$\times$		-1	
2			
	-2	2	
-3			-9

(b)

$\times$	-2		
	10		
-2		6	
3			-12

5. Copy these calculations, filling in the missing numbers:

(a) $\ldots\ldots \times 5 = -20$

(b) $(-80) \div \ldots\ldots = 4$

(c) $16 \times \ldots\ldots = -32$

(d) $(-4) \times \ldots\ldots = 32$

(e) $\ldots\ldots \times (-3) = 12$

(f) $40 \div \ldots\ldots = -8$

(g) $-8 \times \ldots\ldots = 48$

(h) $-32 \div \ldots\ldots = 4$

(i) $15 \times \ldots\ldots = -60$

(j) $100 \div \ldots\ldots = -25$

6. Write down the next 3 terms in each sequence:

(a) 1, -2, 4, -8, 16, ...

(b) -1, 2, -4, 8, -16, ...

(c) 1, -10, 100, -1000, ...

(d) 1, -3, 9, -27, ...

(e) -1, 5, -25, 125, ...

For each sequence, describe the rule that is used to calculate the next term.

7. Make 2 copies of this multiplication table and fill in the missing numbers in 2 different ways:

$\times$				
	1			
		4		
			9	
				25

8. Calculate:

(a) $3 \times (-8) \times (-4)$
(b) $(-4) \times (-8) \times (-2)$
(c) $(-2) \times (-2) \times 2$
(d) $4 \times (-7) \times 2$
(e) $(-2) \times 8 \times (-4)$
(f) $(-6) \times (-2) \times (-1)$

9. Calculate:

(a) $\dfrac{(-3) \times (-4)}{(-2)}$
(b) $\dfrac{5 \times (-6)}{(-2)}$
(c) $\dfrac{(-7) \times (-5) \times (-2)}{5}$
(d) $\dfrac{8 \times (-9) \times 6}{(-2) \times (-3)}$
(e) $\dfrac{(-6) \times (-4)}{2}$
(f) $\dfrac{(-4) \times (-7) \times 3}{(-12)}$

10. Calculate:

(a) $(-6 + 10) \div (-2)$
(b) $(12 - 24) \div (-2)$
(c) $(6 + (-8)) \times (4 - 7)$
(d) $((-2) + 8) \times ((-4) + 2)$
(e) $((-4) \times 2) + (6 \times (-9))$
(f) $(8 \times (-2)) - ((-4) \times 8)$

11. Calculate:

(a) $(-6) \times (-3) + (-4)$
(b) $(-5) \times 4 - (-3)$
(c) $(-8) \times (-7) - 8 \times 7$
(d) $(-11) \times 4 + (-8) \times (-3)$

16 Algebra: Linear Equations

16.1 Fundamental Algebraic Skills

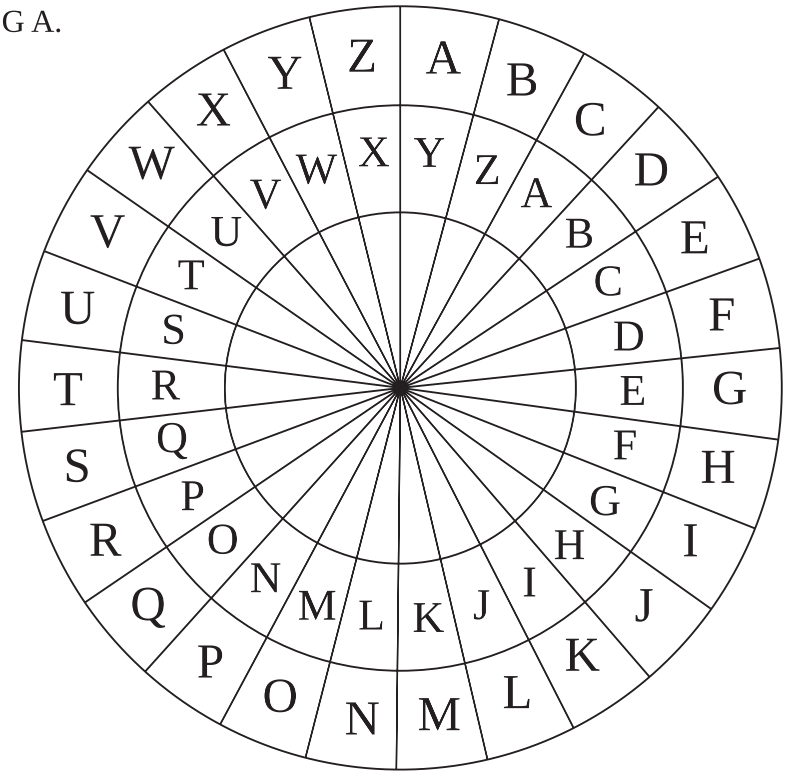
This section looks at some fundamental algebraic skills by examining codes and how to use formulae.

Example 1

Use this code wheel, which codes A on the outer ring as Y on the inner ring, to:

(a) code the word M A T H S,

(b) decode Q M L G A.



Solution

(a) Look for M on the outside circle of letters; this is coded as K which is the letter on the inside circle. Coding the other letters in the same way gives:

M	A	T	H	S
↓	↓	↓	↓	↓
K	Y	R	F	Q

(b) Look for Q on the inside circle. This decodes as S, which is the letter on the outside circle. Decoding the other letters in the same way gives:

Q	M	L	G	A
↓	↓	↓	↓	↓
S	O	N	I	C

Example 2

If $a = 4$, $b = 7$ and $c = 3$, calculate:

(a) $6 + b$ (b) $2a + b$ (c) ab (d) $a(b - c)$

Solution

(a) $6 + b = 6 + 7$
$= 13$

(b) $2a + b = 2 \times 4 + 7$ since $2a = 2 \times a$
$= 8 + 7$
$= 15$

(c) $ab = 4 \times 7$ since $ab = a \times b$
$= 28$

(d) $a(b - c) = 4 \times (7 - 3)$ since $a(b - c) = a \times (b - c)$
$= 4 \times 4$
$= 16$

Example 3

Simplify where possible:

(a) $2x + 4x$ (b) $5p + 7q - 3p + 2q$

(c) $y + 8y - 5y$ (d) $3t + 4s$

Solution

(a)
$$\begin{aligned} 2x + 4x &= 2 \times x + 4 \times x \\ &= (x + x) + (x + x + x + x) \\ &= 6 \times x \\ &= 6x \end{aligned}$$

(b)
$$\begin{aligned} 5p + 7q - 3p + 2q &= 5p - 3p + 7q + 2q \\ &= (5 - 3)p + (7 + 2)q \\ &= 2p + 9q \end{aligned}$$

(c) $y + 8y - 5y = 1y + 8y - 5y$

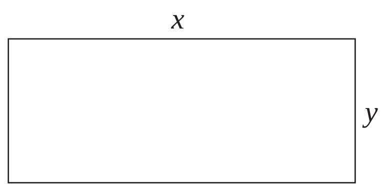
$= (1 + 8 - 5)y$

$= 4y$

(d) $3t + 4s$ cannot be simplified.

Example 4

Write down formulae for the area and perimeter of this rectangle:



Solution

Area $= x \times y$

$= xy$

Perimeter $= x + y + x + y$

$= 2x + 2y$

Exercises

1. Use the code wheel of Example 1 to:

   (a) code this message,

   M E E T M E A T H O M E

   (b) decode this message,

   M T C P R M W M S

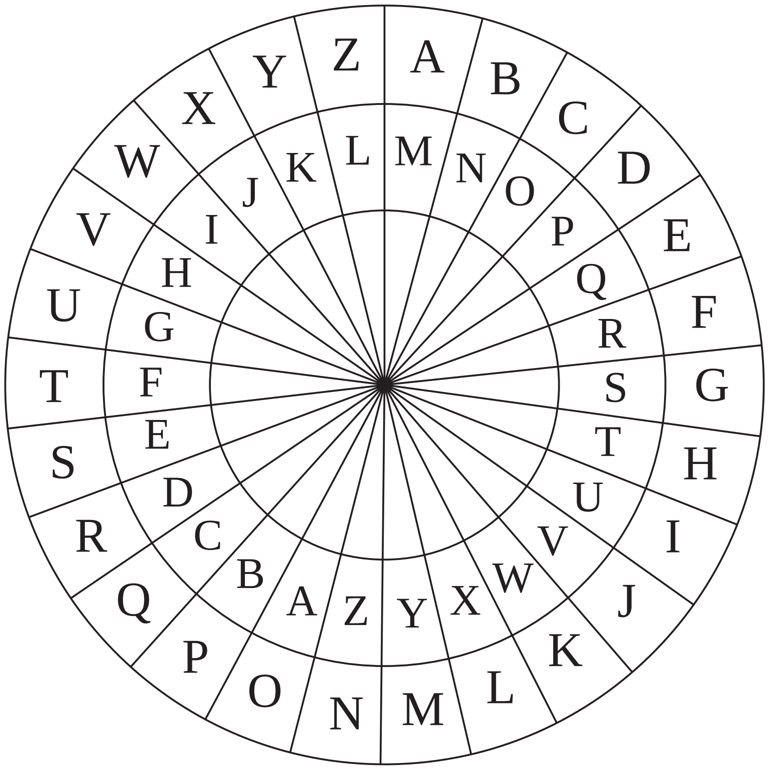
2. Use the code wheel opposite to:

   (a) code

   G O N E F I S H I N G,

   (b) decode

   T U S T R U H Q



3. Laura used a code wheel similar to the one above, but with the outer ring of letters rotated. She used her code wheel to code

	S	E	V	E	N		U	P
as	↓	↓	↓	↓	↓		↓	↓
	V	H	Y	H	Q		X	S

(a) Draw the code wheel that she used.

(b) Use the code wheel to decode:

G D Q J H U   D K H D G

4. If $a = 2$, $b = 6$, $c = 10$ and $d = 3$, calculate:

(a) $a + b$	(b) $c - b$	(c) $d + 7$
(d) $3a + d$	(e) $4a$	(f) ad
(g) $3b$	(h) $2c$	(i) $3c - b$
(j) $6a + b$	(k) $3a + 2b$	(l) $4a - d$

5. If $a = 3$, $b = -1$, $c = 2$ and $d = -4$, calculate:

(a) $a - b$ (b) $a + d$ (c) $b + d$

(d) $b - d$ (e) $3d$ (f) $a + b$

(g) $c - d$ (h) $2c + d$ (i) $3a - d$

(j) $2d + 3c$ (k) $4a - 2d$ (l) $5a + 3d$

6. If $a = 7$, $b = 5$, $c = -3$ and $d = 4$, calculate:

(a) $2(a + b)$ (b) $4(a - b)$ (c) $6(a - d)$

(d) $2(a + c)$ (e) $5(b - c)$ (f) $5(d - c)$

(g) $a(b + c)$ (h) $d(b + a)$ (i) $c(b - a)$

(j) $a(2b - c)$ (k) $d(2a - 3b)$ (l) $c(d - 2)$

7. Use the formula $s = \frac{1}{2}(u + v)t$ to find *s*, when $u = 10$, $v = 20$ and $t = 4$.

8. Use the formula $v = u + at$ to find *v*, if $u = 20$, $a = -2$ and $t = 7$.

9. Simplify, where possible:

(a) $2a + 3a$ (b) $5b + 8b$

(c) $6c - 4c$ (d) $5d + 4d + 7d$

(e) $6e + 9e - 5e$ (f) $8f + 6f - 13f$

(g) $9g + 7g - 8g - 2g - 6g$ (h) $5p + 2h$

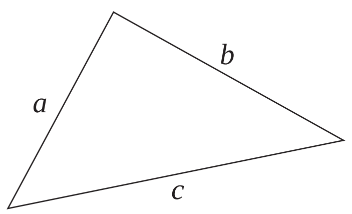
(i) $3a + 4b - 2a$ (j) $6x + 3y - 2x - y$

(k) $8t - 6t + 7s - 2s$

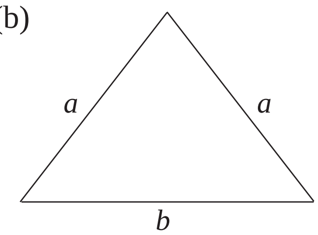
(l) $11m + 3n - 5p + 2q - 2n + 9q - 8m + 14p$

10. Write down formulae for the perimeter of each of these shapes:

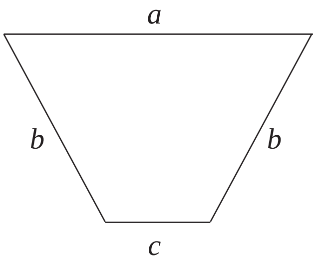
(a)



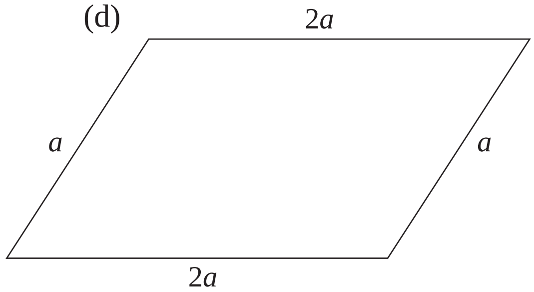
(b)



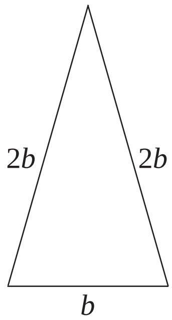
(c)



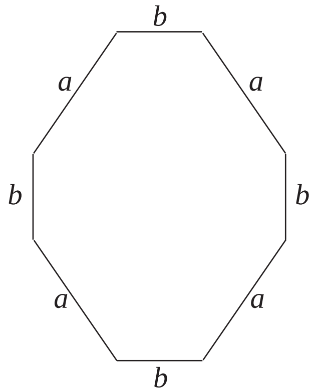
(d)



(e)



(f)



11. Sam asks her friend to think of a number, multiply it by 2 and then add 5. If the number her friend starts with is x, write down a formula for the number her friend gets.

12. A removal firm makes a fixed charge of £50, plus £2 for every mile travelled. Write down the formula for the cost of a removal when travelling x miles.

13. A taxi driver charges passengers £1, plus 50p per mile. Write down a formula for the cost of travelling x miles.

16.2 Function Machines

In this section we look at how to find the input and output of function machines, building on the work on number machines in Book Y7A.

INPUT ⟶ [FUNCTION MACHINE] ⟶ OUTPUT

Example 1

Calculate the output of each of these function machines:

(a) 4 ⟶ [$\times$ 5] ⟶ ?

(b) 5 ⟶ [$\times$ 2] ⟶ [$-$ 1] ⟶ ?

(c) $-3 \rightarrow \boxed{+\ 8} \rightarrow \boxed{\times\ 7} \rightarrow ?$

Solution

(a) The input is simply multiplied by 5 to give 20:

$$4 \rightarrow \boxed{\times\ 5} \rightarrow 20$$

(b) The input is multiplied by 2 to give 10, and then 1 is subtracted from this to give 9:

$$5 \rightarrow \boxed{\times\ 2} \xrightarrow{10} \boxed{-\ 1} \rightarrow 9$$

(c) Firstly, 8 is added to the input to give 5, and this is then multiplied by 7 to give 35:

$$-3 \rightarrow \boxed{+\ 8} \xrightarrow{5} \boxed{\times\ 7} \rightarrow 35$$

Example 2

Calculate the input for each of these function machines:

(a) $? \rightarrow \boxed{\times\ 4} \rightarrow 8$

(b) $? \rightarrow \boxed{+\ 2} \rightarrow \boxed{\times\ 5} \rightarrow 25$

(c) $? \rightarrow \boxed{-\ 5} \rightarrow \boxed{\times\ 3} \rightarrow 6$

Solution

The missing inputs can be found by reversing the machines and using the inverse (i.e. opposite) operations in each machine:

(a) $? \rightarrow \boxed{\times\ 4} \rightarrow 8$

$$2 \leftarrow \boxed{\div\ 4} \leftarrow 8$$

(b) $? \rightarrow \boxed{+\ 2} \rightarrow \boxed{\times\ 5} \rightarrow 25$

$$3 \leftarrow \boxed{-\ 2} \xleftarrow{5} \boxed{\div\ 5} \leftarrow 25$$

(c) $? \rightarrow \boxed{-\ 5} \rightarrow \boxed{\times\ 3} \rightarrow 6$

$$7 \leftarrow \boxed{+\ 5} \xleftarrow{2} \boxed{\div\ 3} \leftarrow 6$$

Note that:

Operation	*Inverse Operation*
+	–
–	+
×	÷
÷	×

Exercises

1. What is the output of each of these function machines:

   (a) 4 → [+ 6] → ?

   (b) 3 → [× 10] → ?

   (c) 10 → [− 7] → ?

   (d) 14 → [÷ 2] → ?

   (e) 21 → [÷ 3] → ?

   (f) 100 → [× 5] → ?

2. What is the output of each of these function machines:

   (a) 3 → [× 4] → [− 7] → ?

   (b) 10 → [− 8] → [× 7] → ?

   (c) 8 → [− 5] → [× 5] → ?

   (d) –2 → [× 6] → [+ 20] → ?

   (e) 7 → [+ 2] → [÷ 3] → ?

   (f) –5 → [+ 8] → [× 9] → ?

3. What is the output of each of these function machines:

(a) ? → $\times 5$ → 30

(b) ? → $+ 8$ → 12

(c) ? → $- 9$ → 11

(d) ? → $\div 4$ → 5

(e) ? → $+ 12$ → 21

(f) ? → $\times 7$ → 42

4. What is the input of each of these *double function* machines:

(a) ? → $+ 1$ → $\times 4$ → 12

(b) ? → $+ 7$ → $\div 6$ → 4

(c) ? → $\times 4$ → $+ 9$ → 37

(d) ? → $\times 9$ → $- 20$ → 34

(e) ? → $\div 6$ → $- 1$ → 7

(f) ? → $- 6$ → $\div 7$ → 9

(g) ? → $+ 8$ → $\times 4$ → 24

(h) ? → $\times 2$ → $+ 7$ → -3

5. Here is a *triple function* machine:

Input → $\times 7$ → $- 5$ → $\div 2$ → Output

(a) What is the *output* if the input is 8.

(b) What is the *input* if the output is 22.

(c) What is the *input* if the output is -13.

6. A number is multiplied by 10, and then 6 is added to get 36.
What was the number?

7. Karen asks her teacher, Miss Sharp, how old she is. Miss Sharp replies that if you double her age, add 7 and then divide by 3, you get 21. How old is Miss Sharp?

8. Sally is given her pocket money. She puts half in the bank and then spends £3 in one shop and £2.50 in another shop. She goes home with £1.25. How much pocket money was she given?

9. A bus has its maximum number of passengers when it leaves the bus station. At the first stop, half of the passengers get off. At the next stop 7 people get on and at the next stop 16 people get off. There are now 17 people on the bus. How many passengers were on the bus when it left the bus station?

10. Prakesh buys a tomato plant. In the first week it doubles its height. In the second week it grows 8 cm. In the third week it grows 5 cm. What was the height of the plant when Prakesh bought it if it is now 35 cm in height?

16.3 Linear Equations

An *equation* is a statement, such as $3x + 2 = 17$, which contains an unknown number, in this case, x. The aim of this section is to show how to find the unknown number, x.

All equations contain an 'equals' sign.

To solve the equation, you need to reorganise it so that the unknown value is by itself on one side of the equation. This is done by performing operations on the equation. When you do this, in order to keep the equality of the sides, you must remember that

> *whatever you do to one side of an equation, you must also do the same to the other side*

Example 1

Solve these equations:

(a) $x + 2 = 8$ (b) $x - 4 = 3$ (c) $3x = 12$

(d) $\dfrac{x}{2} = 7$ (e) $2x + 5 = 11$ (f) $3 - 2x = 7$

Solution

(a) To solve this equation, subtract 2 from each side of the equation:

$$x + 2 = 8$$

$$x + 2 - 2 = 8 - 2$$

$$x = 6$$

(b) To solve this equation, add 4 to both sides of the equation:

$$x - 4 = 3$$
$$x - 4 + 4 = 3 + 4$$
$$x = 7$$

(c) To solve this equation, divide both sides of the equation by 3:

$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

(d) To solve this equation, multiply both sides of the equation by 2:

$$\frac{x}{2} = 7$$
$$2 \times \frac{x}{2} = 2 \times 7$$
$$x = 14$$

(e) This equation must be solved in 2 stages.
First, subtract 5 from both sides:

$$2x + 5 = 11$$
$$2x + 5 - 5 = 11 - 5$$
$$2x = 6$$

Then, divide both sides of the equation by 2:

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

(f) First, subtract 3 from both sides:

$$3 - 2x = 7$$
$$3 - 2x - 3 = 7 - 3$$
$$-2x = 4$$

Then divide both sides by (-2):

$$\frac{-2x}{-2} = \frac{4}{-2}$$
$$x = -2$$

Example 2

Solve these equations:

(a) $3x + 2 = 4x - 3$

(b) $2x + 7 = 8x - 11$

Solution

These equations contain *x* on both sides. The first step is to change them so that *x* is on only *one* side of the equation. Choose the side which has the most *x*; here, the right hand side.

(a) Subtract $3x$ from both sides of the equation:

$$3x + 2 = 4x - 3$$

$$3x + 2 - 3x = 4x - 3 - 3x$$

$$2 = x - 3$$

Then add 3 to both sides of the equation:

$$2 = x - 3$$

$$2 + 3 = x - 3 + 3$$

$$5 = x$$

so $x = 5$

> Note: *it is conventional to give the answer with the unknown value, x, on the left hand side, and its value on the right hand side.*

(b) First, subtract $2x$ from both sides of the equation:

$$2x + 7 = 8x - 11$$

$$2x + 7 - 2x = 8x - 11 - 2x$$

$$7 = 6x - 11$$

Next, add 11 to both sides of the equation:

$$7 + 11 = 6x - 11 + 11$$

$$18 = 6x$$

Then divide both sides by 6:

$$\frac{18}{6} = \frac{6x}{6}$$

$$3 = x$$

so $x = 3$

Example 3

You ask a friend to think of a number. He then multiplies it by 5 and subtracts 7. He gets the answer 43.

(a) Use this information to write down an equation for x, the unknown number.

(b) Solve your equation for x.

Solution

(a) As x = number your friend thought of, then

$$x \longrightarrow \boxed{\times\ 5} \xrightarrow{5x} \boxed{-\ 7} \longrightarrow 5x - 7$$

So $5x - 7 = 43$

(b) First, add 7 to both sides of the equation to give

$$5x = 50$$

Then divide both sides by 5 to give

$$x = 10$$

and this is the number that your friend thought of.

Exercises

1. Solve these equations:

(a) $x + 2 = 8$	(b) $x + 5 = 11$	(c) $x - 6 = 2$
(d) $x - 4 = 3$	(e) $2x = 18$	(f) $3x = 24$
(g) $\frac{x}{6} = 4$	(h) $\frac{x}{5} = 9$	(i) $6x = 54$
(j) $x + 12 = 10$	(k) $x + 5 = 3$	(l) $x - 22 = -4$
(m) $\frac{x}{7} = -2$	(n) $10x = 0$	(o) $\frac{x}{2} + 4 = 5$

2. Solve these equations:

(a) $2x + 4 = 14$	(b) $3x + 7 = 25$	(c) $4x + 2 = 22$
(d) $6x - 4 = 26$	(e) $5x - 3 = 32$	(f) $11x - 4 = 29$
(g) $3x + 4 = 25$	(h) $5x - 8 = 37$	(i) $6x + 7 = 31$
(j) $3x + 11 = 5$	(k) $6x + 2 = -10$	(l) $7x + 44 = 2$

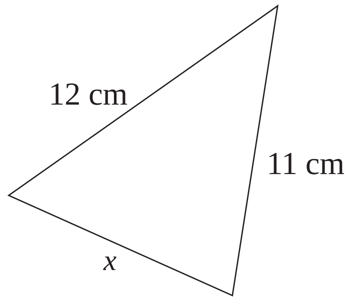
3. Solve these equations, giving your answers as fractions or mixed numbers:

(a) $3x = 4$ (b) $5x = 7$ (c) $2x + 8 = 13$

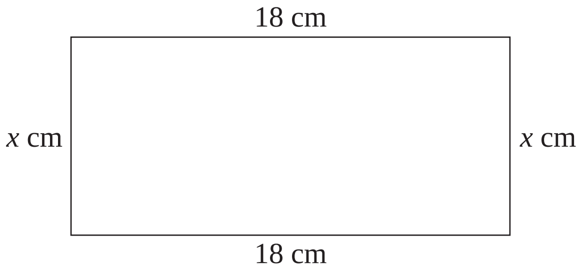
(d) $8x + 2 = 5$ (e) $2x + 6 = 9$ (f) $4x - 7 = 10$

4. The perimeter of this triangle is 31 cm.

Use this information to write down an equation for x and solve it to find x.



5. (a) Write down an expression for the length of the perimeter of this rectangle:



(b) Find x if the perimeter length is 48 cm.

(c) Find x if the perimeter length is 45 cm.

6. Tom asks each of his friends to think of their age, double it and then take away 10.

Here are the answers he is given:

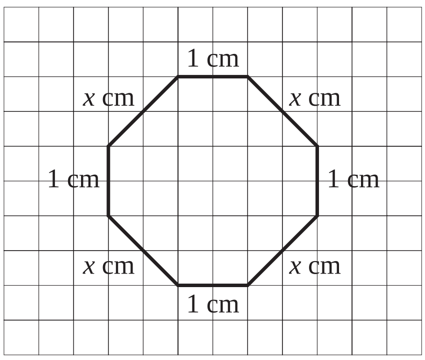
Ben	*Ian*	*Adam*	*Sergio*
8	10	14	11

(a) Using x to represent Ben's age, write down an equation for x and solve it to find Ben's age.

(b) Write down and solve equations to find the ages of Ian, Adam and Sergio.

7. The perimeter of this octagon is 9.6 cm.

Write down an equation and solve it to find x.



8. Solve these equations:

(a) $x + 2 = 2x - 1$ (b) $8x - 1 = 4x + 11$

(c) $5x + 2 = 6x - 4$ (d) $11x - 4 = 2x + 23$

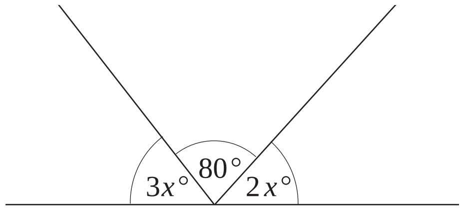
(e) $5x + 1 = 6x - 8$ (f) $3x + 2 + 5x = x + 44$

(g) $6x + 2 - 2x = x + 23$ (h) $2x - 3 = 6x + x - 58$

(i) $3x + 2 = x - 8$ (j) $4x - 2 = 2x - 8$

(k) $3x + 82 = 10x + 12$ (l) $6x - 10 = 2x - 14$

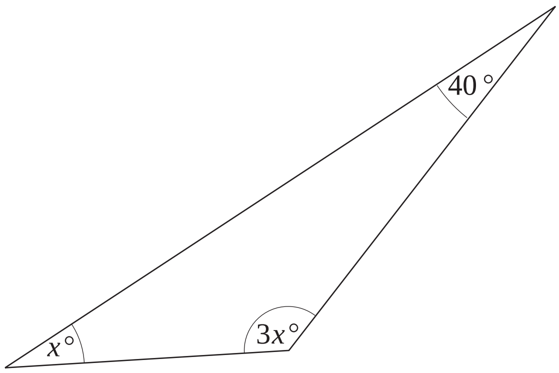
9. The diagram below shows three angles on a straight line:



(a) Write down an equation and use it to find x.

(b) Write down the sizes of the two unknown angles and check that the three angles shown add up to $180°$.

10. Use an equation to find the sizes of the unknown angles in this triangle:



11. Karen thinks of a number, multiplies it by 3 and then adds 10. Her answer is 11 more than the number she thought of. If x is her original number, write down an equation and solve it to find x.

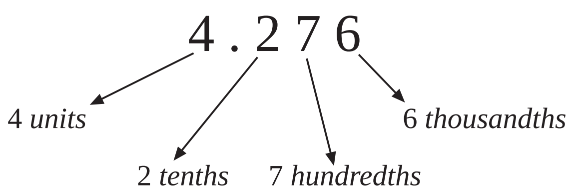
17 Arithmetic: Decimals, Fractions and Percentages

17.1 Conversion: Decimals into Fractions

In this section we revise ideas of decimals and work on writing decimals as fractions.

Recall that the number

means



The table below shows how to write the fractions you need to know in order to write decimals as fractions:

Decimal	*Words*	*Fraction*
0.1	1 tenth	$\frac{1}{10}$
0.01	1 hundredth	$\frac{1}{100}$
0.001	1 thousandth	$\frac{1}{1000}$

Example 1

Write these numbers in order, with the smallest first:

0.7, 0.17, 0.77, 0.71, 0.701, 0.107

Solution

Note: It is, perhaps, easier to see this if we first write *all* the numbers to 3 decimal places,

i.e. 0.700, 0.170, 0.770, 0.710, 0.701, 0.107

The required order is:

0.107, 0.17, 0.7, 0.701, 0.71, 0.77

Example 2

Write these numbers as fractions, where possible giving them in their simplest form:

(a) 0.7 (b) 0.09 (c) 0.004

(d) 0.47 (e) 0.132 (f) 1.75

Solution

(a) $0.7 = \frac{7}{10}$

(b) $0.09 = \frac{9}{100}$

(c) $0.004 = \frac{4}{1000} = \frac{1}{250}$

(d) $0.47 = \frac{47}{100}$

(e) $0.132 = \frac{132}{1000} = \frac{33}{250}$

(f) $1.75 = \frac{175}{100} = \frac{7}{4}$ (note that fractions larger than 1, such as this, are often referred to as *improper* or *vulgar fractions)*

Exercises

1. What is the value of the 7 in each of these numbers:

   (a) 0.714 (b) 0.070 (c) 7.042

   (d) 0.007 (e) 0.471 (f) 0.157

2. Write each list of numbers in order with the smallest first:

   (a) 0.61, 0.16, 0.601, 0.106, 0.661, 0.616

   (b) 0.47, 0.82, 0.4, 0.78, 0.28

   (c) 0.32, 0.23, 0.2, 0.301, 0.3

   (d) 0.17, 0.19, 0.9, 0.91, 0.79

3. Write each of these decimals as a fraction, giving them in their simplest form:

(a)	0.1	(b)	0.9	(c)	0.3
(d)	0.07	(e)	0.25	(f)	0.001
(g)	0.05	(h)	0.003	(i)	0.017
(j)	0.71	(k)	0.87	(l)	0.201

4. Write each of these decimals as a fraction and simplify where possible:

(a)	0.4	(b)	0.08	(c)	0.54
(d)	0.006	(e)	0.012	(f)	0.162
(g)	0.048	(h)	0.84	(i)	0.328
(j)	0.014	(k)	0.006	(l)	0.108

5. Write down the missing numbers:

(a)	$0.6 = \frac{?}{5}$	(b)	$0.14 = \frac{?}{50}$	(c)	$0.18 = \frac{?}{50}$
(d)	$0.008 = \frac{?}{125}$	(e)	$0.012 = \frac{?}{250}$	(f)	$0.016 = \frac{?}{125}$

6. Write these numbers as *improper fractions* in their simplest form:

(a)	1.2	(b)	3.02	(c)	4.12
(d)	3.62	(e)	4.008	(f)	5.015

7. Calculate, giving your answers as decimals *and* as fractions:

(a)	$0.7 + 0.6$	(b)	$0.8 - 0.3$
(c)	$0.71 + 0.62$	(d)	$8.21 - 0.31$
(e)	$0.06 + 0.3$	(f)	$1.7 + 0.21$
(g)	$8.06 - 0.2$	(h)	$0.42 - 0.002$

8. Write the missing numbers as decimals and convert them to fractions in their simplest form:

(a)	$0.20 + \ ? \ = 0.81$	(b)	$0.42 + \ ? \ = 0.53$
(c)	$0.91 - \ ? \ = 0.47$	(d)	$0.92 - \ ? \ = 0.58$

9. Convert these decimals to fractions:

(a) 0.0001 (b) 0.0009

(c) 0.00021 (d) 0.123491

10. Convert these decimals to fractions in their simplest form:

(a) 0.00008 (b) 0.02222

(c) 0.00102 (d) 0.000004

(e) 0.000224 (f) 0.0000002

17.2 Conversion: Fractions into Decimals

In this section we consider how to write fractions as decimals.

Example 1

Write these fractions as decimals:

(a) $\frac{7}{10}$ (b) $\frac{81}{100}$ (c) $\frac{9}{1000}$ (d) $\frac{407}{1000}$

Solution

(a) $\frac{7}{10} = 0.7$

(b) $\frac{81}{100} = 0.81$

(c) $\frac{9}{1000} = 0.009$

(d) $\frac{407}{1000} = 0.407$

Example 2

Write these fractions as decimals:

(a) $\frac{2}{5}$ (b) $\frac{3}{50}$ (c) $\frac{6}{25}$ (d) $\frac{5}{4}$ (e) $\frac{7}{250}$

Solution

In each case, determine the equivalent fraction with the denominator as either 10, 100 or 1000. The fractions can then be written as decimals.

(a) $\frac{2}{5} = \frac{4}{10} = 0.4$ (b) $\frac{3}{50} = \frac{6}{100} = 0.06$

(c) $\frac{6}{25} = \frac{24}{100} = 0.24$ (d) $\frac{5}{4} = \frac{125}{100} = 1.25$

(e) $\frac{7}{250} = \frac{28}{1000} = 0.028$

Example 3

(a) Calculate $18 \div 5$, then write $\frac{18}{5}$ as a decimal.

(b) Calculate $5 \div 8$, then write $\frac{5}{8}$ as a decimal.

Solution

(a) $18 \div 5 = 3.6$, since $\begin{array}{r} 3.6 \\ 5\overline{\smash{)}\,1{}^{3}8.0} \end{array}$

So $\frac{18}{5} = 18 \div 5$

$= 3.6$

(b) $5 \div 8 = 0.625$, since $\begin{array}{r} 0.625 \\ 8\overline{\smash{)}\,5.0{}^{2}0{}^{4}0} \end{array}$

So $\frac{5}{8} = 5 \div 8$

$= 0.625$

Exercises

1. Write these fractions as decimals:

(a) $\frac{3}{10}$ (b) $\frac{7}{100}$ (c) $\frac{9}{1000}$

(d) $\frac{13}{100}$ (e) $\frac{131}{1000}$ (f) $\frac{47}{1000}$

(g) $\frac{21}{100}$ (h) $\frac{183}{1000}$ (i) $\frac{19}{100}$

(j) $\frac{19}{1000}$ (k) $\frac{11}{100}$ (l) $\frac{81}{1000}$

2. Calculate the missing numbers:

(a) $\frac{?}{2} = \frac{5}{10}$ (b) $\frac{?}{20} = \frac{35}{100}$

(c) $\frac{?}{25} = \frac{8}{100}$ (d) $\frac{?}{4} = \frac{25}{100}$

(e) $\frac{2}{?} = \frac{4}{100}$ (f) $\frac{6}{?} = \frac{12}{1000}$

(g) $\frac{8}{?} = \frac{32}{100}$ (h) $\frac{7}{?} = \frac{28}{100}$

3. Write these fractions as decimals:

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{9}{50}$

(d) $\frac{3}{25}$ (e) $\frac{3}{20}$ (f) $\frac{3}{500}$

(g) $\frac{1}{250}$ (h) $\frac{7}{20}$ (i) $\frac{61}{200}$

(j) $\frac{18}{25}$ (k) $\frac{9}{125}$ (l) $\frac{1}{4}$

4. Write these improper fractions as decimals:

(a) $\frac{12}{10}$ (b) $\frac{212}{100}$ (c) $\frac{5218}{1000}$

(d) $\frac{2008}{100}$ (e) $\frac{2008}{1000}$ (f) $\frac{418}{10}$

5. Write these improper fractions as decimals:

(a) $\frac{7}{2}$ (b) $\frac{21}{20}$ (c) $\frac{33}{20}$

(d) $\frac{31}{25}$ (e) $\frac{16}{5}$ (f) $\frac{1001}{500}$

6. Write as a fraction and as a decimal:

(a) $3 \div 5$ (b) $3 \div 8$ (c) $25 \div 4$

(d) $16 \div 5$ (e) $26 \div 4$ (f) $30 \div 8$

7. (a) Calculate $7 \div 8$.

(b) Write $\frac{7}{8}$ as a decimal.

8. (a) Calculate $41 \div 5$.

(b) Write $\frac{41}{5}$ as a decimal.

9. Write $\frac{1}{8}$ as a decimal by using division.

10. Write $\frac{13}{16}$ as a decimal.

17.3 Introduction to Percentages

The word 'percentage' means 'per hundred'. In this section we look at how percentages can be used as an alternative to fractions or decimals.

$$100\% = \frac{100}{100} = 1$$

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$1\% = \frac{1}{100}$$

Example 1

Draw diagrams to show:

(a) 71%

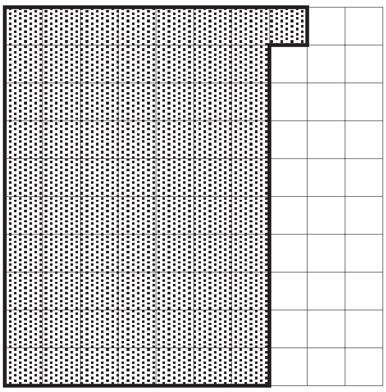
(b) 20%

(c) 5%

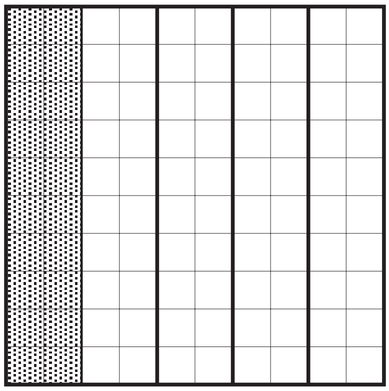
Solution

These percentages can be shown by shading a suitable fraction of a 10 by 10 square shape.

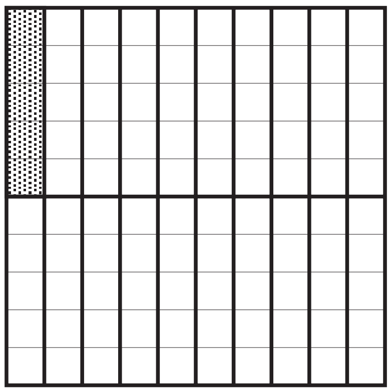
(a) $71\% = \frac{71}{100}$, so $\frac{71}{100}$ of a shape needs to be shaded:



(b) $20\% = \frac{20}{100} = \frac{1}{5}$, so $\frac{1}{5}$ of a shape needs to be shaded:



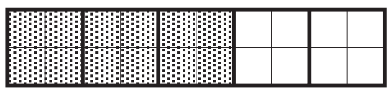
(c) $5\% = \frac{5}{100} = \frac{1}{20}$, so $\frac{1}{20}$ of a shape needs to be shaded:



Example 2

(a) What percentage of this shape is shaded?

(b) What percentage of this shape is *not* shaded?



Solution

(a) $\frac{3}{5}$ of the shape is shaded, and

$$\frac{3}{5} = \frac{6}{10}$$

$$= \frac{60}{100},$$

so 60% is shaded.

(b) $100 - 60 = 40$, so 40% is *not* shaded.

Example 3

Find:

(a) 5% of 100 kg,

(b) 20% of 40 m,

(c) 25% of £80.

Solution

(a) 5% of 100 kg $= \frac{5}{100} \times 100$

$= \frac{1}{20} \times 100$

$= 5$ kg

(b) 20% of 40 m $= \frac{20}{100} \times 40$

$= \frac{1}{5} \times 40$

$= 8$ m

(c) 25% of £80 $= \frac{25}{100} \times 80$

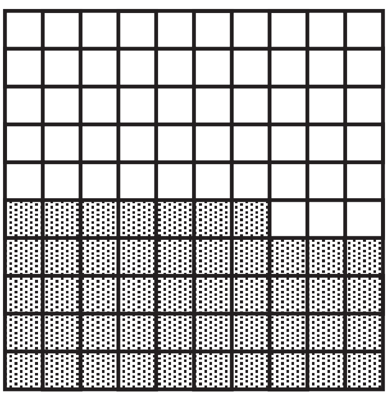
$= \frac{1}{4} \times 80$

$=$ £20

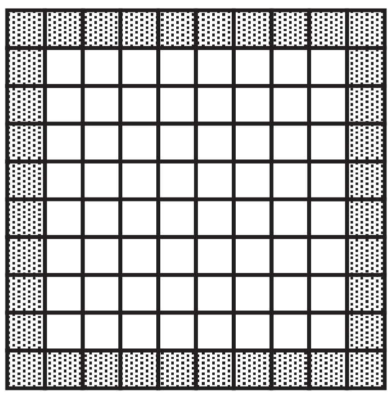
Exercises

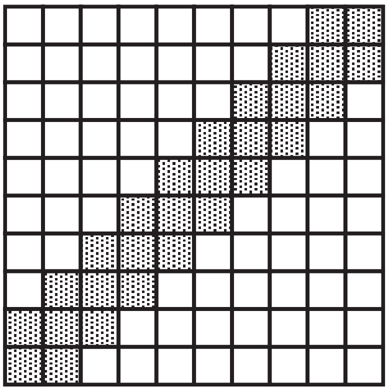
1. For each diagram, state the percentage that is shaded:

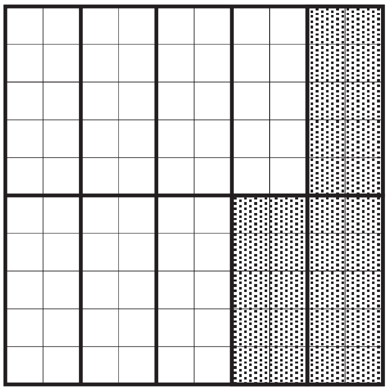
(a)

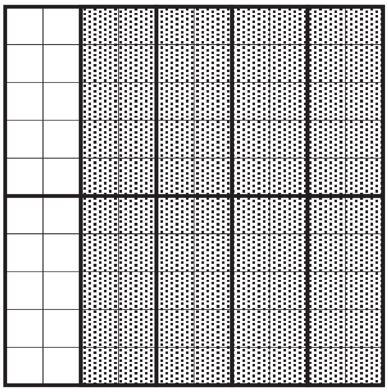


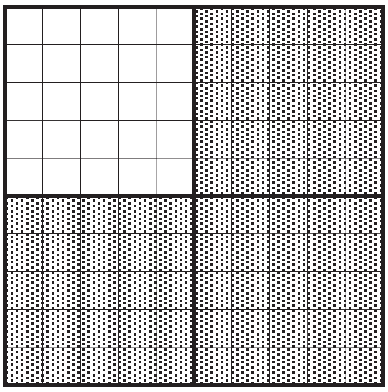
(b)



(c) 

(d) 

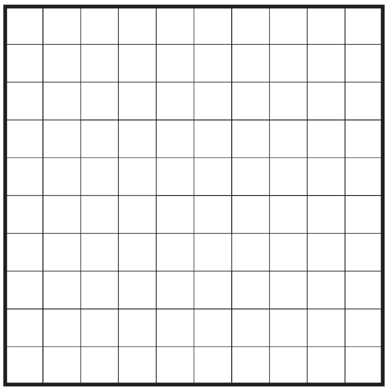
(e) 

(f) 

2. For each diagram in question 1, state the percentage that is *not* shaded.

3. If 76% of a rectangle is shaded, what percentage is *not* shaded?

4. Make 4 copies of this diagram and shade the percentage stated:



   (a) 23%

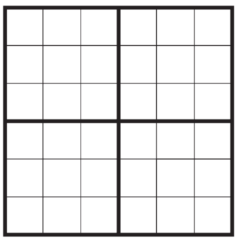
   (b) 50%

   (c) 79%

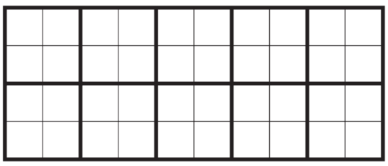
   (d) 87%

5. Copy each diagram and shade the percentage stated:

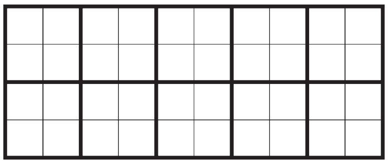
   (a) 25%



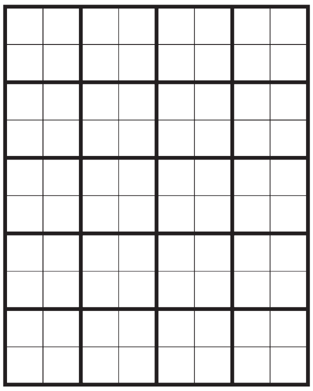
   (b) 30%



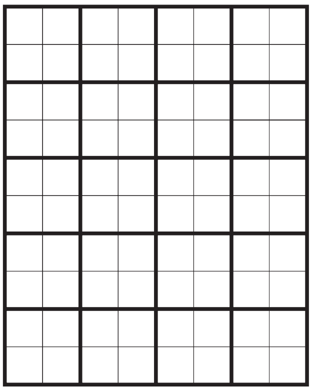
(c) 90%



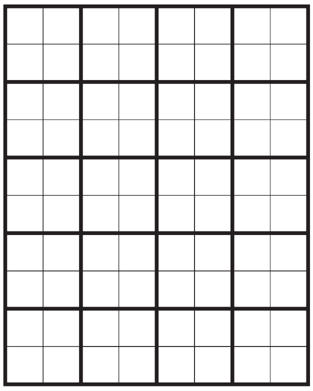
(d) 5%



(e) 15%

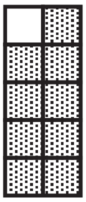


(f) 65%

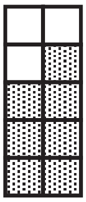


6. State the shaded percentage of each shape:

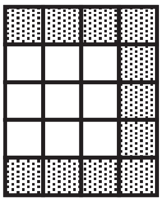
(a)



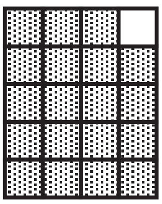
(b)



(c)

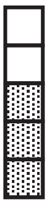


(d)

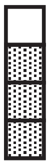


(e)

(f)



(g)



7. If 35% of a class are girls, what percentage are boys?

8. If 88% of a class pass a maths test, what percentage fail the test?

9. Calculate:

(a) 50% of £200,

(b) 30% of 500 kg,

(c) 60% of 50p,

(d) 5% of £2,

(e) 15% of 10 kg,
(f) 25% of 120 m,
(g) 2% of £400,
(h) 26% of £2,
(i) 20% of £300,
(j) 75% of 200 kg.

10. Ben and Adam spend their Saturdays cleaning cars. They agree that Adam will have 60% of the money they earn and that Ben will have the rest.

(a) What percentage of the money will Ben have?

(b) How much do they each have if they earn £25?

(c) How much do they each have if they earn £30?

17.4 Decimals, Fractions and Percentages

In this section we concentrate in converting between decimals, fractions and percentages.

Example 1

Write these percentages as decimals:

(a) 72% (b) 3%

Solution

(a) $72\% = \dfrac{72}{100}$

$= 0.72$

(b) $3\% = \dfrac{3}{100}$

$= 0.03$

Example 2

Write these decimals as percentages:

(a) 0.71 (b) 0.4 (c) 0.06

Solution

(a) $0.71 = \dfrac{71}{100}$

$= 71\%$

(b) $0.4 = \frac{4}{10}$

$= \frac{40}{100}$

$= 40\%$

(c) $0.06 = \frac{6}{100}$

$= 6\%$

Example 3

Write these percentages as fractions in their simplest possible form:

(a) 90% (b) 20% (c) 5%

Solution

(a) $90\% = \frac{90}{100}$

$= \frac{9}{10}$

(b) $20\% = \frac{20}{100}$

$= \frac{1}{5}$

(c) $5\% = \frac{5}{100}$

$= \frac{1}{20}$

Example 4

Write these fractions as percentages:

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{7}{20}$

Solution

(a) $\frac{1}{2} = \frac{50}{100}$

$= 50\%$

(b) $\dfrac{2}{5} = \dfrac{40}{100}$

$= 40\%$

(c) $\dfrac{7}{20} = \dfrac{35}{100}$

$= 35\%$

Exercises

1. Write these percentages as decimals:

(a)	42%	(b)	37%	(c)	20%
(d)	5%	(e)	8%	(f)	10%
(g)	22%	(h)	3%	(i)	15%

2. Write these decimals as percentages:

(a)	0.14	(b)	0.72	(c)	0.55
(d)	0.4	(e)	0.03	(f)	0.9
(g)	0.18	(h)	0.04	(i)	0.7

3. Write these percentages as fractions in their simplest forms:

(a)	50%	(b)	30%	(c)	80%
(d)	70%	(e)	15%	(f)	25%
(g)	64%	(h)	98%	(i)	56%

4. Write these fractions as percentages:

(a)	$\dfrac{7}{100}$	(b)	$\dfrac{18}{100}$	(c)	$\dfrac{3}{50}$
(d)	$\dfrac{17}{50}$	(e)	$\dfrac{3}{20}$	(f)	$\dfrac{7}{25}$
(g)	$\dfrac{3}{5}$	(h)	$\dfrac{7}{10}$	(i)	$\dfrac{3}{4}$
(j)	$\dfrac{1}{20}$	(k)	$\dfrac{1}{2}$	(l)	$\dfrac{3}{25}$

5. Copy and complete this table:

Fraction	*Decimal*	*Percentage*
	0.04	
		10%
$\frac{1}{2}$		
		45%
$\frac{7}{50}$		
	0.84	

6. There are 200 children in a school hall, eating lunch. Of these children, 124 have chosen chips as part of their lunch.

   (a) What *fraction* of the children have chosen chips?

   (b) What *percentage* of the children have chosen chips?

   (c) What percentage of the children have *not* chosen chips?

7. In a survey, $\frac{9}{10}$ of the children in a school said that maths was their favourite subject. What percentage of the children *did not* say that maths was their favourite subject?

8. In a Year 7 class, $\frac{3}{4}$ of the children can swim more than 400 m and only $\frac{1}{10}$ of the children can not swim more than 200 m.

   What percentage of the class can swim:

   (a) more than 400 m,

   (b) less than 200 m,

   (c) a distance between 200 m and 400 m?

9. In the school canteen, children can choose chips, baked potato or rice. One day 50% choose chips and 26% choose baked potatoes.

   (a) What percentage choose rice?

   (b) What fraction of the children choose rice?

10. In a car park, 40% of the cars are red and $\frac{7}{20}$ of the cars are blue.

   (a) What *percentage* are blue?

   (b) What *percentage* are *neither* red *nor* blue?

   (c) What *percentage* are red or blue?

   (d) What *fraction* are red?

   (e) What *fraction* are *neither* red *nor* blue?

   (f) What *fraction* are red or blue?

18 Quantitative Data

18.1 Presentation

In this section we look at how vertical line diagrams can be used to display discrete quantitative data. (Remember that discrete data can only take specific numerical values.)

Example 1

The marks below were scored by the children in a class on their maths test. The marks are all out of a possible total of 10 marks.

8	6	8	7	7
7	10	9	6	8
8	4	3	2	5
8	8	6	5	6
4	9	8	4	7
7	5	3	7	6

Draw a vertical line diagram to illustrate these data.

Use your diagram to answer these questions:

(a) What is the *most common* mark?

(b) What is the *highest* mark?

(c) What is the *lowest* mark?

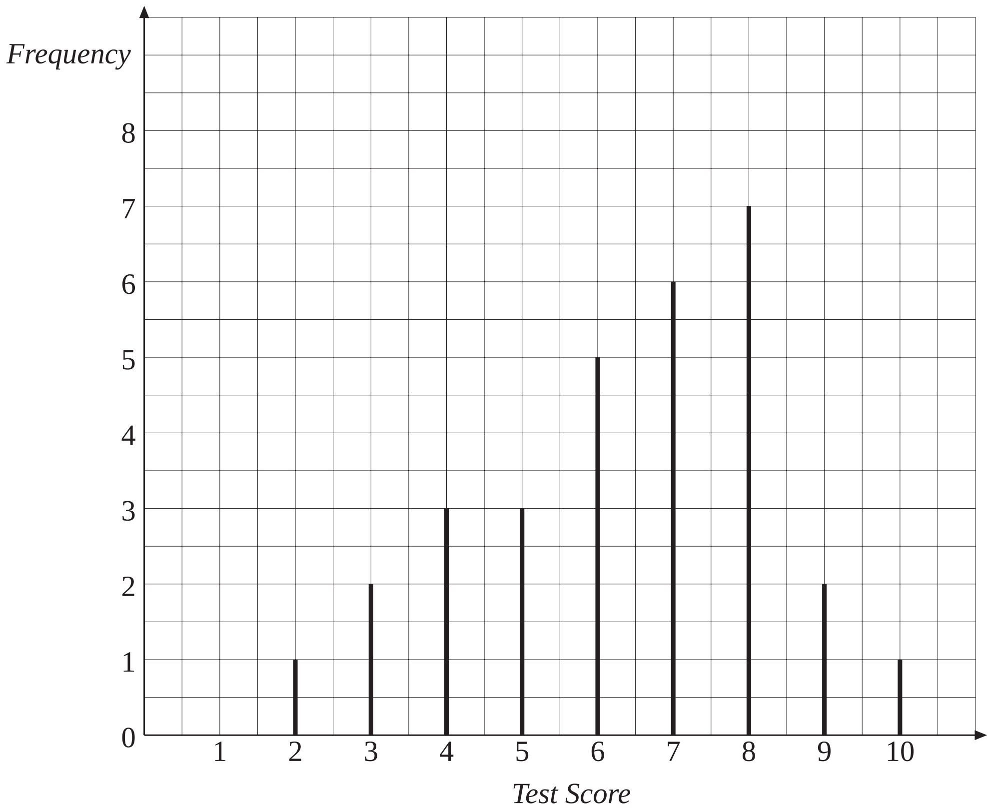
(d) What is the difference between the *highest* and *lowest* marks?

Solution

The first step is to organise the data using a *tally chart*, as shown here:

Mark	*Tally*	*Frequency*
2	|	1
3	||	2
4	|||	3
5	|||	3
6	卌	5
7	卌 |	6
8	卌 ||	7
9	||	2
10	|	1

The diagram can then be drawn as shown below. The height of each line is the same as the frequency; that is, the number of times it occurs in the data list.



(a) The *most common* mark is 8, which occurred 7 times.

(b) The *highest* mark is 10.

(c) The *lowest* mark is 2.

(d) The difference between the *highest* and *lowest* marks is $10 - 2 = 8$.

Note: a *vertical line diagram* is an appropriate way to represent information that consists of distinct, single values, each with its own frequency. A *bar graph* is more suitable for grouped numerical data.

Exercises

1. A teacher gives the children in her class a test, and lists their scores in this table:

Mark	*Frequency*
1	1
2	4
3	1
4	3
5	6
6	8
7	4
8	2

(a) Draw a vertical line diagram to illustrate these results.

(b) What is the *most common* mark?

(c) How many children are there in the class?

2. The staff in a shoe shop keep a record of the sizes of all the shoes they sell in one day. These are listed below:

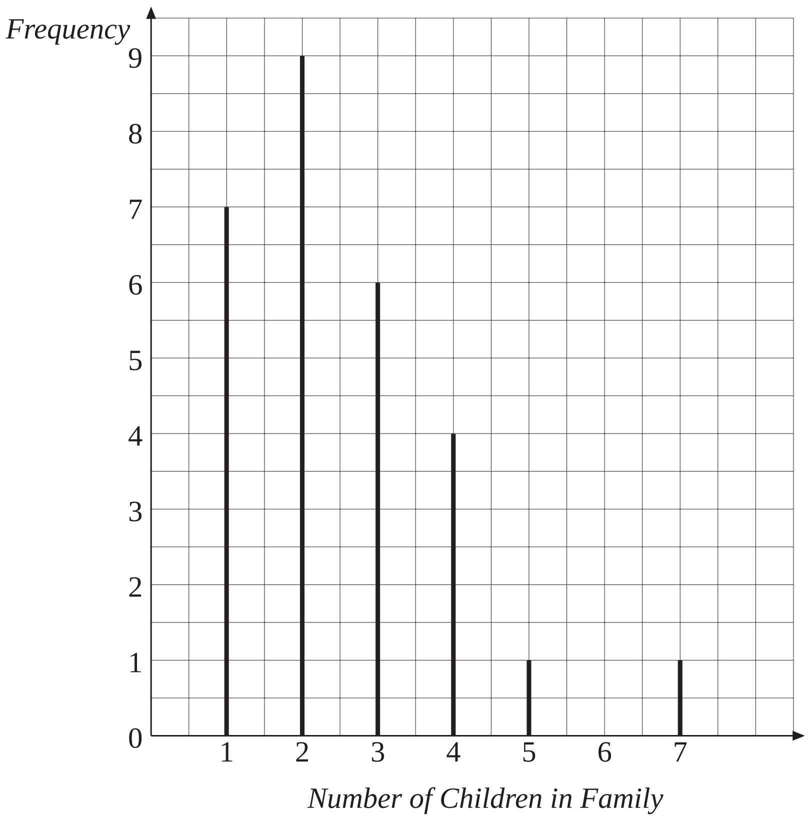
8	7	6	6	8	7	5	4	3	1
11	7	8	9	5	6	6	5	6	4
3	10	8	9	7	6	6	5	4	2
6	9	11	3	5	6	7	8	8	3
4	6	7	8	9	8	8	7	6	4

(a) Complete a tally chart for these data.

(b) Draw a vertical line diagram for these data.

(c) What advice could you give the shop staff about which size shoes they should keep in stock?

3. The vertical line diagram below is based on data collected by a class about the number of children in their families:



(a) What is the *most common* number of children per family?

(b) How many children are there in the class?

4. (a) Collect data on the number of children in the families of the pupils in your mathematics class.

   (b) Draw a vertical line diagram like the one in question 3.

   (c) Compare your vertical line diagram with the one for question 3. What *similarities* are there? What *differences* are there?

5. Mr Graddon says that his class is better at tables than Mr Hall's class. The two classes each take a tables test, and the results are given below. The scores are out of 10.

Mr Graddon's Class						*Mr Hall's Class*					
5	6	7	8	9	10	4	7	8	3	5	6
0	1	3	6	9	2	7	4	5	6	6	5
5	1	2	2	0	1	5	5	6	7	4	3
6	4	0	1	10	9	4	5	6	6	7	8
1	2	3	5	10	9	6	7	5	6	4	5

   (a) Draw a vertical line diagram for each class.

   (b) Which features of the two diagrams would Mr Graddon use to support his claim that his class is better at tables?

   (c) How would Mr Hall use the diagrams to argue the other way?

   (d) Which class do *you* think is better at tables?

6. A gardener keeps a record of the number of tomatoes he picks from the plants in his greenhouse during August. The number of tomatoes picked each day is listed below:

7	10	3	6	8	9	5	10	4	7	9
6	10	11	12	13	7	8	4	3	6	9
7	9	10	11	14	13	7	8	9		

   (a) Draw a vertical line diagram for these data.

   (b) What is the *largest* number of tomatoes picked on one day?

   (c) What is the *smallest* number of tomatoes picked on one day?

   (d) What is the number of tomatoes that was picked *most often*?

7. A sample of children were asked how many pets they had, and their responses are listed below:

4	1	1	0	2	0	1	3	4	0
1	0	1	2	0	1	1	3	0	5

(a) Draw a vertical line diagram for these data.

(b) How many pets were in the sample?

(c) How many children owned at least one pet?

(d) Is it true that, in this sample, there are more children who own pets than children who do not?

8. A rail company keeps a record of how many trains are late each day. The data for January are listed below:

2	0	3	0	1	1	2	0	3	0	4
6	1	0	0	0	2	1	3	1	0	
0	0	1	2	3	1	1	1	2	3	

The data for February are listed below:

3	2	4	7	0	1	2	0	1	2
0	0	0	1	0	1	2	1	2	0
0	2	1	3	1	2	1	1		

(a) Draw vertical line diagrams for each month.

(b) Comment on whether the trains were on time more often in February than in January.

9. A traffic warden keeps a record of the number of parking tickets that she issues on 20 working days.

0	3	7	8	12	0	1	3	4	5
6	5	4	0	1	3	4	6	7	5

(a) Draw a vertical line diagram for these data.

(b) How many blank parking tickets do you think she should take with her when she starts her daily traffic patrol? Explain your answer.

10. Graham uses his calculator to generate random numbers. He decides to investigate if the numbers are really random. Using his calculator, he produces the following numbers:

9	9	1	5	4	7	0	3	9	2
7	9	2	3	0	9	1	0	5	8
9	2	2	1	0	7	0	4	3	9
0	8	6	2	9	7	3	2	9	9

(a) Draw a vertical line diagram for these data.

(b) Do you think that the numbers that Graham's calculator produces are really random? Explain your answer.

18.2 Measures of Central Tendency

In this section we will consider three different types of 'average'. These are the *mean*, the *median* and the *mode*, and statisticians refer to them as *measures of central tendency*.

$$\textit{Mean} = \frac{\textit{sum of all values}}{\textit{total number of values}}$$

Median = *middle value (when the data are arranged in order)*

Mode = *most common value*

Measures of central tendency are single values chosen as being representative of a whole data set. When we select which of the mean, the median or the mode to use, we choose the one that we think is most typical of the data and appropriate for the context.

Example 1

What is:

(a) the *mean*, (b) the *median* and (c) the *mode*

of the numbers:

4, 7, 8, 4, 5

Solution

(a) $$\textit{Mean} = \frac{4+7+8+4+5}{5} = \frac{28}{5} = 5.6$$

(b) To calculate the *median*, write the numbers in order,

4, 4, (5), 7, 8

The middle number is 5,

median = 5

(c) The most common number is 4, so

mode = 4

Example 2

What number is the *median* of the numbers:

4, 7, 11, 4, 6, 7, 2, 9

Solution

First write the numbers in order:

2, 4, 4, (6, 7,) 7, 9, 11

In this case there are two middle numbers, 6 and 7. The *median* is the mean of these two numbers:

$$\textit{Median} = \frac{6+7}{2}$$

$$= 6.5$$

Note: where there is an *odd* number of data items, there will be a single value in the middle and that will be the median – provided you have arranged the data in order. When there is an *even* number of data items, there will be two values in the middle and you must find their mean to get the median of the full data set.

Example 3

David keeps a record of the number of carrier bags that he is given when he does his weekly shopping. The data he collects over 10 weeks is listed below:

9 8 5 9 12 8 7 6 5 9

(a) Calculate: (i) the *mean*, (ii) the *median*, (iii) the *mode*?

(b) Explain why the mean is not very useful in this context.

(c) Which value might be used by an environmental group who think that supermarkets cause pollution by giving out too many carrier bags?

(d) Which value might be used by a shopper who thinks that the supermarket doesn't give him enough carrier bags for his shopping?

Solution

(a) (i) $\textit{Mean} = \frac{9+8+5+9+12+8+7+6+5+9}{10}$

$$= \frac{78}{10}$$

$$= 7.8$$

(ii) To find the *median*, put the numbers in order, and find the middle numbers:

5 5 6 7 8 8 9 9 9 12

$$\textit{Median} = \frac{8+8}{2}$$

$$= 8$$

(iii) The most common number is 9:

Mode = 9

(b) The mean is not very useful as no one would ever actually use 7.8 plastic bags.

(c) The mode, as this is the largest of the three values.

(d) The mean, as this is the smallest of the three values.

Exercises

1. Find the *mean, median* and *mode* of each set of numbers:

   (a) 4 4 6 8 5

   (b) 6 7 7 7 7 5 6 2 9 8

   (c) 8 4 3 3 5 7

   (d) 6 6 7 7 4 9 1 7 10

2. The owner of a shoe shop recorded the sizes of the feet of all the customers who bought shoes in his shop in one morning. These sizes are listed below:

   8 7 4 5 9 13 10 8 8 7

   6 5 3 11 10 8 5 4 8 6

   (a) What are the *mean, median* and *mode* shoe sizes?

   (b) Which of these values would be most sensible for the shop owner to use when ordering shoes for his shop? Explain your choice.

3. Eight people work in a shop. They are paid hourly rates of

   £4 £15 £6 £5 £4 £5 £4 £4

   Would you use the *mean, median* or *mode* to show that they were:

   (a) *well* paid, (b) *badly* paid?

4. A newspaper reports that the average number of children per family is 2.4.

   (a) Which type of value has the newspaper used?

   (b) Explain how you can tell which value was used.

   (c) Would your answer to (b) be the same if the newspaper had reported the average as 2.5 children?

5. The mean of six numbers is 9. If five of the numbers are 10, 12, 7, 6 and 9, what is the sixth number?

6. The table below gives the number of accidents each year at a particular road junction:

1991	1992	1993	1994	1995	1996	1997	1998
4	5	4	2	10	5	3	5

   (a) Calculate the *mean*, *median* and *mode*.

   (b) Describe which value would be most sensible for a road safety group to use, if they want the junction to be made less dangerous.

   (c) The council do not want to spend money on the road junction. Which value do you think they should use?

7. One day the number of minutes that trains were late to arrive at a station was recorded. The times are listed below:

0	7	0	0	1	2	5	0	0	0
6	0	1	52	0	10	1	1	8	22

   (a) Calculate the *mean*, *median* and *mode* of these data.

   (b) Explain which value would be the best to use to argue that the trains arrive late too often.

   (c) Explain who might use the mode and why it might be an advantage to them.

8. Mr Hall grows two different types of tomato plant in his greenhouse. One week he keeps a record of the number of tomatoes he picks from each type of plant.

Day	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*	*Sat*	*Sun*
Type A	5	5	4	1	0	2	5
Type B	3	3	3	3	7	9	6

(a) Calculate the *mean*, *median* and *mode* for each type of plant.

(b) Use one value to argue that type A is the best plant.

(c) Use a different value to argue that type B is the best plant.

9. The heights of eight children are given below, to the nearest cm:

   158 162 142 155 163 157 160 112

   (a) Explain why the mode is *not* a suitable value to use for these data.

   (b) Calculate the median and the mean of these data.

   (c) Explain why the mean is less than the median.

10. A set contains four positive numbers.

    The *mode* of these numbers is 1.

    The *mean* of these numbers is 2.5.

    The *median* of these numbers is 1.5.

    What are the four numbers?

18.3 Measures of Dispersion

The *range* of a set of data is the difference between the largest and the smallest values in the data set. The range gives a measure of the dispersion of the data, or, more simply, describes the spread of the data.

Example 1

Calculate the *range* of this set of data:

4 7 6 8 3 9 14 22 3

Solution

The largest value is 22.

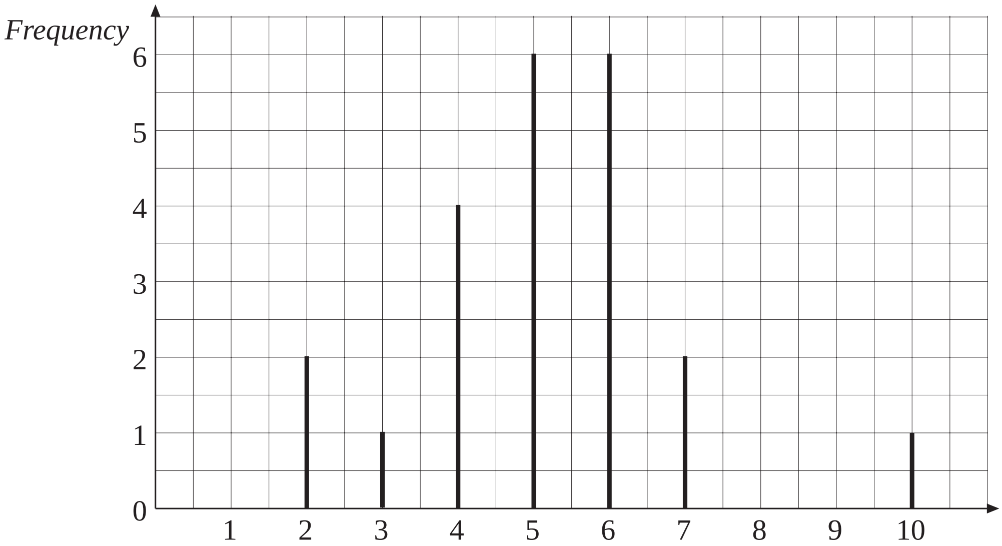
The smallest value is 3.

Range = 22 – 3

= 19

Example 2

What is the *range* of the data illustrated in this vertical line diagram?



Solution

Largest value = 10

Smallest value = 2

Range = 10 – 2

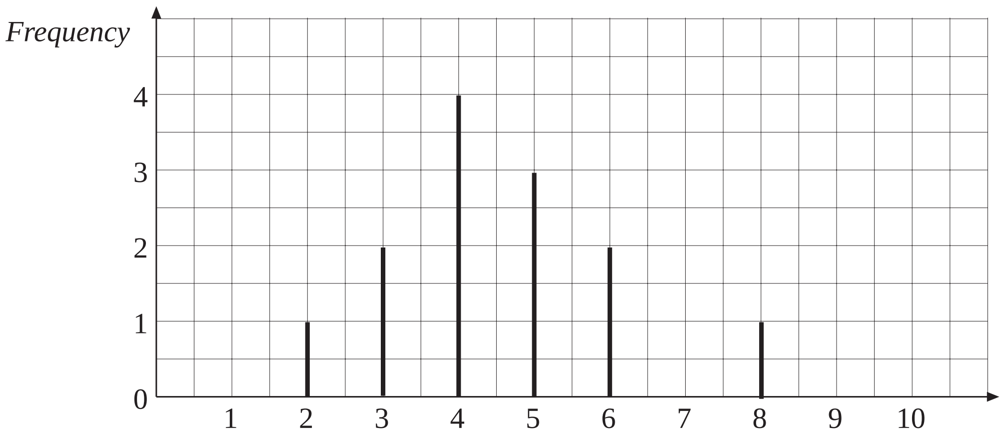
= 8

Exercises

1. Calculate the *range* of each of these sets of data:

(a)	4	7	6	3	9	12	7	12
(b)	6	5	5	16	12	21	42	7
(c)	0	2	4	1	3	0	6	
(d)	3	7	8	9	4	7	11	

2. Calculate the *range* of the data illustrated in this vertical line diagram:



3. The range of a set of data is 12 and the smallest number in the set of data is 5.

   What is the *largest* number in the set of data?

4. The largest number in a set of data is 86. The range of the set of data is 47. What is the *smallest* number in the set of data?

5. The heights of 10 students were measured to the nearest centimetre and are listed below:

144	162	173	158	143
159	164	182	162	158

   What is the *range* of this set of data?

6. Rafiq keeps a record of the amount of money he spends each day. The amounts for one week are listed below:

   47p 10p 36p 85p 22p 30p

   There are only 6 amounts because he forgets to include one day.

   (a) What is the *range* of the numbers listed above?

   (b) If the range was 90p, what was the missing amount?

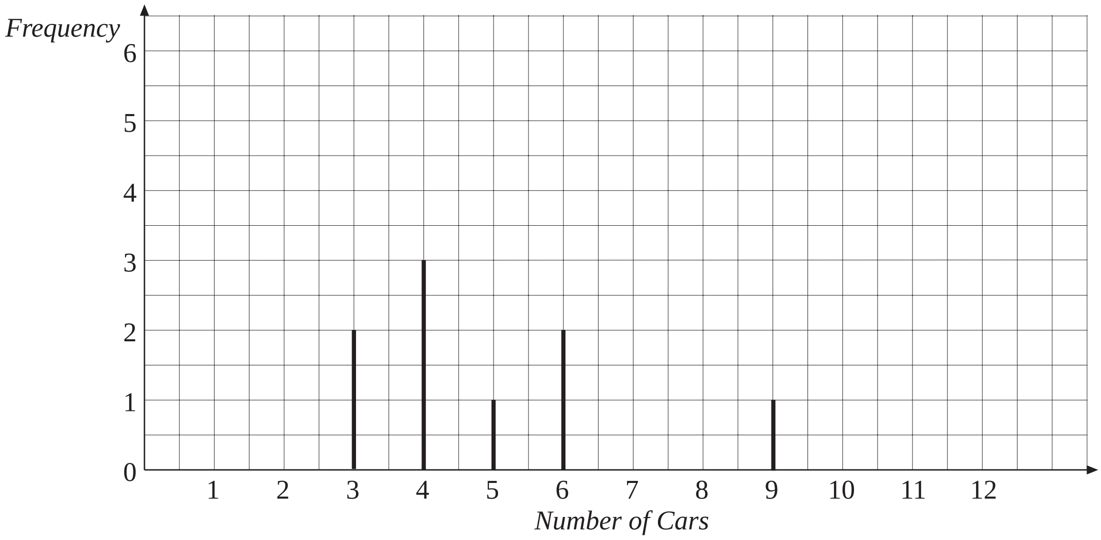
   (c) If the range was double your answer to (a), what was the missing amount?

   (d) Explain why the range must be equal to or greater than your answer to part (a).

7. The vertical line diagram on the following page is for a data set that has one missing value.

   What can you say about the missing value if the range is:

   (a) 7, (b) 9, (c) 6 ?



8. What is the range of this set of temperatures:

$-4\ °C \quad 3\ °C \quad 5\ °C \quad -1\ °C \quad -3\ °C \quad 6\ °C$?

9. The range of a set of temperatures is $8\ °C$. If the *maximum* temperature in the set is $6\ °C$, what is the *minimum* temperature?

10. The range of a set of temperatures is $7\ °C$. If the *minimum* temperature in the set is $-11\ °C$ what is the *maximum* temperature?

18.4 Comparing Data

In this section we consider how averages and the range can be used to compare sets of data.

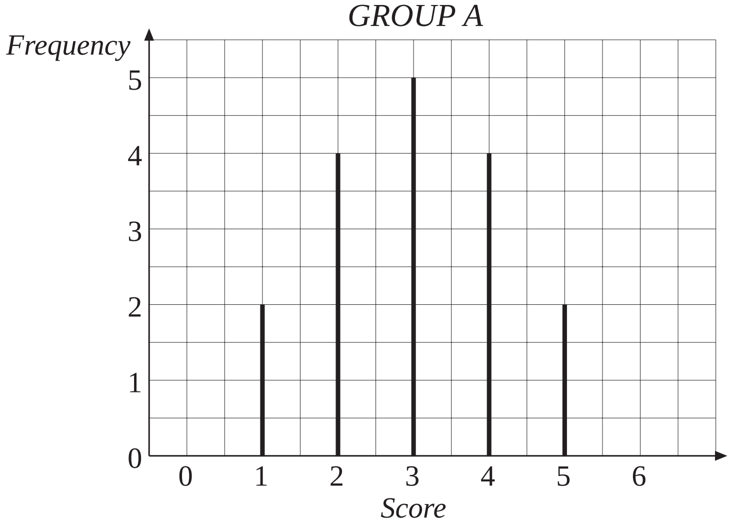
Example 1

The two line diagrams on the next page illustrate data that was collected about the scores of two groups of children in a short test.

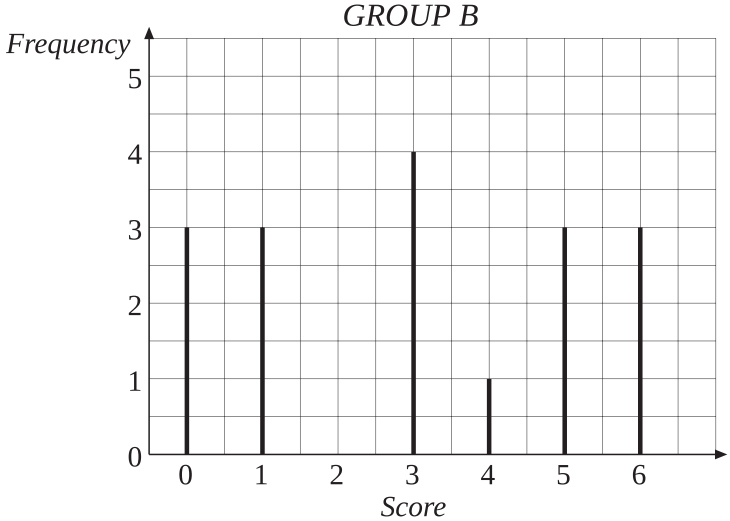
(a) Calculate the *mode* and *range* for each group.

(b) Describe the differences between the groups.

GROUP A



GROUP B



Solution

(a) *Group A* | *Group B*

Group A		*Group B*	
Mode	$= 3$	*Mode*	$= 3$
Range	$= 5 - 1$		$= 6 - 0$
	$= 4$		$= 6$

(b) Both groups have the same mode but different ranges. The range is greater for group B.

The low range for group A indicates that the scores for those students are reasonably similar. The higher range for group B shows that their scores are much more varied. This can be seen from the line diagrams, where none of group A get the extreme scores of 0 and 6, while these are obtained by several students in group B.

Example 2

Kathryn plants two different types of tomato plant. She records the number of tomatoes that she picks from each plant every day for 10 days. Her records are shown below:

Plant A	4	6	7	3	5	2	1	3	6	5
Plant B	5	6	7	6	8	9	6	7	8	9

Compare the two plants and recommend which type she should buy next year.

Solution

First consider the mean and range for each plant:

PLANT A

$$\begin{aligned} \textit{Mean} &= \frac{4+6+7+3+5+2+1+3+6+5}{10} \\ &= \frac{42}{10} \\ &= 4.2 \end{aligned}$$

$$\begin{aligned} \textit{Range} &= 7-1 \\ &= 6 \end{aligned}$$

PLANT B

$$\begin{aligned} \textit{Mean} &= \frac{5+6+7+6+8+9+6+7+8+9}{10} \\ &= \frac{71}{10} \\ &= 7.1 \end{aligned}$$

$$\begin{aligned} \textit{Range} &= 9-5 \\ &= 4 \end{aligned}$$

As plant B has a higher mean, this suggests that using plant B will produce more tomatoes than using plants of type A. The fact the plant B has the lower range suggests that it will also be more consistent in the number of tomatoes that it produces than type A. Type A will have some productive days but it will also have some poor days.

Exercises

1. (a) Calculate the *mean* and *range* of these two data sets:

A	5	10	0	1	9	5
B	5	6	4	3	7	5

   (b) Describe the difference between the two sets.

2. (a) Calculate the *mean* and *range* of these two data sets:

A	4	6	7	8	5	6.
B	5	7	7	8	9	6

   (b) Describe the difference between the two sets.

3. (a) Calculate the *mean* and *range* of these two data sets:

A	4	6	10	3	5	2
B	6	7	9	9	5	3

   (b) Describe the differences between the two sets.

4. (a) Calculate the *mean* and *range* of these 3 sets of data:

A	4	7	8	6	5	
B	0	10	12	1	3	
C	8	8	9	10	9	8

   (b) Describe the differences between the three sets.

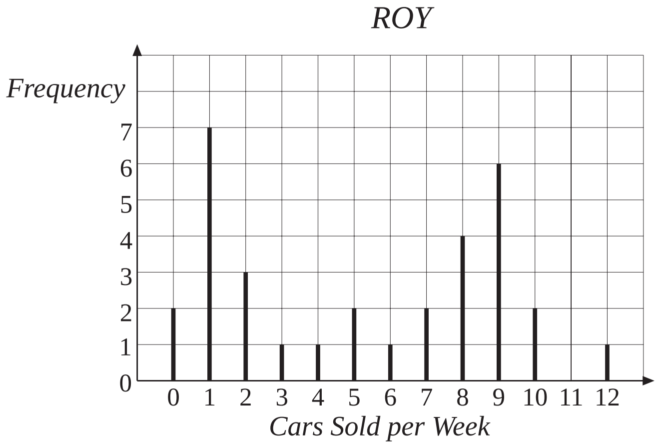
5. Roy and Frank are second-hand car salesmen. The following vertical line diagrams show how many cars they have sold per week over a period of time.

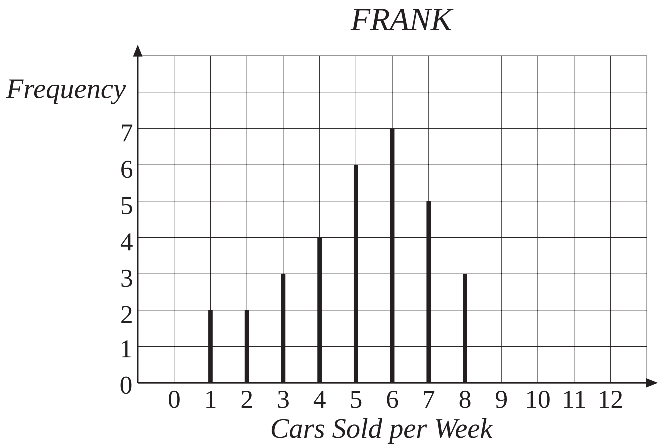
   (a) Write down the *mode* for Roy and for Frank.

   (b) Calculate the *range* for Roy and for Frank.

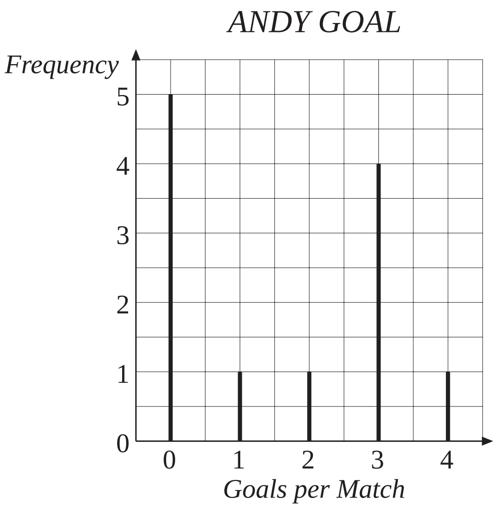
   (c) Who sold more cars?

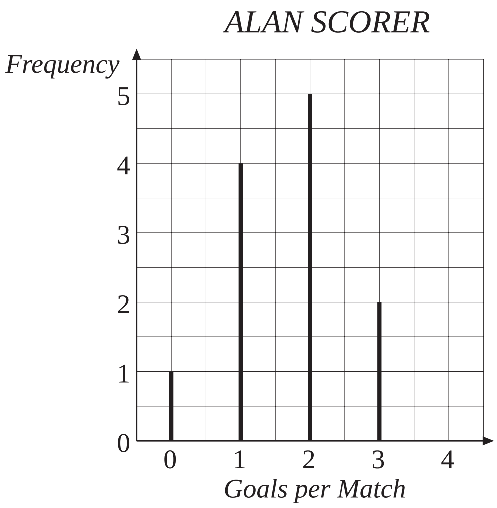
   (d) Who you think is the better salesman? Explain why.





6. The two vertical line diagrams show the number of goals scored per match by two top footballers.





(a) Calculate the *mean* and *range* for each player.

(b) Describe the differences between the two players.

(c) Which of these players would you like to have on your favourite team? Explain why.

7. Miss Sharp's class decide to have a spelling competition with Mr Berry's class. They have a test and the scores for each class are listed below:

Miss Sharp's Class						*Mr Berry's Class*					
10	1	5	8	5	7	5	5	7	6	7	8
2	6	8	7	5	9	5	4	3	3	2	5
2	4	8	0	5	3	4	5	6	5	4	6
5	10	2	5	7	1	7	7	6	4	3	5
5	5	3	3	0	9	3	5	5	6	4	5

(a) Calculate the *mean* for each class.

(b) Calculate the *range* for each class.

(c) Comment on the differences between the two classes.

8. A bus company keeps records of the number of buses that were late each day in February and in July in the same year:

February

6	7	5	4	3	0	0	1	2	5
9	10	5	4	3	6	7	1	0	0
0	0	1	2	1	0	4	1		

July

3	0	1	0	3	1	2	3	4	9	1
2	0	4	1	1	2	3	4	1	5	
7	2	1	2	3	0	4	1	0	2	

(a) Calculate the *mean*, *median* and *mode* for each month.

(b) Calculate the *range* for each month.

(c) Do you think the bus company improved its service to customers between February and July? Give reasons for your answer.

9. "Do boys have bigger feet than girls?"

(a) Collect data from your class.

(b) Draw separate vertical line diagrams for the boys' data and the girls' data.

(c) Calculate the *mode*, *mean*, *median* and *range* for each set of data.

(d) Use your diagrams and calculations to decide, for your class, the answer to the question above.

10. Investigate whether girls eat more fruit than boys.

18.5 Trends

Moving averages can be used to make predictions. They do this by smoothing out monthly, seasonal or other periodic variations.
For example, an ice-cream seller might expect to sell more in the summer than he does in the winter. He could use a moving average over the four seasons to find out if his sales are increasing for each 12 month period.

$$\textit{1st moving average} = \frac{\text{spring 1} + \text{summer 1} + \text{autumn 1} + \text{winter 1}}{4}$$

$$\textit{2nd moving average} = \frac{\text{summer 1} + \text{autumn 1} + \text{winter 1} + \text{spring 2}}{4}$$

$$\textit{3rd moving average} = \frac{\text{autumn 1} + \text{winter 1} + \text{spring 2} + \text{summer 2}}{4}$$

$$\textit{4th moving average} = \frac{\text{winter 1} + \text{spring 2} + \text{summer 2} + \text{autumn 2}}{4}$$

and so on. In each case, the oldest piece of data is replaced by the newest one. So, for the *fifth moving average*, the ice-cream seller would replace the winter sales figure for the first year with the winter sales figure for the second year, and so on. Because the mean of four items of data is being found every time, this is called a *4 point* moving average.

Example 1

(a) Calculate the 4 point moving averages for this list of data:

6 5 7 4 6.1 5.1 7.1 4.1

(b) Estimate the next two values in the list.

Solution

(a) *1st moving average* $= \frac{6 + 5 + 7 + 4}{4}$
$= 5.5$

2nd moving average $= \frac{5 + 7 + 4 + 6.1}{4}$
$= 5.525$

3rd moving average $= \frac{7 + 4 + 6.1 + 5.1}{4}$
$= 5.55$

4th moving average $= \frac{4 + 6.1 + 5.1 + 7.1}{4}$
$= 5.575$

5th moving average $= \dfrac{6.1 + 5.1 + 7.1 + 4.1}{4}$

$= 5.6$

(b) Note that the moving averages increase by 0.025 at each step.

The next moving average will be expected to be 5.625, so

$$5.625 \times 4 = 5.1 + 7.1 + 4.1 + x$$

where x is the next term.

$$x = 5.625 \times 4 - 5.1 - 7.1 - 4.1$$
$$= 6.2$$

To estimate the next value, we use

$$5.65 \times 4 - 7.1 - 4.1 - 6.2 = 5.2$$

Example 2

The table below gives the average daytime temperatures for each of the four seasons over a two-year period.

Year 1				*Year 2*			
Spring	*Summer*	*Autumn*	*Winter*	*Spring*	*Summer*	*Autumn*	*Winter*
12.1	18.6	11.2	8.1	12.4	19.0	11.8	8.6

Use a 4 point moving average to predict the temperature for Spring and Summer of Year 3.

Solution

(a) *1st moving average* $= \dfrac{12.1 + 18.6 + 11.2 + 8.1}{4}$

$= 12.5$

2nd moving average $= \dfrac{18.6 + 11.2 + 8.1 + 12.4}{4}$

$= 12.575$

3rd moving average $= \dfrac{11.2 + 8.1 + 12.4 + 19}{4}$

$= 12.675$

$$\textit{4th moving average} = \frac{8.1 + 12.4 + 19 + 11.8}{4}$$
$$= 12.825$$

$$\textit{5th moving average} = \frac{12.4 + 19 + 11.8 + 8.6}{4}$$
$$= 12.95$$

The differences between the moving averages are

0.075, 0.1, 0.15, 0.125

$$\text{The mean difference} = \frac{0.075 + 0.1 + 0.15 + 0.125}{4}$$
$$= 0.1125$$

We can now predict:

$$\textit{6th moving average} = 12.95 + 0.1125$$
$$= 13.0625$$

$$\textit{7th moving average} = 13.0625 + 0.1125$$
$$= 13.175$$

$$\text{Year 3 } \textit{Spring} \text{ temperature} = 13.0625 \times 4 - 8.6 - 11.8 - 19.0$$
$$= 12.85$$

$$\text{Year 3 } \textit{Summer} \text{ temperature} = 13.175 \times 4 - 12.85 - 8.6 - 11.8$$
$$= 19.45$$

Exercises

1. (a) Calculate the 3 point moving averages for this set of data:

   4 3 5 4 3 5

   (b) What do you notice about the moving averages?

2. (a) Calculate the 4 point moving averages for this set of data:

   6 2 7 1 8 4 9 3 10

   (b) Describe what is happening to the moving average.

   (c) Predict the next *two* values using a 4 point moving average.

3. (a) Calculate the 4 point moving averages for this data:

16 7 20 5 14.2 7.2 19.2 4.2

(b) Use your results to predict the next 2 values.

4. Use a 3 point moving average to estimate the next 2 entries in this list:

4 6 5 5.5 7.5 6.5 . . . . . .

5. The first value from a list of data is missing:

☐ 3.8 6.2 5.8 4.6 4.2 6.6 6.2

(a) Calculate the 4 point moving averages for the data given.

(b) Estimate the missing value.

6. The sales of an ice-cream company are given in the table below, in thousands of ice-creams:

1996				*1997*			
Spring	*Summer*	*Autumn*	*Winter*	*Spring*	*Summer*	*Autumn*	*Winter*
3.6	9.7	3.2	4.1	3.6	9.8	3.4	4.4

Use a 4 point moving average to estimate the number of ice-creams sold each season in 1998.

7. The value, in pence, of a single share in a company is given in the table below:

1997				*1998*			
January	*April*	*July*	*October*	*January*	*April*	*July*	*October*
58	62	74	81	67	70	81	89

Use a 4 point moving average to estimate the value of the share for January, April, July and October 1999.

8. A company keeps a record of its total profits, in £10 000's, for the first, second, third and fourth quarters of each year.

1997				*1998*			
1st	*2nd*	*3rd*	*4th*	*1st*	*2nd*	*3rd*	*4th*
24.1	26.3	28.4	20.4	29.3	31.9	35.2	28.4

Use a 4 point moving average to estimate the profits for:

(a) 1999, (b) 1996.

9. A school tuck shop keeps a record of the number of cans of drink it sells over a 3-week period.

Week 1					*Week 2*					*Week 3*				
Mon	*Tues*	*Wed*	*Thurs*	*Fri*	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*	*Mon*	*Tues*	*Wed*	*Thurs*	*Fri*
18	22	9	7	15	19	23	9	8	16	21	23	10	10	16

Use a 5 point moving average to estimate the sales of cans for week 4.

10. The amount of fuel used in a school in the 4 seasons is shown in the table below (in 1000s of litres).

1997				*1998*			
Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
5.3	4.4	5.4	7.3	6.6	5.6	6.5	8.3

Use an appropriate moving average to estimate the amount of fuel used each season in 1999.

19 Scale Drawing

19.1 Measuring Lengths

In this section we consider which units to use when measuring lengths, estimating lengths, and the errors made when measuring.

1 cm	=	10 mm
1 m	=	100 cm
1 km	=	1000 m

Example 1

Which unit of length is the most appropriate to measure:

(a) your height,

(b) the height of a block of flats,

(c) the length of your foot,

(d) the thickness of your maths book,

(e) the distance between your school and the nearest other school?

Solution

(a) cm

(b) m

(c) cm

(d) mm

(e) probably km

Example 2

Estimate the lengths of each of the following:

(a) a car,

(b) the width of your thumbnail,

(c) the length of your pen.

Solution

(a) This will depend on the type of car, but answers between 2 m and 4 m are reasonable.

(b) Between 1 cm and $1\frac{1}{2}$ cm (or between 10 mm and 15 mm).

(c) 15 cm.

Example 3

A line is measured to the nearest centimetre as 12 cm.

(a) What is the *shortest* possible length of the line?

(b) What length must the actual length of the line be *less than*?

Solution

(a) 11.5 cm, as this is the shortest length that rounds to 12 cm.

(b) 12.5 cm, as this is the shortest length that rounds to 13 cm, rather than to 12 cm.

The answers in Example 3 are called *upper* and *lower bounds*

Note: the answer to Example 3 (a) is called the *lower bound* of the length; the answer to Example 3 (b) is called the *upper bound.*

Exercises

1. Which unit of length would be the most appropriate if you were measuring:

   (a) the distance between two towns,

   (b) the height of your classroom,

   (c) the length of a calculator,

   (d) the thickness of a dictionary,

   (e) the height of your desk?

2. Choosing suitable units estimate the following distances:

   (a) the length of your classroom,

   (b) the height of your teacher,

   (c) the height of your classroom,

   (d) the length of your little finger,

   (e) the width of your desk.

3. Are each of the following statements likely to be *true* or *false*?

   (a) John's height is 242 cm.

   (b) The height of a desk is 1.2 m.

   (c) The height of a door is 190 cm.

   (d) The length of a football pitch is 800 m.

   (e) The length of a finger is 8 cm.

   (f) The thickness of a sheet of paper is 1 mm.

4. The length of a line is measured as 10 cm, to the nearest cm. What are the upper and lower bounds of its actual length?

5. The length of a pen is measured as 16 cm, to the nearest cm. What is the minimum possible length of the pen?

6. The distance between 2 airports is 1700 km, correct to the nearest 100 km.

   (a) What are the upper and lower bounds of the actual distance?

   (b) What would be your answers to (a) if the information was correct to the nearest 10 km?

7. The end of Andy's tape measure is broken, and all the distances that he measures are 1 cm shorter than he thinks. Is this error significant when he measures:

   (a) 5 cm, (b) 5 m,

   (c) 20 cm, (d) 1 m?

8. (a) Estimate the lengths of each of these lines to the nearest cm:

   (i)

   (ii)

   (iii)

   (iv)

   (v)

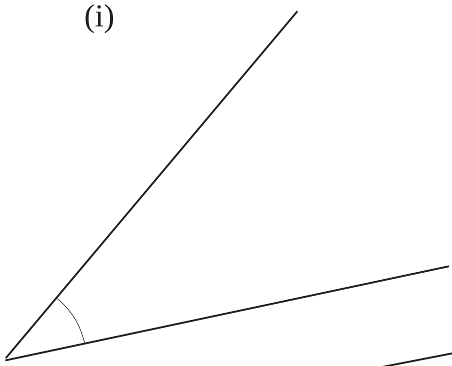
   (vi)

   (b) Measure the lengths of each line, correct to the nearest cm.

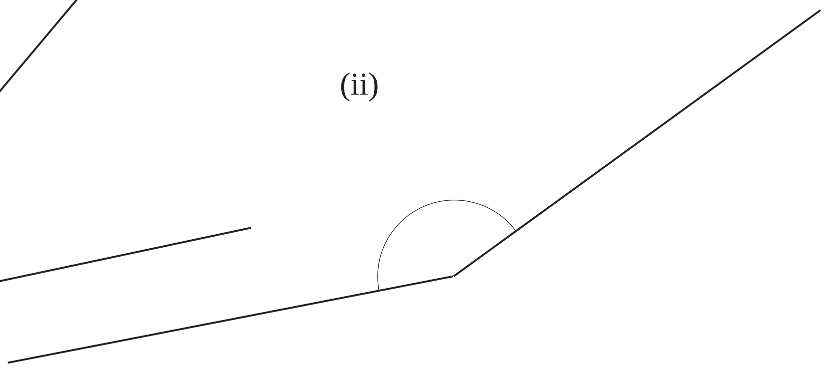
   (c) Explain why you would get more sensible results if you measured the lines to the nearest mm.

9. (a) Estimate the size of each of these angles:

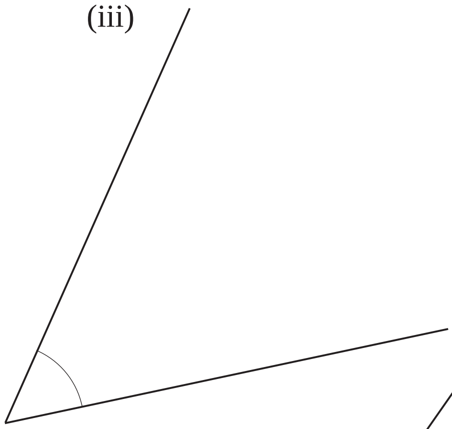
(i)



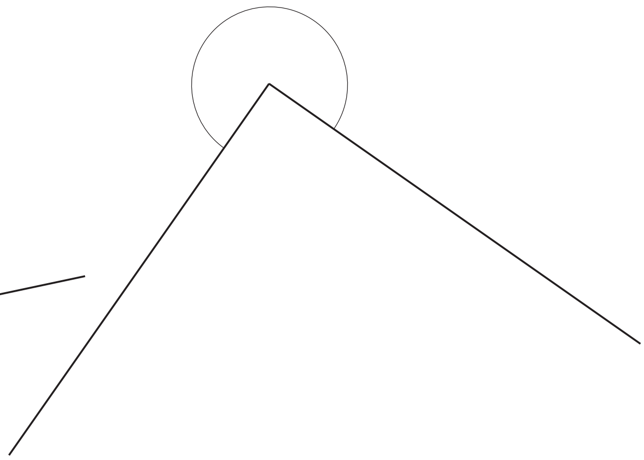
(ii)



(iii)

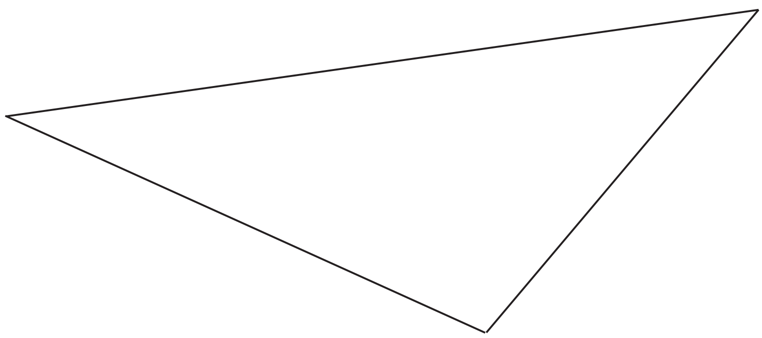


(iv)



(b) Measure the size of each angle to see how good your estimates were.

10. (a) Measure the 3 angles in this triangle:



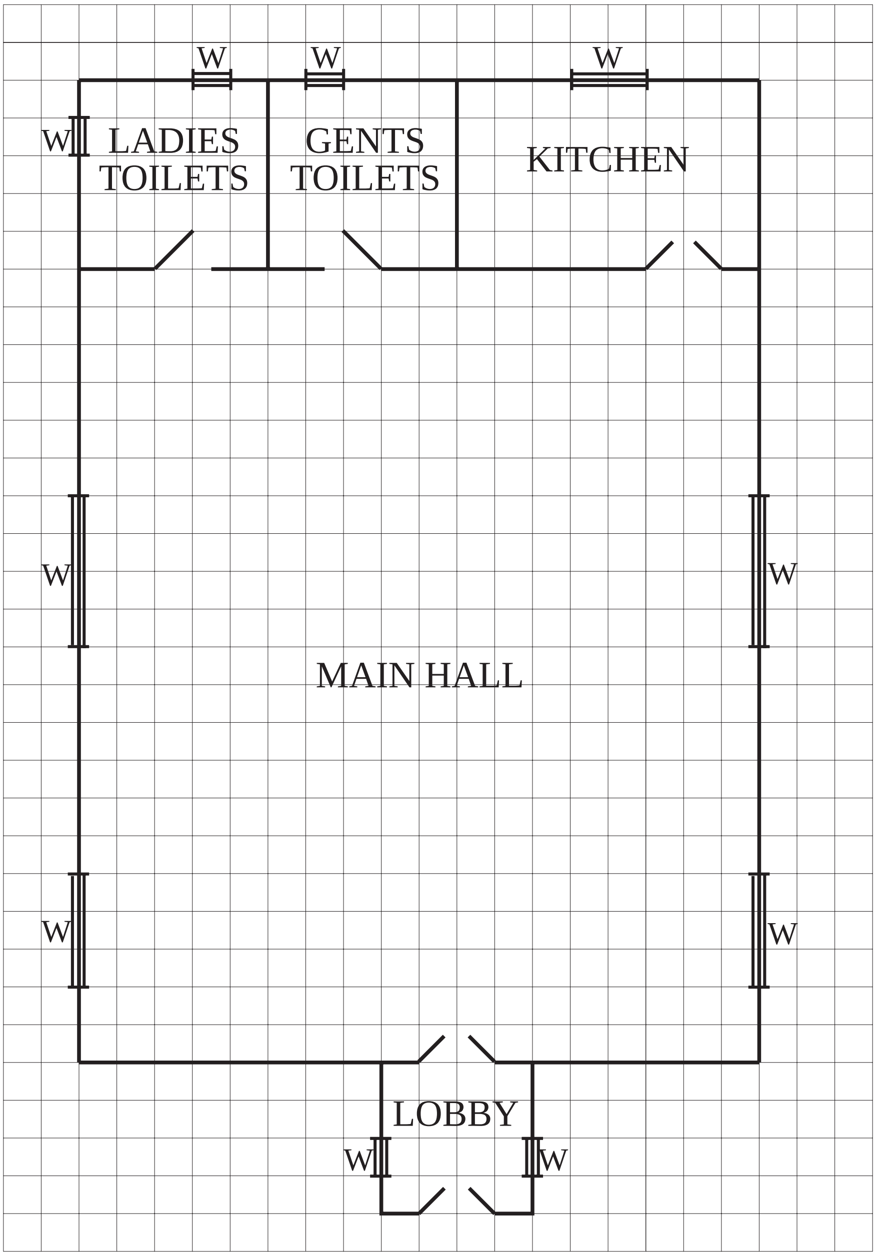
(b) Check that they add up to $180°$.

19.2 Plans

Plans are drawn using a scale such as 1 : 100. This means that 1 cm on the plan represents 100 cm, or 1 m, in real life. In this section we consider how to take measurements from plans and how to draw plans.

Example 1

The diagram shows the plan of a village hall, on a scale of 1 : 100.



What are the 'real life' measurements for:

(a) the dimensions of the kitchen,

(b) the dimensions of the gents toilet,

(c) the length of the main hall,

(d) the area of the lobby,

(e) the length of the kitchen window?

Solution

(a) On the plan the kitchen is 4 cm by 2.5 cm. In reality these distances are 100 times larger, that is, $4 \times 100 = 400$ cm by $2.5 \times 100 = 250$ cm, or 4 m by 2.5 m.

(b) On the plan the gents toilet is 2.5 cm by 2.5 cm. In reality it is 250 cm by 250 cm, or 2.5 m by 2.5 m.

(c) The length of the main hall is 10.5 cm on the plan. In reality it will be 1050 cm, or 10.5 m.

(d) On the plan the lobby is 2 cm by 2 cm. This corresponds to actual dimensions of 2 m by 2 m, so

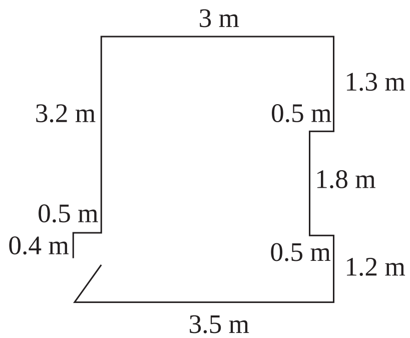
$$\begin{aligned} \text{area} &= 2 \times 2 \\ &= 4 \text{ m}^2 \end{aligned}$$

(e) The length of this window on the plan is 1 cm. The actual length will be 100 cm, or 1 m.

Example 2

Veronica makes a rough sketch of her bedroom and measures the lengths of the walls.

Draw an accurate plan of Veronica's bedroom, using a scale of 1 : 50.



Solution

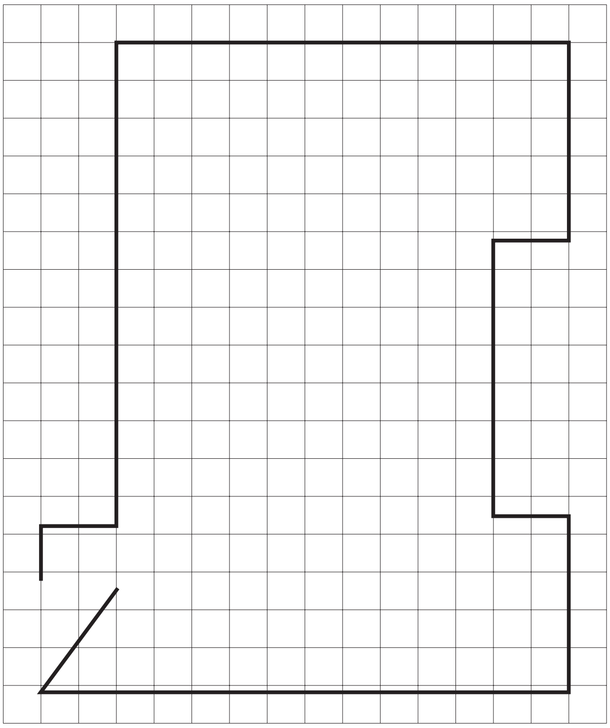
A scale of 1 : 50 means that 1 cm on the plan will represent an actual distance of 50 cm.

All the distances on the sketch must be divided by 50 to find the distances that should be used on the plan.

The table shows these distances:

Actual Size in m	*cm*	*Size on Plan in cm*
3	300	$300 \div 50 = 6$
1.3	130	$130 \div 50 = 2.6$
1.8	180	$180 \div 50 = 3.6$
1.2	120	$120 \div 50 = 2.4$
3.5	350	$350 \div 50 = 7$
0.4	40	$40 \div 50 = 0.8$
0.5	50	$50 \div 50 = 1$
3.2	320	$320 \div 50 = 6.4$

The plan can then be drawn accurately, as shown in the diagram.



Exercises

1. This is a plan of a temporary building at a school. It is drawn using a scale of 1 : 100.

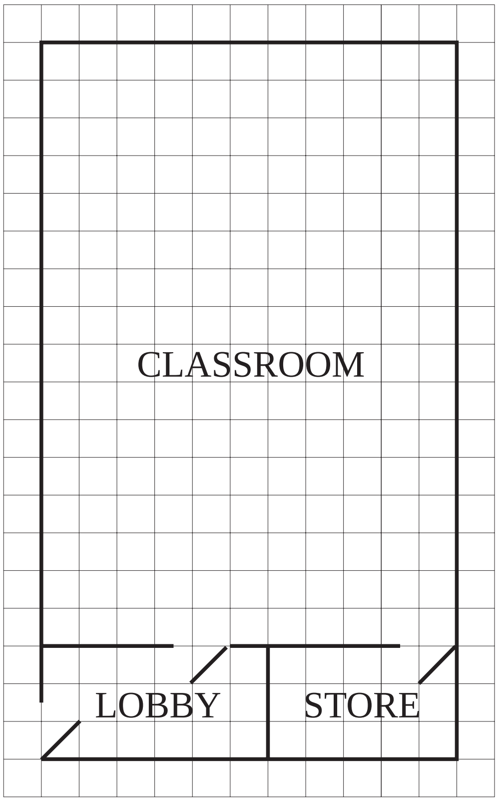
   What are the actual measurements of:

   (a) the length of the classroom,

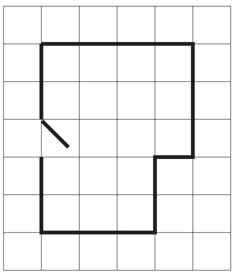
   (b) the width of the classroom,

   (c) the dimensions of the lobby,

   (d) the dimensions of the store?



2. Adam draws a plan of his bedroom using a scale of 1 : 200. Find the actual lengths of the walls in his bedroom.



3. Jai's garage is 3 m wide and 8 m long. What would be the dimensions of this garage on a plan with a scale of:

   (a) 1 to 100,

   (b) 1 to 200,

   (c) 1 to 50,

   (d) 1 to 10,

   (e) 1 to 20?

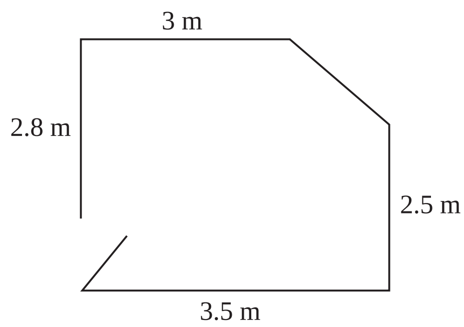
4. On a plan with a scale of 1 : 50, the floor of a rectangular cupboard is shown with dimensions 2.5 cm by 3.6 cm. What are the actual dimensions of the floor? Give your answers in metres.

5. Alice draws this sketch of her bedroom.

   The doorway is 0.7 m wide.

   (a) Draw a plan of this room using a scale of 1 : 50.

   (b) Calculate the actual length of the wall that is *not* at right angles to the other walls.



6. On a plan, an actual length of 5 m is represented by 25 cm. What is the scale of the plan?

7. A rectangular room has dimensions 4 cm by 5 cm on a plan with a scale of 1 : 120.

   (a) What are the actual dimensions of the room in metres?

   (b) What is the floor area of the room?

   (c) What is the length of the longest straight line that could be drawn on the floor of the room?

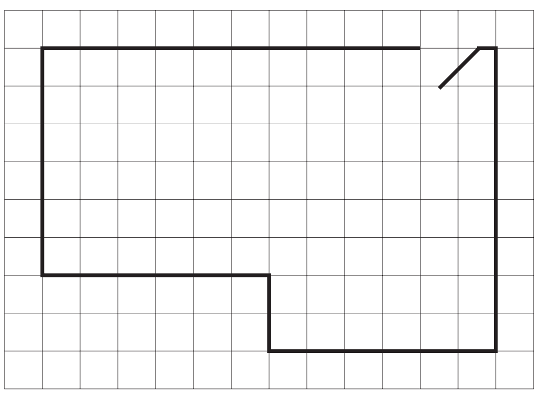
8. This diagram shows the plan of a room, drawn using a scale of 1 : 200.

   Calculate:

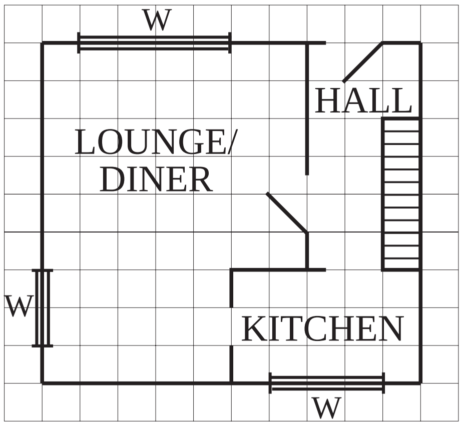
   (a) the perimeter of the room,

   (b) the total floor area of the room,

   (c) the length of the longest straight line that could be drawn on the floor of the room.



9. The diagram shows the plan of the ground floor of a house, using a scale of 1 : 120.



Calculate the following:

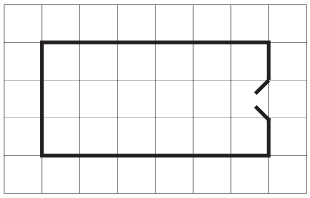
(a) the total length of the two windows in the lounge/diner,

(b) the floor area of the kitchen,

(c) the total floor area of the lounge/diner,

(d) the floor area of the hall, excluding the stairs.

10. The diagram shows the plan of a workshop.



The area of the workshop floor is 72 m^2.

(a) What actual area does each small square on the grid represent?

(b) What length does 5 mm on the plan represent?

(c) What is the scale of the plan?

19.3 Maps

Scales are used on maps in the same way that they are used in plans. A scale of 1 : 50 000 is used on many Ordnance Survey maps. This means that 1 cm on the map represents an actual distance of 50 000 cm (or 500 m or 0.5 km).

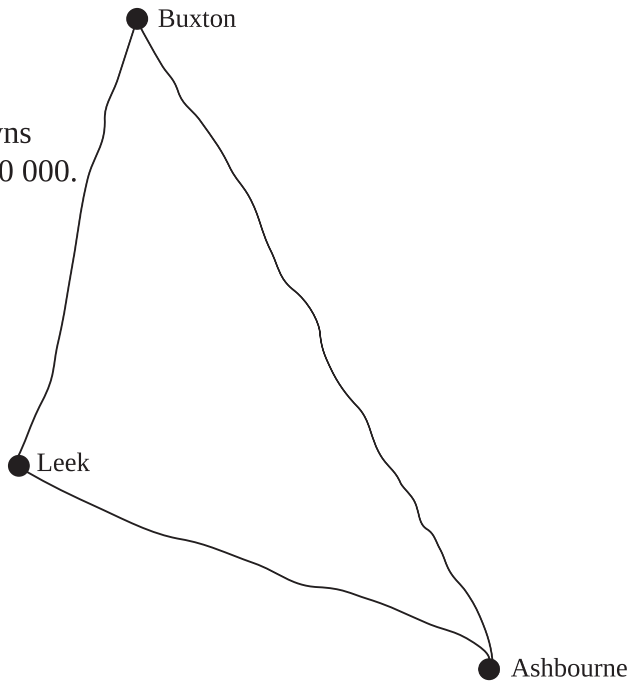
Example 1

This map shows roads linking 3 towns and is drawn using a scale of 1 : 300 000.

What is the actual distance on a straight line between:

(a) Buxton and Ashbourne,

(b) Buxton and Leek?



Solution

(a) The distance between Buxton and Ashbourne is 9.8 cm on the map.

Actual distance $= 9.8 \times 300\,000$

$= 2\,940\,000$ cm

$= 29\,400$ m

$= 29.4$ km

(b) The distance between Buxton and Leek on the map is 6.1 cm.

Actual distance $= 6.1 \times 300\,000$

$= 1\,830\,000$ cm

$= 18.3$ km

Example 2

The distance between two towns is 3.5 km. How far apart would these towns be on a map with a scale of 1 : 50 000 ?

Solution

Actual distance $= 3.5$ km

$= 3500$ m

$= 350\,000$ cm

Distance on map $= \dfrac{350\,000}{50\,000}$

$= 7$ cm

Alternative solution: a scale of 1 : 50 000 means that 1 cm on the map represents 50 000 cm or 500 m or 0.5 km, in reality.

Distance on map $= \dfrac{3.5}{0.5}$

$= 7$ cm

Exercises

1. The following map shows some places in Kent and is drawn using a scale of 1 : 300 000.

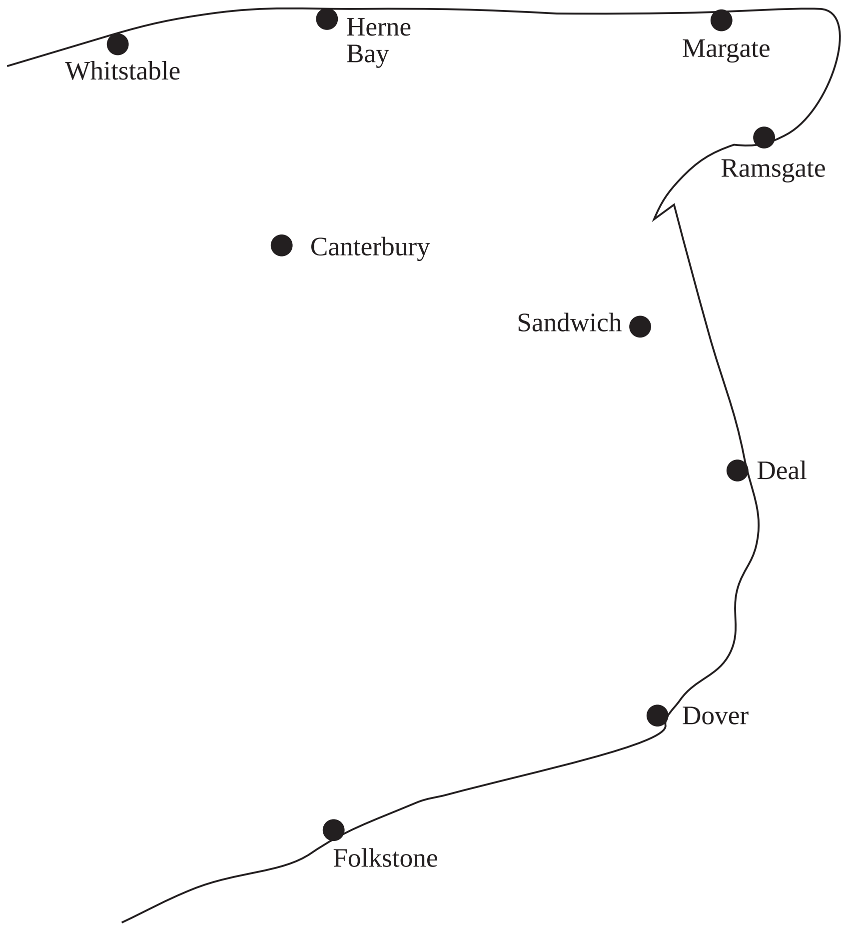
   What is the actual distance on a straight line, in km, between:

   (a) Canterbury and Dover,

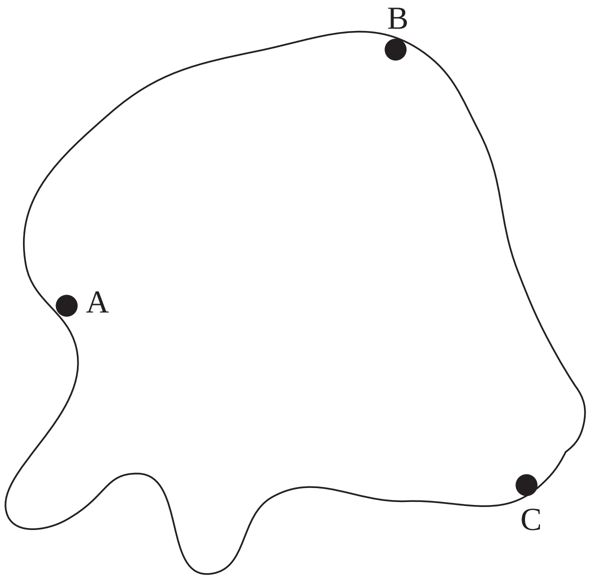
   (b) Whitstable and Margate,

   (c) Sandwich and Deal,

   (d) Herne Bay and Folkstone?



2. The map below shows a small island drawn using a scale of 1 : 25 000. There are three lookout posts, at A, B and C.



A person walks from A to B, from B to C, and from C back to A. He always walks in a straight line between the lookout posts. What is the total distance that the person walks?

3. A map has a scale of 1 : 50 000. What are the actual distances, in km, that are represented by each of these lengths on the map:

   (a) 4 cm,

   (b) 10 cm,

   (c) 3.2 cm,

   (d) 5.1 cm?

4. The distance between two places on a map is 6 cm. If the map has a scale of 1 : 40 000, what is the actual distance between the two places?

5. On a map with a scale of 1 : 3 000 000, the distance between Edinburgh and London is 18 cm. What is the actual distance, in km, between these cities?

6. Two towns are 15 km apart. What would be the distance between the two towns on a map with a scale of 1 : 300 000?

7. A tower is 2 km due north of a church. A windmill is 5 km east of the tower.

   A map is to be drawn with a scale of 1 : 25 000.

   (a) What will be the distance on the map between the church and the tower?

   (b) What will be the distance on the map between the tower and the windmill?

   (c) Draw the map, and use it to calculate the actual distance between the church and the windmill.

8. The distance between London and Birmingham is 165 km. What would be the distance between these two cities on a map with a scale of:

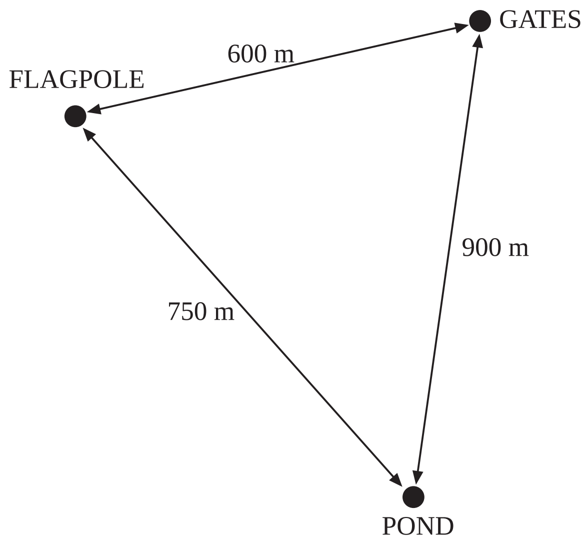
   (a) 1 : 500 000,

   (b) 1 : 1 000 000,

   (c) 1 : 300 000,

   (d) 1 : 110 000 ?

9. A student measures the distances between various points shown in her school grounds. The points are shown in the diagram, which is *not* drawn to scale.



(a) Draw a map to show this information, using a scale of 1 : 10 000.

(b) A person is exactly halfway between the gates and the pond. How far are they from the flagpole?

(c) Another student stands at the gates looking towards the flagpole. They turn anticlockwise so that they are looking at the pond. What angle does the student turn through?

10. On a map, a distance of 40 km, is represented by 32 cm. What actual distance would be represented by 14 cm on the map?

11. A map has a scale of 1 : 50 000. A park is shown on the map as a rectangle measuring 6 cm by 4.2 cm. What is the actual area of the park?

20 Arithmetic: Fractions

20.1 Revision: Whole Numbers and Decimals

In this section we revise *addition*, *subtraction*, *multiplication* and *division* of whole numbers and decimals, before starting to work with *fractions*.

Example 1

Calculate:

(a) $18 + 49$ (b) $1.6 + 0.84$ (c) $3.82 - 1.6$

Solution

(a)
$$\begin{array}{r} 18 \\ +\ 49 \\ \hline 67 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 1.60 \\ +\ 0.84 \\ \hline 2.44 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 3.82 \\ -\ 1.60 \\ \hline 2.22 \\ \hline \end{array}$$

Example 2

Calculate:

(a) 18×34 (b) 1.7×2.6

Solution

(a)
$$\begin{array}{r} 18 \\ \times\ \ 34 \\ \hline 72 \\ 540 \\ \hline 612 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 17 \\ \times\ \ 26 \\ \hline 102 \\ 340 \\ \hline 442 \\ \hline \end{array}$$

Hence $1.7 \times 1.6 = 4.42$

Example 3

Calculate:

(a) $165 \div 5$ (b) $4.26 \div 3$

Solution

(a)
$$\begin{array}{r|l} & 33 \\ \hline 5 & 1\,{}^{1}6\,5 \end{array}$$

so $165 \div 5 = 33$

(b)
$$\begin{array}{r|l} & 1.42 \\ \hline 3 & 4.\,{}^{1}2\,6 \end{array}$$

so $4.26 \div 3 = 1.42$

Exercises

1. Calculate:

(a) $182 + 57$	(b) $32 + 168$	(c) $1807 + 94$
(d) $3.2 + 4.7$	(e) $18.2 + 1.9$	(f) $3.71 + 4.2$
(g) $0.26 + 1.2$	(h) $11.4 + 6.21$	(i) $0.09 + 0.123$
(j) $38 + 4.7$	(k) $0.71 + 2.8$	(l) $4.52 + 9.89$

2. Calculate:

(a) $192 - 71$	(b) $486 - 234$	(c) $620 - 108$
(d) $0.9 - 0.2$	(e) $1.8 - 0.3$	(f) $2.42 - 1.23$
(g) $0.8 - 0.11$	(h) $8.9 - 1.12$	(i) $3.7 - 2.15$
(j) $28 - 3.7$	(k) $52 - 6.9$	(l) $4.07 - 3.88$

3. Calculate:

(a) 18×3	(b) 42×5	(c) 63×7
(d) 12×15	(e) 26×14	(f) 39×23
(g) 0.7×5	(h) 1.9×6	(i) 4.29×3
(j) 1.8×2.9	(k) 3.5×2.6	(l) 1.42×1.6

4. Calculate:

(a) $468 \div 2$	(b) $578 \div 2$	(c) $145 \div 5$
(d) $345 \div 5$	(e) $78 \div 3$	(f) $981 \div 3$
(g) $6.84 \div 4$	(h) $14.7 \div 7$	(i) $7.92 \div 6$

5. There were 52 people on a bus and 17 got off. How many people were still on the bus?

6. Floppy disks cost 34p each. How much would 6 floppy disks cost?

7. It costs £5.20 for one adult to go into a theme park. How much would it cost in total for 24 adults to go into the theme park?

8. Tickets for a show cost £3 each. To cover the cost of putting on the show, £378 is needed. How many tickets must be sold to cover the cost of the show?

9. An 8 m length of rope is cut into 5 pieces of equal length. How long is each of the 5 pieces?

10. A PE department has £30 to spend on footballs which cost £4 each.

    (a) How many footballs can they buy?

    (b) How much money will they have left?

20.2 Addition and Subtraction of Fractions

In this section we consider how to add and subtract fractions. The key step in this process is to make sure that both fractions have the *same denominator.*

Example 1

Calculate:

(a) $\frac{1}{5} + \frac{2}{5}$ (b) $\frac{5}{6} - \frac{1}{6}$

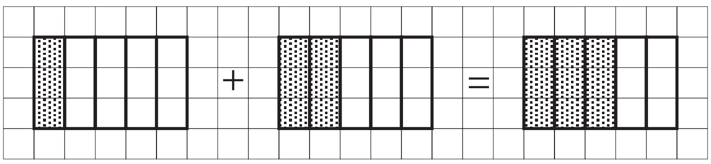
Solution

(a) As the denominator is the same in both fractions, we simply add the numbers on the top of the fraction to give

$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5}$$

$$= \frac{3}{5}$$

This addition is shown in the diagram below:



$\frac{1}{5}$ + $\frac{2}{5}$ = $\frac{3}{5}$

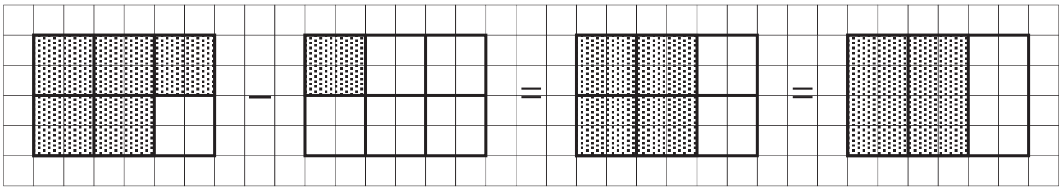
(b) The denominator is the same in both fractions, so

$$\frac{5}{6} - \frac{1}{6} = \frac{5-1}{6}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

This is shown in the diagram below:



$$\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Example 2

Calculate:

(a) $\frac{1}{4} + \frac{2}{5}$ (b) $\frac{2}{3} - \frac{1}{4}$

Solution

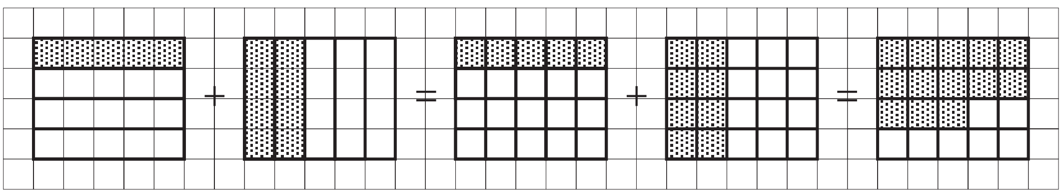
(a) These fractions do not have the same denominator, so the first step is to change them so that they do. In this case, we can use 20 as the common denominator.

$$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20}$$

$$= \frac{5+8}{20}$$

$$= \frac{13}{20}$$

This is illustrated in the diagram below:



$$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

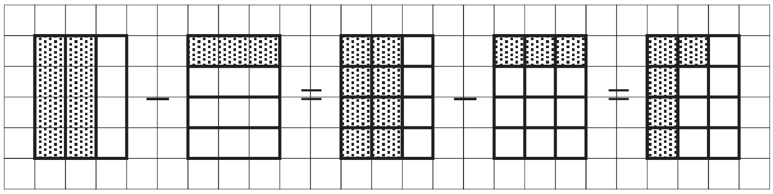
(b) In this case we can use a common denominator of 12.

$$\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12}$$

$$= \frac{8-3}{12}$$

$$= \frac{5}{12}$$

This is illustrated in the diagram below:



$$\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$

Example 3

Calculate:

(a) $1\frac{1}{8} + 3\frac{1}{3}$ (b) $4\frac{3}{8} - 1\frac{3}{4}$ (c) $2\frac{2}{3} + 1\frac{1}{2}$

Solution

(a) $1 + 3 = 4$

$$\frac{1}{8} + \frac{1}{3} = \frac{3}{24} + \frac{8}{24}$$

$$= \frac{3+8}{24}$$

$$= \frac{11}{24}$$

So $1\frac{1}{8} + 3\frac{1}{3} = 4\frac{11}{24}$

(b) $4\frac{3}{8} - 1\frac{3}{4} = \frac{35}{8} - \frac{7}{4}$

$$= \frac{35}{8} - \frac{14}{8}$$

$$= \frac{35-14}{8}$$

$$= \frac{21}{8}$$

$$= 2\frac{5}{8}$$

Note: It is usually easier to convert the mixed numbers into improper fractions.

(c) $$2\frac{2}{3} + 1\frac{1}{2} = \frac{8}{3} + \frac{3}{2}$$

$$= \frac{16}{6} + \frac{9}{6}$$

$$= \frac{16 + 9}{6}$$

$$= \frac{25}{6}$$

$$= 4\frac{1}{6}$$

Exercises

1. Calculate:

(a) $\frac{3}{7} + \frac{1}{7}$ (b) $\frac{3}{8} + \frac{1}{8}$ (c) $\frac{1}{9} + \frac{7}{9}$

(d) $\frac{3}{10} + \frac{7}{10}$ (e) $\frac{1}{5} + \frac{3}{5}$ (f) $\frac{2}{7} + \frac{4}{7}$

(g) $\frac{1}{4} + \frac{3}{4}$ (h) $\frac{5}{8} - \frac{3}{8}$ (i) $\frac{7}{9} - \frac{5}{9}$

(j) $\frac{9}{10} - \frac{7}{10}$ (k) $\frac{8}{11} - \frac{3}{11}$ (l) $\frac{4}{15} - \frac{2}{15}$

(m) $\frac{6}{13} - \frac{3}{13}$ (n) $\frac{4}{7} - \frac{3}{7}$ (o) $\frac{6}{25} - \frac{2}{25}$

2. Fill in the missing numbers:

(a) $\frac{1}{2} + \frac{1}{5} = \frac{?}{10} + \frac{?}{10} = \frac{?}{10}$

(b) $\frac{4}{5} + \frac{2}{3} = \frac{?}{15} + \frac{?}{15} = \frac{?}{15}$

(c) $\frac{1}{6} + \frac{4}{5} = \frac{5}{?} + \frac{24}{?} = \frac{?}{?}$

(d) $\frac{4}{7} - \frac{1}{3} = \frac{?}{21} - \frac{?}{21} = \frac{?}{21}$

(e) $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{?}{6} = \frac{?}{6}$

3. Calculate:

(a) $\frac{1}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{2}{3}$ (c) $\frac{1}{5}+\frac{1}{4}$

(d) $\frac{3}{5}+\frac{2}{3}$ (e) $\frac{5}{8}+\frac{1}{4}$ (f) $\frac{1}{3}+\frac{1}{6}$

(g) $\frac{4}{5}+\frac{2}{7}$ (h) $\frac{1}{7}+\frac{2}{3}$ (i) $\frac{1}{2}+\frac{1}{10}$

(j) $\frac{6}{7}+\frac{2}{3}$ (k) $\frac{5}{6}-\frac{1}{2}$ (l) $\frac{7}{8}-\frac{3}{4}$

(m) $\frac{8}{9}-\frac{3}{4}$ (n) $\frac{3}{7}-\frac{1}{3}$ (o) $\frac{4}{5}-\frac{3}{4}$

4. A birthday cake is divided into 12 equal parts. Andrew eats $\frac{3}{12}$ of the cake and Timothy eats $\frac{1}{12}$ of the cake.

(a) What fraction of the cake is left?

(b) How many pieces of cake are left?

5. A garden has an area of $\frac{3}{4}$ hectare. The owner buys an extra $\frac{3}{5}$ of a hectare of land.

(a) What is the area of the garden now?

(b) How much more land would the owner need to have a garden with an area of 2 hectares?

6. Steve and Sheila buy a computer. Steve fills $\frac{2}{5}$ of the hard disk with his programs. Sheila fills $\frac{1}{3}$ of the hard disk with her programs.

(a) What fraction of the hard disk is full?

(b) What fraction of the hard disk is empty?

(c) Steve deletes one of his programs that takes up $\frac{1}{10}$ of the hard disk.
What fraction of the hard disk do his programs fill now?

7. If $\frac{9}{10}$ of all men in the UK own cars, and $\frac{2}{3}$ of all men in the UK own *more than one* car, what fraction of men in the UK:

(a) do *not* own a car,

(b) own only *one* car?

8. Calculate:

(a) $1\frac{1}{2} + 1\frac{1}{3}$ (b) $1\frac{3}{4} + 2\frac{1}{2}$ (c) $4\frac{2}{5} + 3\frac{1}{2}$

(d) $1\frac{4}{7} + 1\frac{3}{8}$ (e) $1\frac{1}{2} - \frac{2}{3}$ (f) $3\frac{1}{4} - 1\frac{3}{5}$

(g) $2\frac{1}{2} - 1\frac{5}{8}$ (h) $4\frac{1}{7} + 3\frac{2}{3}$ (i) $4\frac{3}{5} - 2\frac{7}{8}$

(j) $6\frac{1}{4} - 1\frac{2}{5}$ (k) $3\frac{1}{2} - 1\frac{3}{4}$ (l) $5\frac{1}{4} - 2\frac{1}{2}$

9. Ron wins $£1\frac{1}{4}$ million. He gives $£\frac{3}{5}$ million to his daughter and $£\frac{1}{3}$ million to his wife. How much does he have left?

10. An old-fashioned gardener measures the height of a plant as $6\frac{3}{8}$ inches. A week later the height is measured as $8\frac{3}{5}$ inches. How much did the plant grow during the week?

20.3 Multiplying Fractions

In this section we extend the ideas of Unit 10, where you multiplied fractions by numbers, to now include multiplying fractions by fractions.

Example 1

Calculate:

(a) $\frac{1}{3}$ of £24, (b) $\frac{2}{5}$ of £40, (c) $\frac{3}{7}$ of 35 m.

Solution

(a) $\frac{1}{3}$ of £24 $= \frac{24}{3}$

$= £8$

(b) $\frac{1}{5}$ of £40 $= \frac{40}{5}$

$= £8$

$\frac{2}{5}$ of £40 $= 2 \times 8$

$= £16$

$\left\{ \text{or} \quad \frac{2}{5} \text{ of } £40 = \frac{2 \times 40}{5} = \frac{80}{5} = £16 \right\}$

(c) $\frac{1}{7}$ of 35 m $= \frac{35}{7}$

$= 5$ m

$\frac{3}{7}$ of 35 m $= 3 \times 5$

$= 15$ m

$\left\{ \text{or} \quad \frac{3}{7} \text{ of } 35 \text{ m} = \frac{3 \times 35}{7} = \frac{105}{7} = 15 \text{ m} \right\}$

Example 2

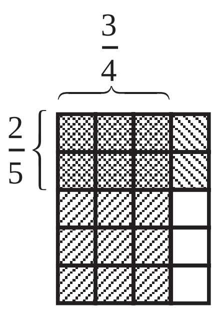
Calculate $\frac{2}{5} \times \frac{3}{4}$ and illustrate this on a diagram.

Solution

(c) $\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4}$

$= \frac{6}{20}$

$= \frac{3}{10}$



Note that 6 of the small squares are shaded twice, so $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

Note that we are using the rule:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Example 3

Calculate:

(a) $\frac{4}{7} \times \frac{3}{5}$ (b) $1\frac{3}{4} \times \frac{2}{3}$ (c) $1\frac{1}{2} \times 3\frac{1}{3}$

Solution

(a) $\frac{4}{7} \times \frac{3}{5} = \frac{4 \times 3}{7 \times 5}$

$= \frac{12}{35}$

(b) $1\frac{3}{4} \times \frac{2}{3} = \frac{7}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{3}$ (*Note*: it is usually quicker to cancel down at this stage rather than at the end.)

$= \frac{7 \times 1}{2 \times 3}$

$= \frac{7}{6}$

$= 1\frac{1}{6}$

(c) $2\frac{1}{4} \times 3\frac{1}{3} = \frac{\cancel{9}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{10}^{5}}{\cancel{3}_{1}}$

$= \frac{3 \times 5}{2 \times 1}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Exercises

1. Calculate:

(a) $\frac{1}{5} \times 15$ (b) $\frac{1}{8} \times 32$ (c) $\frac{7}{8} \times 16$

(d) $\frac{3}{7} \times 14$ (e) $\frac{3}{4} \times 28$ (f) $\frac{4}{5} \times 30$

(g) $\frac{5}{7} \times 21$ (h) $24 \times \frac{5}{8}$ (i) $18 \times \frac{5}{9}$

(j) $66 \times \frac{2}{3}$ (k) $34 \times \frac{4}{17}$ (l) $\frac{5}{19} \times 57$

2. Calculate:

(a) $\frac{1}{2} \times \frac{1}{3}$ (b) $\frac{1}{2} \times \frac{1}{2}$ (c) $\frac{1}{3} \times \frac{1}{4}$

(d) $\frac{2}{3} \times \frac{3}{4}$ (e) $\frac{3}{7} \times \frac{4}{5}$ (f) $\frac{3}{8} \times \frac{3}{4}$

(g) $\frac{4}{7} \times \frac{2}{9}$ (h) $\frac{6}{7} \times \frac{3}{8}$ (i) $\frac{5}{6} \times \frac{5}{7}$.

(j) $\frac{3}{10} \times \frac{3}{7}$ (k) $\frac{1}{2} \times \frac{3}{19}$ (l) $\frac{4}{11} \times \frac{2}{3}$

3. Calculate:

(a) $1\frac{1}{2} \times \frac{3}{4}$ (b) $4\frac{1}{2} \times \frac{2}{3}$ (c) $1\frac{3}{4} \times \frac{2}{5}$

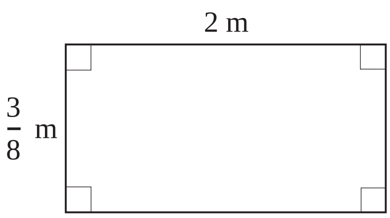
(d) $1\frac{3}{7} \times \frac{1}{2}$ (e) $4\frac{1}{4} \times \frac{1}{5}$ (f) $3\frac{1}{7} \times \frac{1}{3}$

(g) $4\frac{1}{2} \times \frac{3}{5}$ (h) $1\frac{1}{2} \times 1\frac{1}{2}$ (i) $1\frac{1}{3} \times 1\frac{1}{2}$

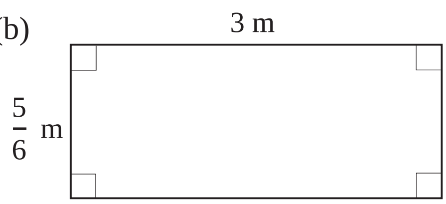
(j) $1\frac{1}{4} \times 2\frac{1}{2}$ (k) $3\frac{1}{4} \times 2\frac{1}{3}$ (l) $1\frac{1}{4} \times 2\frac{1}{5}$

4. Calculate the area of each of these rectangles:

(a)



(b)



5. A cake recipe requires $\frac{3}{4}$ kg of flour. How much flour is needed to make:

   (a) 2 cakes,

   (b) 6 cakes,

   (c) 10 cakes?

6. Jan buys $\frac{3}{4}$ kg cheese. She keeps $\frac{2}{3}$ of it and gives $\frac{1}{3}$ to her sister.
   What is the weight of:

   (a) the cheese Jan keeps,

   (b) the cheese Jan gives to her sister?

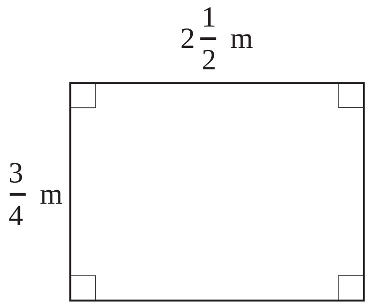
7. A large company makes £$\frac{3}{5}$ million profit. They spend $\frac{1}{4}$ of this on new equipment.

   (a) How much does the company spend on new equipment?

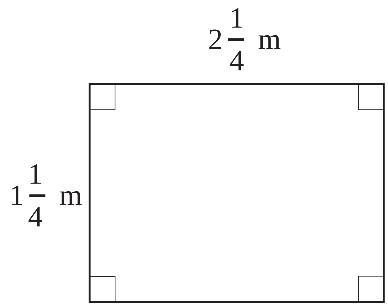
   (b) How much is left?

8. Calculate the area of each of these rectangles:

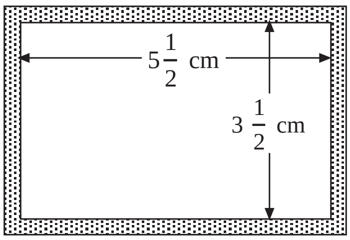
   (a)



   (b)



9. The diagram shows a small picture frame. The shaded border is $\frac{3}{4}$ cm wide.



   What is the area of the shaded border?

10. A petrol can holds $3\frac{1}{2}$ litres. Sanjit fills up a lawn mower and uses $\frac{1}{3}$ of the petrol from the full can.

    (a) How much petrol does the lawn mower hold?

    (b) How much petrol is left in the can?

    Later, Sanjit uses another $\frac{3}{4}$ litres of petrol from the can.

    (c) How much petrol has he now used?

20.4 Dividing Fractions

In this section we consider how to divide fractions and whole numbers by either whole numbers or fractions.

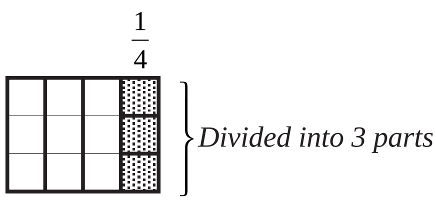
Example 1

Calculate $\frac{1}{4} \div 3$.

Solution

You can deal with this problem by thinking about the fraction being divided into 3 parts.

$\frac{1}{4}$ of the diagram has been divided into 3 parts:



Each of these parts is $\frac{1}{12}$ of the whole, so

$$\frac{1}{4} \div 3 = \frac{1}{12}$$

We can also obtain the result in this way:

$$\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$

which uses the rule:

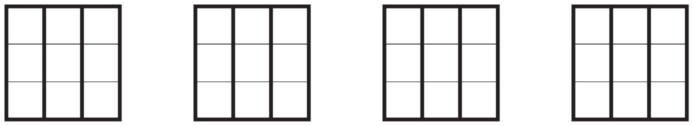
$$\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$$

Example 2

Calculate: (a) $4 \div \frac{1}{3}$, (b) $4 \div \frac{2}{5}$.

Solution

(a) The problem is to calculate how many $\frac{1}{3}$s there are in 4 whole units. The four whole units are shown below, and each is divided into $\frac{1}{3}$s.



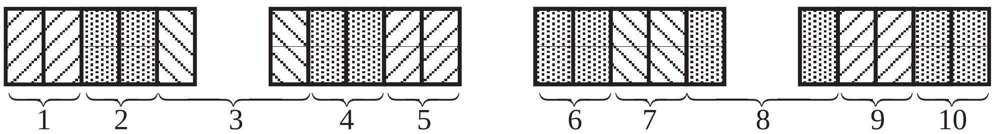
The diagram shows 12 $\frac{1}{3}$s, so

$$4 \div \frac{1}{3} = 12$$

We can obtain this result from

$$4 \div \frac{1}{3} = 4 \times 3$$
$$= 12$$

(b) The problem is to calculate how many $\frac{2}{5}$s there are in 4 whole units.



The diagram shows 10 $\frac{2}{5}$s, so

$$4 \div \frac{2}{5} = 10$$

We can also obtain this result from

$$4 \div \frac{2}{5} = 4 \times \frac{5}{2}$$
$$= \frac{20}{2}$$
$$= 10$$

using the rule:

$$a \div \frac{b}{c} = \frac{a \times c}{b}$$

Example 3

Calculate: (a) $\frac{3}{4} \div \frac{1}{5}$ (b) $\frac{5}{7} \div \frac{2}{3}$ (c) $\frac{3}{4} \div \frac{9}{10}$

Solution

These problems can be tackled using the same approach as when a whole number is divided by a fraction.

(a)
$$\frac{3}{4} \div \frac{1}{5} = \frac{3}{4} \times \frac{5}{1}$$
$$= \frac{15}{4}$$
$$= 3\frac{3}{4}$$

(b)
$$\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2}$$
$$= \frac{15}{14}$$
$$= 1\frac{1}{14}$$

(c)
$$\frac{3}{4} \div \frac{9}{10} = \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{9}}}$$
$$= \frac{5}{6}$$

[*Note:* You can cancel *only* when the 2nd fraction has been turned upside-down.]

We are using the rule:

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}}$$

Exercises

1. Calculate:

(a) $\frac{1}{2} \div 3$ (b) $\frac{3}{4} \div 2$ (c) $\frac{1}{8} \div 2$

(d) $\frac{3}{4} \div 4$ (e) $\frac{5}{8} \div 3$ (f) $\frac{4}{5} \div 2$

(g) $\frac{6}{7} \div 3$ (h) $\frac{4}{5} \div 9$ (i) $\frac{1}{8} \div 3$

(j) $\frac{5}{6} \div 4$ (k) $\frac{9}{10} \div 6$ (l) $\frac{4}{5} \div 7$

2. Calculate:

(a) $6 \div \frac{1}{2}$ (b) $9 \div \frac{1}{3}$ (c) $8 \div \frac{1}{3}$

(d) $2 \div \frac{1}{4}$ (e) $5 \div \frac{2}{3}$ (f) $4 \div \frac{3}{4}$

(g) $8 \div \frac{1}{7}$ (h) $5 \div \frac{5}{7}$ (i) $9 \div \frac{3}{7}$

(j) $6 \div \frac{2}{3}$ (k) $14 \div \frac{7}{9}$ (l) $11 \div \frac{1}{13}$

3. Calculate:

(a) $\frac{1}{2} \div \frac{1}{3}$ (b) $\frac{3}{8} \div \frac{1}{2}$ (c) $\frac{3}{4} \div \frac{2}{3}$

(d) $\frac{4}{5} \div \frac{2}{3}$ (e) $\frac{3}{4} \div \frac{1}{8}$ (f) $\frac{3}{8} \div \frac{1}{2}$

(g) $\frac{5}{7} \div \frac{2}{5}$ (h) $\frac{5}{7} \div \frac{5}{9}$ (i) $\frac{1}{8} \div \frac{2}{9}$

(j) $\frac{3}{4} \div \frac{1}{9}$ (k) $\frac{1}{7} \div \frac{1}{3}$ (l) $\frac{4}{5} \div \frac{5}{8}$

4. By using *improper fractions*, calculate:

(a) $1\frac{1}{2} \div 3\frac{1}{4}$ (b) $3\frac{1}{2} \div 1\frac{1}{4}$ (c) $1\frac{5}{8} \div \frac{5}{7}$

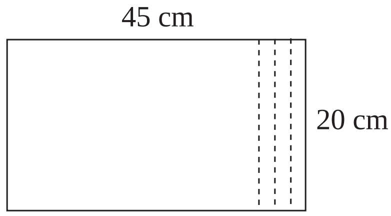
(d) $3\frac{1}{2} \div 1\frac{1}{2}$ (e) $5\frac{1}{2} \div \frac{2}{3}$ (f) $4\frac{1}{5} \div \frac{5}{7}$

5. Ahmed has $\frac{3}{4}$ kg of sweets. He divides these into 3 equal parts so that he can share them with his two brothers. What fraction of a kg does each boy get?

6. Sandra has $\frac{1}{4}$ litre of orange squash to make 10 drinks. How much orange squash should she put in each drink?

7. A large cake uses 3 times as much flour as a small cake. A large cake needs $1\frac{1}{8}$ kg of flour. How much flour does a small cake need?

8. A piece of leather is 20 cm wide and 45 cm long.



How many bookmarks, $2\frac{1}{2}$ cm wide, can be made if the leather is:

(a) cut as shown above, to make bookmarks 20 cm long,

(b) cut the other way to make bookmarks 45 cm long?

9. A recipe for a cake requires $\frac{1}{4}$ kg of sugar. How many cakes can be made with:

(a) $1\frac{1}{4}$ kg of sugar.,

(b) $2\frac{3}{4}$ kg of sugar,

(c) $3\frac{1}{3}$ kg of sugar?

10. A car uses $1\frac{1}{4}$ litres of petrol for every 10 miles it travels. How far can the car travel on:

(a) 5 litres of petrol,

(b) $7\frac{1}{2}$ litres of petrol,

(c) 9 litres of petrol?

21 Probability of One Event

21.1 Introduction to Probability

A *probability* describes mathematically how likely it is that something will happen. We can talk about the probability that it will rain tomorrow or the probability that England will win their next football match.

Example 1

Which of the words,

certain, likely, unlikely or *impossible*

best describes how likely each of the events below is to take place?

(a) It will rain tomorrow.

(b) It will snow tomorrow.

(c) Manchester United will win the next FA Cup.

(d) Exeter City will win the next FA Cup.

(e) It is your teacher's birthday tomorrow.

(f) You will obtain a 7 when rolling a dice.

Solution

(a) This can be *likely* or *unlikely* depending on the current weather pattern.

(b) This is *unlikely.*

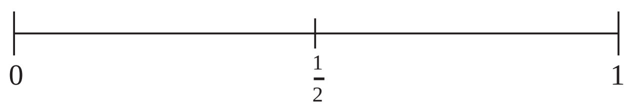
(c) This is *likely.*

(d) This is *unlikely,* some would say *impossible*!

(e) This is *unlikely,* but it could be true.

(f) *Impossible* (as only numbers 1 to 6 can be obtained).

Probability Line



Probability is 0 for an *impossible* event

Probability is 1 for a *certain* event

Probability is $\frac{1}{2}$ for an *even chance*
(for example, getting a head when you toss an unbiased coin)

Example 2

For each event below, mark an estimate of the probability that it will happen on a probability scale.

(a) It will snow on 19 August next year in London.

(b) Your maths teacher will give you homework this week.

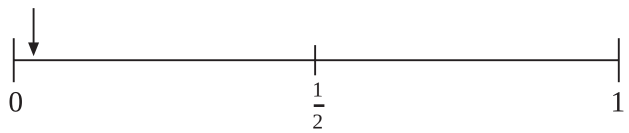
(c) You will go to school tomorrow.

(d) You will go to bed before midnight tonight.

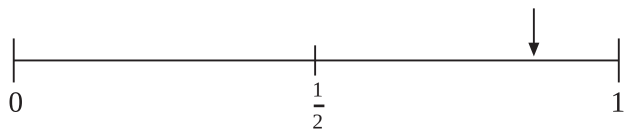
(e) You throw an unbiased dice and get an even number.

Solution

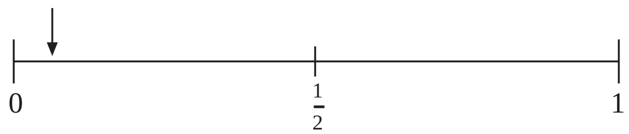
(a) This is virtually impossible, so the probability will be very close to zero.



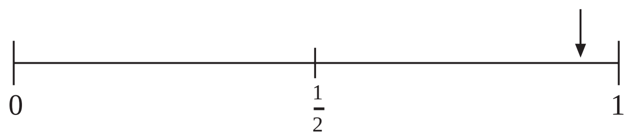
(b) This is very likely (at least for most teachers) and so the probability will be quite high.



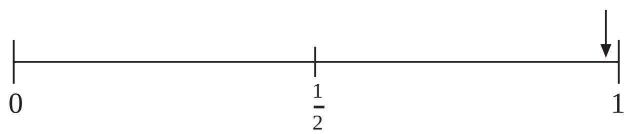
(c) The answer to this depends on what day of the week it is. On a Friday or Saturday it is very unlikely that you will go to school the next day, so the probability is low.



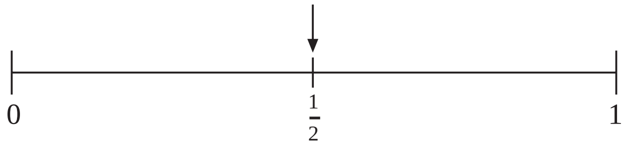
On any other day the probability is much higher as it is very likely that you will go to school the next day.



(d) This is almost certain. The probability is very close to 1.



(e) There are 3 even numbers and 3 odd numbers on a dice, so there is an even chance that you will get an even number. The probability is $\frac{1}{2}$.



Exercises

1. Use one of the words, *certain, likely, unlikely* or *impossible,* to describe each event below. Give a reason for each of your answers.

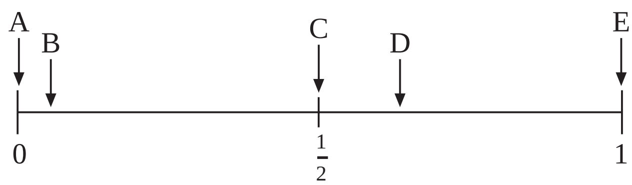
   (a) You are more than 8 years old.

   (b) You will miss the school bus tomorrow.

   (c) Your favourite football team will win their next match.

   (d) You will arrive at school on time tomorrow.

2. The probability line shows the probability of 5 events, A, B, C, D and E.



   (a) Which event is *certain*?

   (b) Which event is *impossible*?

   (c) Which event is *unlikely* but *possible*?

   (d) Which event is *most likely*, but *not certain* to occur?

3. For each event below, draw a probability line and mark in the probability of the event:

   (a) England will win the most medals at the next Olympic Games.

   (b) You will forget to take your packed lunch to school.

   (c) You will get a 6 when you roll an unbiased dice once.

   (d) It will rain tomorrow.

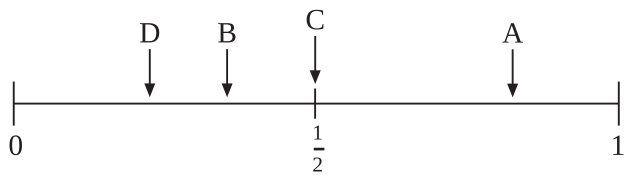
   (e) It will not rain tomorrow.

4. Describe an event that is:

   (a) impossible, (b) certain,

   (c) very unlikely, (d) very likely.

5. The events A, B, C and D have probabilities as shown on this probability line:



(a) Which event is *most likely* to take place?

(b) Which event is *most unlikely* to take place?

(c) Which events are *more likely* to take place than event B?

(d) Which events are *less likely* to take place than event A?

6. (a) What is the probability that a pupil in Year 7 will be 12 years old on their next birthday?

(b) What is the probability that a pupil in Year 7 will be 13 years old on their next birthday?

7. Estimate, by marking a probability line, the probability that you will get all your next maths homework correct.

8. The events A, B, C, D and E are listed below:

A : You will live to be 70 years old.

B : You will live to be 80 years old.

C : You will live to be 90 years old.

D : You will live to be 100 years old.

E : You will live to be 110 years old.

Mark an estimate of the probability of each event on a probability line.

9. When you toss an unbiased coin, the probability of getting a head is $\frac{1}{2}$, because you have an equal (or even) chance of getting a head or a tail.

What other events have a probability of $\frac{1}{2}$?

10. Make a list of some events that have a probability of *more than* $\frac{1}{2}$, but that are *not certain.*

21.2 Calculating the Probability of a Single Event

In this section we calculate the probabilities of single events. We consider cases where all the possible outcomes are equally likely. For example, when you roll a fair dice you are equally likely to get *any* of the six numbers. (The words 'fair' or 'unbiased' mean that all outcomes are equally likely.)

$$\textit{Probability of an event} = \frac{\textit{number of successful outcomes}}{\textit{total number of outcomes}}$$

Example 1

When you roll a fair dice, what is the probability of getting:

(a) a five,

(b) an even number,

(c) a four or a five?

Solution

The possible outcomes when you roll a dice are the scores

$$1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6$$

so there are 6 possible outcomes.

(a) In this case there is only one successful outcome, that is, a 5.

$$\text{Probability of a five} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{1}{6}$$

(b) In this case there are 3 successful outcomes, 2, 4 or 6.

$$\text{Probability of an even number} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

(c) In this case there are 2 successful outcomes, 4 or 5.

$$\text{Probability of a 4 or a 5} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Example 2

A bag of sweets contains 6 mints and 4 eclairs. One sweet is taken at random from the bag. What is the probability that it is:

(a) a mint, (b) an eclair?

Solution

The total number of possible outcomes is 10 as there are 10 sweets in the bag.

(a) As there are 6 mints in the bag, there are 6 successful outcomes.

$$\text{Probability of a mint} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

(b) As there are 4 eclairs, there are 4 successful outcomes.

$$\text{Probability of an eclair} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{4}{10}$$

$$= \frac{2}{5}$$

Exercises

1. When you roll a fair dice, what is the probability that you obtain:

   (a) an odd number,

   (b) a 2,

   (c) a multiple of 3,

   (d) a number less than 5,

   (e) a number greater than 4,

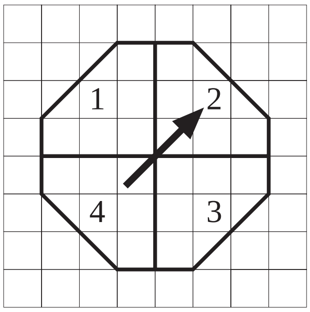
   (f) a 3 or a number less than 3?

2. A bag contains 6 red sweets and 14 blue sweets. A sweet is taken at random from the bag. What is the probability that it is:

(a) a red sweet, (b) a blue sweet?

3. You toss a fair coin. What is the probability that you obtain a tail?

4. The diagram shows a spinner from a game. The black arrow spins and ends up pointing to one of the four numbers.



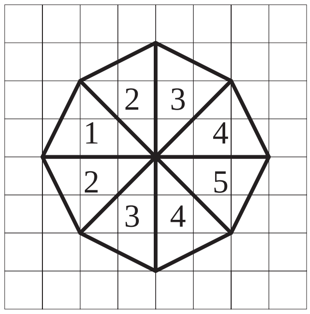
What is the probability that the arrow points to:

(a) the number 1,

(b) an even number.

(c) a multiple of 3?

5. The diagram shows a spinner that is used in a board game. When the spinner is spun, what is the probability that it lands on:



(a) 1,

(b) 5,

(c) 4,

(d) an even number,

(e) a number less than 4?

6. A bag of sweets contains 8 mints, 6 toffees and 2 boiled sweets. A sweet is taken at random from the bag. What is the probability that it is:

(a) a mint,

(b) a toffee,

(c) a boiled sweet,

(d) not a mint,

(e) not a toffee?

7. In a class there are 18 boys and 12 girls. One child is chosen at random to represent the class. What is the probability that this child is:

(a) a girl,

(b) a boy?

8. The diagram shows a piece of card that is folded to form a dice.

When the dice is rolled, what is the probability that it shows:

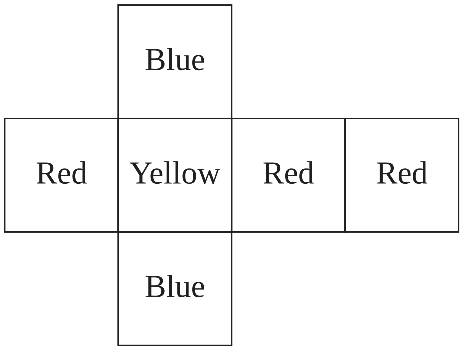
(a) a blue face,

(b) a red face,

(c) a yellow face,

(d) a face that is not red,

(e) a face that is not yellow?



9. The children in a class were asked to name their favourite colour. The results are given in the table:

Colour	*Number of Children*
Red	6
Black	2
Yellow	3
Green	4
Blue	10
Pink	7

If a child is picked at random from the class, what is the probability that their favourite colour is:

(a) red,

(b) yellow,

(c) pink

(d) black,

(e) not pink,

(f) not green?

10. A bag contains 6 red balls and some white balls. When a ball is taken from the bag at random, the probability that it is red is $\frac{3}{5}$. How many white balls are in the bag?

21.3 Relative Frequency

Some probabilities cannot be calculated as in the last section; for example, the probability that it will rain on 20 November cannot be found in this way. Probabilities can, however, be estimated using *relative frequencies* found from observations or from experiments.

$$\textit{Relative frequency} = \frac{\textit{number of successful trials}}{\textit{total number of trials}}$$

Example 1

Matthew decides to try to estimate the probability that toast lands butter-side-down when dropped. He drops a piece of buttered toast 50 times and observes that it lands butter-side-down 30 times.

Estimate the probability that the toast lands butter-side-down.

Solution

An estimate of the probability is given by the relative frequency. In this case this is

$$\frac{30}{50} = \frac{3}{5}$$

Example 2

Sarah tosses a coin 200 times. She gets 108 heads and 92 tails. Using her results, estimate the probability of obtaining:

(a) a head when the coin is tossed,

(b) a tail when the coin is tossed.

Solution

The relative frequency gives an estimate of the probability.

(a) Relative frequency $= \frac{108}{200} = \frac{27}{50}$

(b) Relative Frequency $= \frac{92}{200} = \frac{23}{50}$

We would expect both these probabilities to be $\frac{1}{2}$, and here the estimates are close to that value, indicating that her coin may be a fair one.

Example 3

Rachel was testing a coin to see if it was fair. She tossed the coin 50 times and recorded 36 HEADS. She tossed it another 50 times and recorded 32 HEADS. She continued in this way, and recorded her results on the following table:

No. of Tosses	*No. of Heads*
50	36
50	32
50	30
50	38
50	30
50	36
50	34
50	30

(a) Calculate the total frequency (total number of HEADS) after 50, 100, 150, . . . , 400 throws and also calculate, at each stage, the relative frequency.

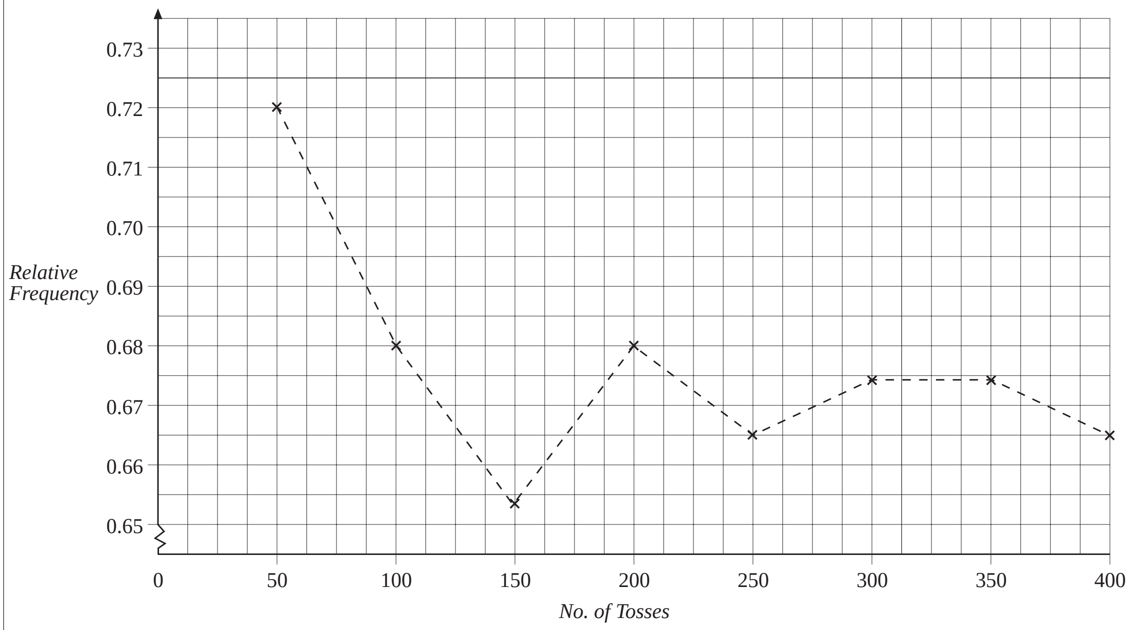
(b) Plot the points on a relative frequency graph, and hence estimate the probability of obtaining a head. What should be Rachel's conclusion?

Solution

(a)

No. of Tosses	50	100	150	200	250	300	350	400
Total Frequency	36	68	98	136	166	202	236	266
Relative Frequency	$\frac{36}{50} = 0.72$	$\frac{68}{100} = 0.68$	0.653	0.68	0.664	0.673	0.674	0.665

(b) The relative frequency graph follows, from which we can see that the probability looks to be about 0.665 (or about $\frac{2}{3}$). Rachel should conclude that her coin is probably *not* fair, i.e. the coin probably *is* biased (since, for a fair coin, we would expect the probability to be 0.5; so although the evidence is fairly strong, Rachel cannot be certain that her coin is biased).



Exercises

1. (a) Toss a coin 100 times. Record your results after every 10 tosses. Plot a relative frequency graph and estimate the probability of obtaining a head when you toss the coin.

   (b) Is your answer to (a) close to $\frac{1}{2}$?

   (c) Put all the results for your class together and obtain a new estimate of the probability of obtaining a head.

   (d) Is your new estimate closer to $\frac{1}{2}$ than the estimate in (a)?

2. A drawing pin can land 'point up' or 'point down' when dropped. Carry out an experiment to find an estimate of the probability that a drawing pin lands 'point up', using a relative frequency graph.

3. (a) Roll a dice 100 times and record the results you obtain.

   (b) Estimate the probability of obtaining each of the numbers on the faces of the dice.

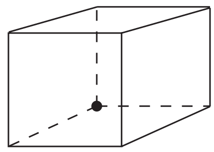
   (c) Do you think that the probabilities that you obtain are reasonable?

   (d) Obtain more results by rolling the dice another 100 times. How do your probability estimates change as you use more results?

4. By considering the people in your class, estimate the probability that a person chosen at random is left-handed.

5. If it rained on 12 days in November last year, estimate the probability that it will rain on 20 November next year.

6. You can make a biased dice out of a hollow cube by sticking a small lump of blu-tac inside the cube.



Make a biased dice and use it to estimate the probability of obtaining a 6 on your dice.

7. A calculator can be used to generate random digits. Halim generates 100 random digits with his calculator. He lists the results in the following table:

0	卌 ||	5	卌 卌
1	卌 |||	6	卌 ||||
2	卌 |	7	卌 卌 ||
3	卌 卌 ||	8	卌 卌
4	卌 卌 ||	9	卌 卌 ||||

Based on Halim's results, estimate the probability that the calculator produces:

(a) 9,

(b) 2,

(c) a digit that is an odd number,

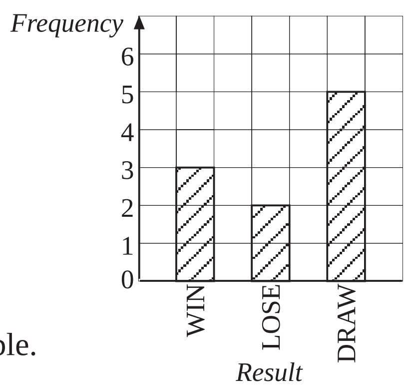
(d) a digit that is a prime number.

8. Tony estimates that the probability that there will be an empty space in the car park when he arrives at work is $\frac{4}{5}$. His estimate is based on 50 observations. On how many of these 50 days was he *unable* to find an empty space in the car park?

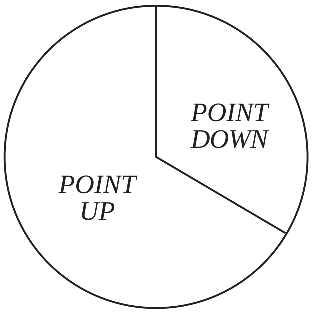
9. Paul draws the bar chart opposite to show the results for his football team so far this season.

(a) Use the bar chart to estimate the probability that his team will win their next match.

(b) Give reasons why this estimate of the probability that they will win their next match may not be very reliable.



10. Sasha carries out the drawing pin experiment described in question 2. She shows her results in this pie chart:



Use her results to estimate the probability that the pin lands 'point up'.

21.4 Complementary Events

Two events are described as *complementary* if they are the *only two* possible outcomes. For example, the events A and B below are complementary:

A : It rains.

B : It does not rain.

> If A is an event and A' is the complementary event,
>
> $$p(\text{A}) + p(\text{A}') = 1$$
>
> or
>
> $$p(\text{A}') = 1 - p(\text{A})$$

Example 1

If the probability that it will rain tomorrow is $\frac{1}{5}$, what is the probability that it will not rain tomorrow?

Solution

As these are two complementary events,

$$\text{probability that it will not rain tomorrow} = 1 - \frac{1}{5}$$

$$= \frac{4}{5}$$

Example 2

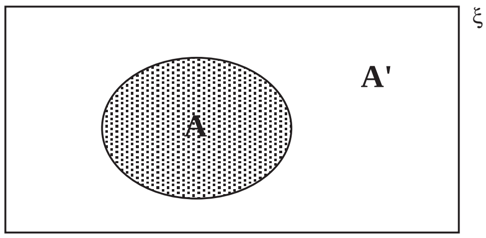
A dice has been renumbered so that the probability of obtaining an even number is now $\frac{2}{3}$. What is the probability of obtaining an odd number?

Solution

As these are two complementary events,

$$\text{probability of obtaining an odd number} = 1 - \frac{2}{3} = \frac{1}{3}$$

Complementary events can be illustrated in a Venn diagram:



Exercises

1. The probability that Scot will win his next tennis match is $\frac{3}{5}$. What is the probability that he will *not* win?

2. The probability that it will snow on Christmas Day is $\frac{1}{8}$. What is the probability that it will *not* snow on Christmas Day?

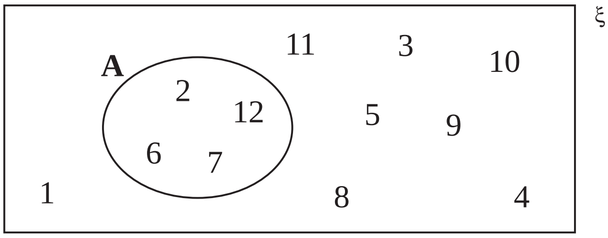
3. The probability that a child is left-handed is $\frac{1}{20}$. What is the probability that a child is right-handed?

4. The probability that Natasha is late for school is 0.1. What is the probability that she is *not* late?

5. The probability that Sergio gets all his spellings correct in his next test is 0.75. What is the probability that he does *not* get them all correct?

6. If you take a card at random from a pack of playing cards, the probability that you get a king is $\frac{1}{13}$. What is the probability that you do *not* get a king?

7. One of the numbers in the Venn diagram below is chosen at random.



(a) What is the probability that the number is in the set A?

(b) What is the probability that the number is *not* in the set A?

8. (a) What is the probability that you will get a prime number when you roll a fair dice?

(b) What is the probability that you will *not* get a prime number when you roll a fair dice?

9. In one year it rains on 12 days in the month of September.

(a) Use this information to estimate the probability that it will rain on 19 September next year.

(b) Use your answer to (a) to make an estimate of the probability that it will not rain on 19 September next year.

10. A bag contains 100 balls, each marked with a number from 1 to 100. A ball is taken from the bag at random.

(a) What is the probability that the number on the ball is a multiple of 3?

(b) What is the probability that the number on the ball is *not* a multiple of 3?

21.5 Estimating the Number of Outcomes

If we know the probability of an event we can estimate the number of times we expect that event to take place.

Expected number of successful outcomes = *probability of success* × *total number of outcomes*

Example 1

You toss an unbiased coin 500 times. How many heads should you expect to obtain?

Solution

Probability of a head $= \frac{1}{2}$

Expected number of heads $= \frac{1}{2} \times 500$

$= 250$

Example 2

You roll a fair dice 120 times. How many times would you expect to obtain:

(a) a 6,

(b) a multiple of 3?

Solution

(a) Probability of a 6 $= \frac{1}{6}$

Expected number of sixes $= \frac{1}{6} \times 120$

$= 20$

(b) Probability of a multiple of 3 $= \frac{1}{3}$

Expected number of multiples of 3 $= \frac{1}{3} \times 120$

$= 40$

Exercises

1. If you roll an unbiased dice 600 times, how many times would you expect to obtain:
   (a) a one,
   (b) an *even* number,
   (c) an *odd* number,
   (d) a number less than 3?

2. A spinner is marked with the numbers 1 to 5, each of which is equally likely to occur when the spinner is spun. If it is spun 200 times, how many times would you expect to obtain:
   (a) a five,
   (b) an even number,
   (c) a number less than 3,
   (d) a prime number?

3. If the probability that it rains on a day in September is $\frac{1}{5}$, on how many days in September would you expect it to rain?

4. When you open a packet of sweets and take one out at random, the probability that it is blue is $\frac{1}{8}$. If you open 40 packets of sweets, how many times would you expect to take out a blue sweet *first*?

5. Some crisp packets contain prizes. The probability that you find a prize in a crisp packet is $\frac{1}{25}$. How many prizes would you expect to find if you opened:
   (a) 50 packets,
   (b) 200 packets,
   (c) 1000 packets?

6. The probability that Joshua misses the school bus is $\frac{3}{10}$. In a school year there are 40 weeks, each of 5 days.
   How many times can you expect Joshua to miss the bus in:
   (a) a 12-week term,
   (b) a school year?

7. The probability that a person, selected at random, has been trained in First Aid is $\frac{1}{50}$. How many people trained in First Aid would you expect to find in:

   (a) a crowd of 50 000 spectators at a football match,

   (b) an audience of 300 at a theatre,

   (c) a group of 50 onlookers at the scene of an accident?

8. The probability that a certain type of seed germinates is 0.7. How many seeds would you expect to germinate if you planted:

   (a) 20 seeds,

   (b) 70 seeds,

   (c) 1000 seeds?

9. The probability that Emma wins a game of 'Freecell' on her computer is $\frac{2}{5}$. She wants to be able to say that she has won 5 games. How many games should she expect to play before she wins 5 games?

10. Prakesh says that the probability that he misses the school bus is $\frac{1}{10}$.

    (a) How many times would you expect him to miss the bus in 4 weeks?

    (b) In 4 weeks he actually misses the bus 3 times, which is not the same as your answer to (a). Explain why your answer to (a) is still correct.

21.6 Addition Law for Mutually Exclusive Events

Two events are *mutually exclusive* if only *one* can take place at any given time. For example, if a bag contains red balls and yellow balls, when a ball is taken out it is either red or yellow, but it cannot be both. The events 'red ball' and 'yellow ball' are therefore *mutually exclusive.*

> If two events, A and B, are mutually exclusive,
>
> $$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$
>
> where $p(\text{A}) =$ probability of A
>
> and $p(\text{B}) =$ probability of B

Example 1

A bag contains 6 red balls, 8 yellow balls and 4 green balls. One ball is taken at random from the bag.

What is the probability that the ball is:

(a) yellow,

(b) green,

(c) yellow or green?

Solution

(a) $p(\text{yellow}) = \frac{8}{18}$

$= \frac{4}{9}$

(b) $p(\text{green}) = \frac{4}{18}$

$= \frac{2}{9}$

(c) $p(\text{yellow or green}) = p(\text{yellow}) + p(\text{green})$

$= \frac{4}{9} + \frac{2}{9}$

$= \frac{6}{9}$

$= \frac{2}{3}$

Example 2

Ten children were asked to state their favourite sport. Their responses are listed in this table:

Favourite Sport	*Number of Children*
Swimming	4
Hockey	1
Tennis	1
Volleyball	1
Football	3

What is the probability that, if one of the children is chosen at random, their favourite sport will be:

(a) volleyball, (b) swimming, (c) volleyball or swimming?

Solution

(a) $p(\text{volleyball}) = \frac{1}{10}$

(b) $p(\text{swimming}) = \frac{4}{10}$

$= \frac{2}{5}$

(c) $p(\text{volleyball or swimming}) = p(\text{volleyball}) + p(\text{swimming})$

$= \frac{1}{10} + \frac{4}{10}$

$= \frac{5}{10}$

$= \frac{1}{2}$

Exercises

1. A bag contains 6 red balls, 5 blue balls and 9 yellow balls. A ball is taken at random from the bag. What is the probability that it is:

   (a) red,

   (b) blue,

   (c) yellow,

   (d) red or blue,

   (e) red or yellow,

   (f) blue or yellow?

2. In a packet of sweets there are 20 mints, 10 fudges and 20 toffees. A sweet is taken at random from the packet. What is the probability that it is:

   (a) a fudge,

   (b) a toffee,

   (c) a mint,

   (d) a mint or a toffee,

   (e) a mint or a fudge,

   (f) a toffee or a fudge?

3. A group of children were asked their ages. These are recorded in the table opposite:

Age (in years)	*Number of Children*
10	4
11	14
12	16
13	6

What is the probability that a child selected at random from the group, is:

(a) age 10,

(b) age 10 or 11,

(c) age 12 or 13,

(d) age 11 or 12?

4. In the school canteen, children can choose one of baked potato, chips or rice for lunch. The probability that a child chooses a baked potato is $\frac{1}{6}$, the probability that they choose chips is $\frac{2}{3}$ and the probability that they choose rice is $\frac{1}{6}$.

What is the probability that a child chooses:

(a) rice or baked potato,

(b) rice or chips,

(c) chips or baked potato?

5. Zahra buys a mobile phone with 5 different coloured covers. The probability of her using each cover is given in the table opposite.

Colour	*Probability*
Red	$\frac{1}{8}$
Blue	$\frac{1}{4}$
Green	$\frac{1}{16}$
Yellow	$\frac{1}{16}$
Pink	$\frac{1}{2}$

What is the probability that she uses:

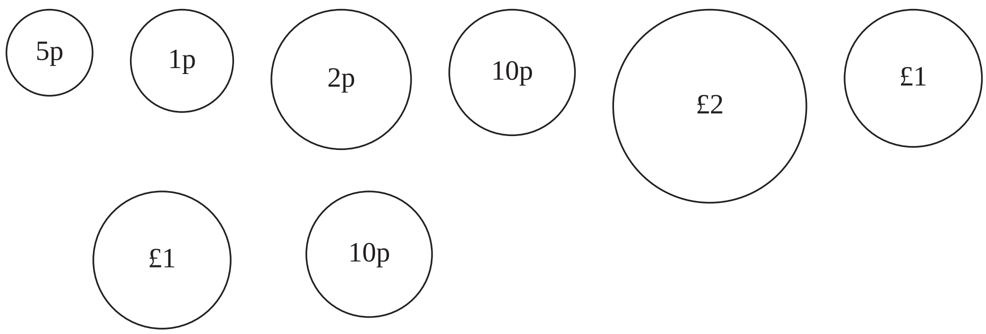
(a) a red or a pink cover,

(b) a green or a red cover,

(c) a yellow or a blue cover,

(d) a pink or a yellow cover?

6. In his pocket, Andy has the coins below:



He takes a coin at random from his pocket.

What is the probability that it is:

(a) a £1 or a £2 coin,

(b) a 1p or a 2p coin,

(c) a 5p or a 10p coin,

(d) a 1p or a £1 coin?

7. Alex can walk to school, cycle to school or go by bus. The probability that he walks or cycles to school is $\frac{3}{4}$.

(a) If the probability that he cycles is $\frac{1}{4}$, what is the probability that he walks?

(b) What is the probability that he goes by bus?

(c) What is the probability that he cycles or goes by bus?

8. A bus can arrive early, on time or late. The probability that it is late is $\frac{1}{4}$.

The probability that it is on time or late is $\frac{2}{3}$.

(a) What is the probability that the bus is on time?

(b) What is the probability that it is early or on time?

9. A bag contains 30 coloured balls. Of these, 10 are red, 6 are blue and the rest are green or yellow.

A ball is taken at random from the bag. The probability that this ball is yellow is $\frac{1}{6}$.

What is the probability that a ball taken at random from the bag is:

(a) green,

(b) green or yellow,

(c) red or blue,

(d) red, blue or yellow?

10. A bag contains red, yellow and green balls. One ball is taken at random from the bag. The probability that it is red or green is $\frac{1}{2}$. The probability that it is yellow or green is $\frac{3}{4}$.

What is the probability that a ball taken at random from the bag is:

(a) green,

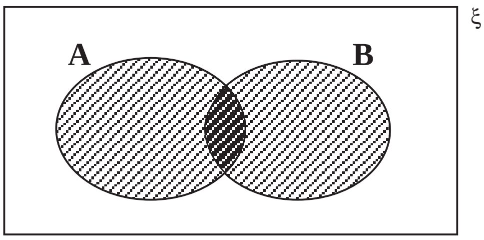
(b) red or yellow?

21.7 General Addition Law

Events may not always be mutually exclusive. For example, if you roll a dice, the events 'getting a six' and 'getting an even number' are *not* mutually exclusive. In this section we consider examples of this type.

$$p(\text{A or B}) = p(\text{A}) + p(\text{B}) - p(\text{A and B})$$

This can be shown using a *Venn diagram*.



The *union* (A or B) is shaded, whilst the *intersection* (A and B) is double shaded.

So, if there are n possible events, with

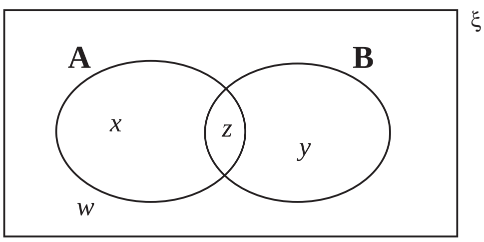
number of events in only A $= x$

number of events in only B $= y$

number of events in both A and B $= z$

number of events in neither A nor B $= w$,

this can be illustrated in the following Venn diagram:



Thus

$$p(\text{A}) = \frac{x+z}{n}$$

$$p(\text{B}) = \frac{y+z}{n}$$

$$p(\text{A and B}) = \frac{z}{n}$$

and $$p(\text{A or B}) = \frac{x+z+y}{n}$$

Now $$p(\text{A}) + p(\text{B}) - p(\text{A and B}) = \left(\frac{x+z}{n}\right) + \left(\frac{y+z}{n}\right) - \left(\frac{z}{n}\right)$$

$$= \frac{x+z+y+z-z}{n}$$

$$= \frac{x+z+y}{n}$$

$$= p(\text{A or B})$$

and so the result is true.

Example 1

One of the numbers 1 to 10 is selected at random. What is the probability that it is:

(a) an even number,

(b) greater than 5,

(c) an even number *and* greater than 5,

(d) an even number *or* greater than 5.

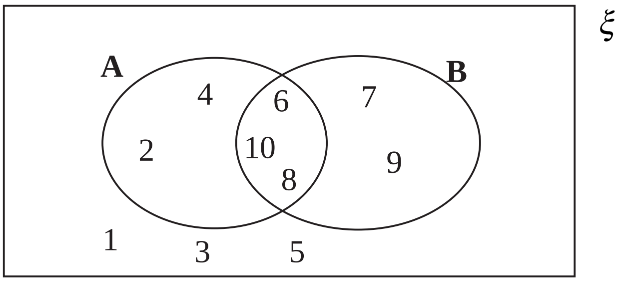
Solution

A Venn diagram is helpful for problems like this.

Let:

A be the set of even numbers

and B be the set of numbers greater than 5.



(a) $p(\text{an even number}) = p(\text{A})$

$= \frac{5}{10}$

$= \frac{1}{2}$

(b) $p(\text{greater than 5}) = \frac{5}{10}$

$= \frac{1}{2}$

(c) $p(\text{even } \textit{and} \text{ greater than 5}) = p(\text{A and B})$

$= \frac{3}{10}$

(d) $p(\text{even } \textit{or} \text{ greater than 5}) = p(\text{A}) + p(\text{B}) - p(\text{A and B})$

$= \frac{1}{2} + \frac{1}{2} - \frac{3}{10}$

$= \frac{7}{10}$

Example 2

A dice is rolled. What is the probability of getting a prime number or an even number?

Solution

$p(\text{even number}) = \frac{3}{6}$ (since there are 3 even numbers, 2, 4 and 6)

$= \frac{1}{2}$

$$p(\text{prime number}) = \frac{3}{6} \quad \text{(since there are 3 prime numbers, 2, 3 and 5)}$$

$$= \frac{1}{2}$$

$$p(\text{even } \textit{and} \text{ prime number}) = \frac{1}{6} \quad \text{(since there is one even, prime number, namely 2)}$$

$$p(\text{even } \textit{or} \text{ prime}) = p(\text{even}) + p(\text{prime}) - p(\text{even and prime})$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6}$$

$$= \frac{5}{6}$$

Exercises

1. One of the numbers 10 to 20 is selected at random. What is the probability that it is:

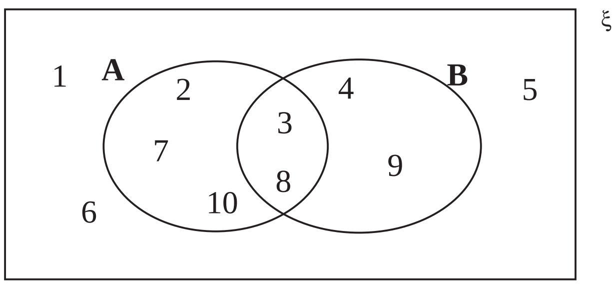
   (a) an even number,

   (b) a multiple of 5,

   (c) an even number *and* a multiple of 5,

   (d) an even number *or* a multiple of 5?

2. The numbers 1 to 10 are sorted into sets as shown in the Venn diagram:



   One of these numbers is selected at random. What is the probability that it is a member of:

   (a) A,

   (b) B,

   (c) A *and* B,

   (d) A *or* B?

3. A fair dice is rolled. What is the probability of getting a prime number *or* an odd number?

4. One of the numbers 1 to 20 is selected at random. What is the probability that the number is a multiple of 3 *or* a multiple of 4?

5. A bingo set contains 100 balls each marked with one of the numbers 1 to 100. One of these balls is selected at random. What it the probability that the number on this ball is a multiple of 7 or of 10?

6. If $p(\text{A}) = \frac{3}{4}$, $p(\text{B}) = \frac{3}{8}$ and $p(\text{A and B}) = \frac{1}{3}$, find $p(\text{A or B})$.

7. If $p(\text{A or B}) = \frac{3}{5}$, $p(\text{A and B}) = \frac{2}{5}$ and $p(\text{A}) = \frac{1}{2}$, find $p(\text{B})$.

8. If $p(\text{A or B}) = \frac{1}{2}$, $p(\text{A}) = \frac{2}{5}$ and $p(\text{B}) = \frac{1}{3}$, find $p(\text{A and B})$.

9. The probability that Jai is late for school is 0.1. The probability that he forgets his lunch is 0.3. The probability that he forgets his lunch *and* is late is 0.05.

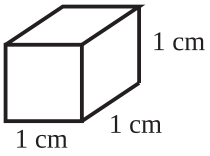
   What is the probability that he forgets his lunch or is late?

10. The faces of a dice are marked with the numbers 1, 2, 3 or 4. This is done so that the probability of rolling a 3 is $\frac{1}{6}$, the probability of rolling a 3 or a 4 is $\frac{1}{3}$ and the probability of rolling a prime or an even number is $\frac{2}{3}$. How often does each number appear on the dice?

22 Volume

22.1 Concept of Volume: the Unit Cube

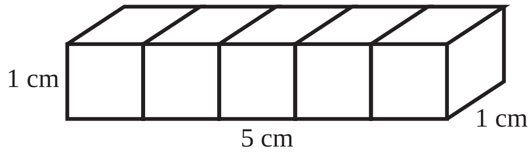
In this section we look at volume for the first time, by counting the number of 1 cm cubes in a solid.



The volume of this cube is 1 cm^3 (1 cubic centimetre)

Example 1

What is the volume of this solid:

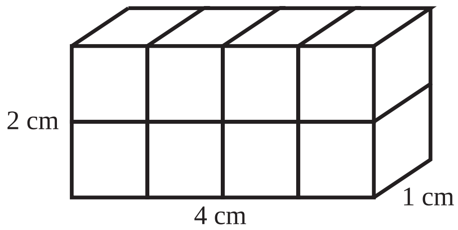


Solution

The solid contains 5 cubes of side 1 cm, so the volume is 5 cm^3.

Example 2

What is the volume of this solid:



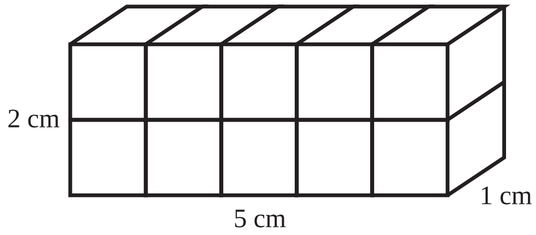
Solution

This solid contains 8 cubes of side 1 cm, so the volume is 8 cm^3.

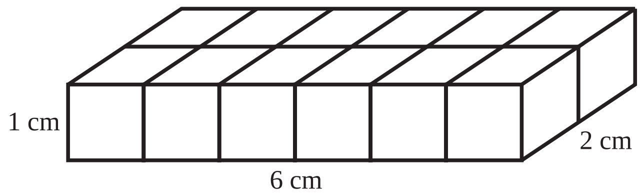
Exercises

1. What is the volume of each of these cuboids:

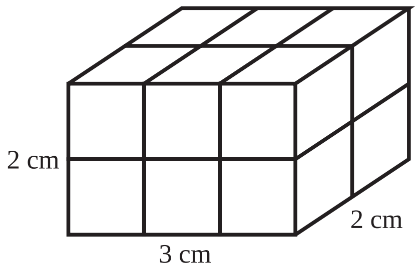
(a)



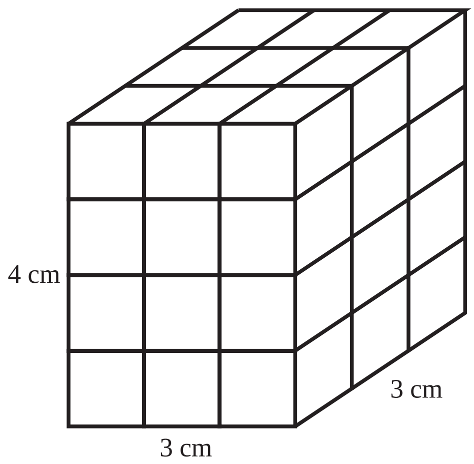
(b)



(c)

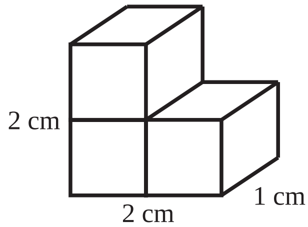


(d)

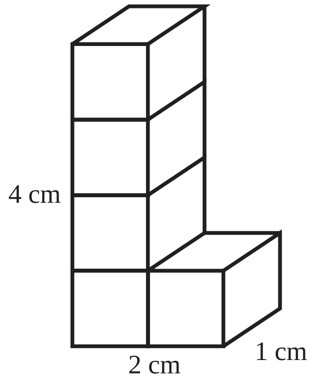


2. What is the volume of each of these solids:

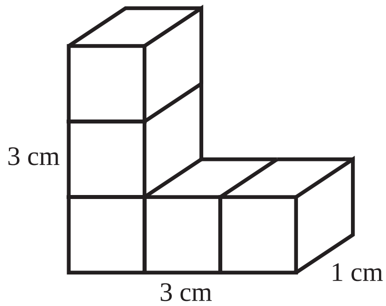
(a)



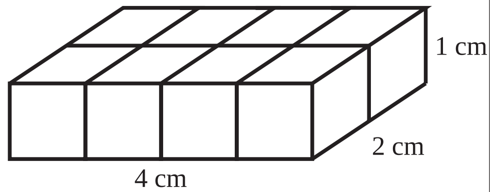
(b)



(c)



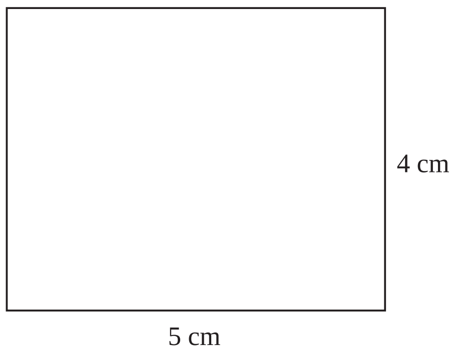
3. The diagram shows the cubes that are used to make the first layer of a cuboid:



(a) How many cubes are there in the first layer?

(b) What is the volume of the cuboid if it is made up of 6 layers?

4. A cuboid is built from 1 cm cubes on top of this rectangular base:



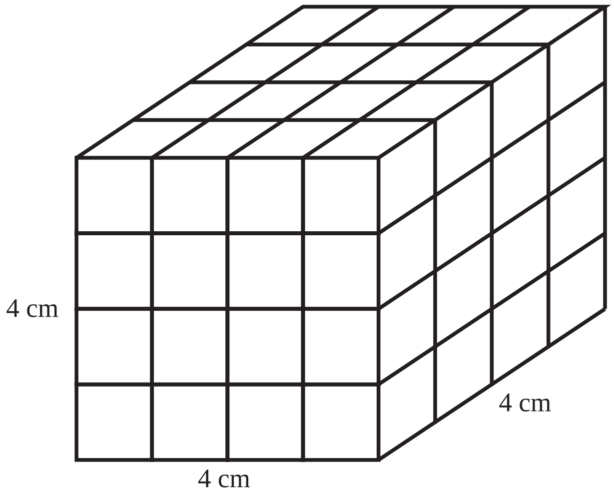
(a) How many cubes are there in the first layer?

(b) If there are 4 layers, what is the volume of the cuboid?

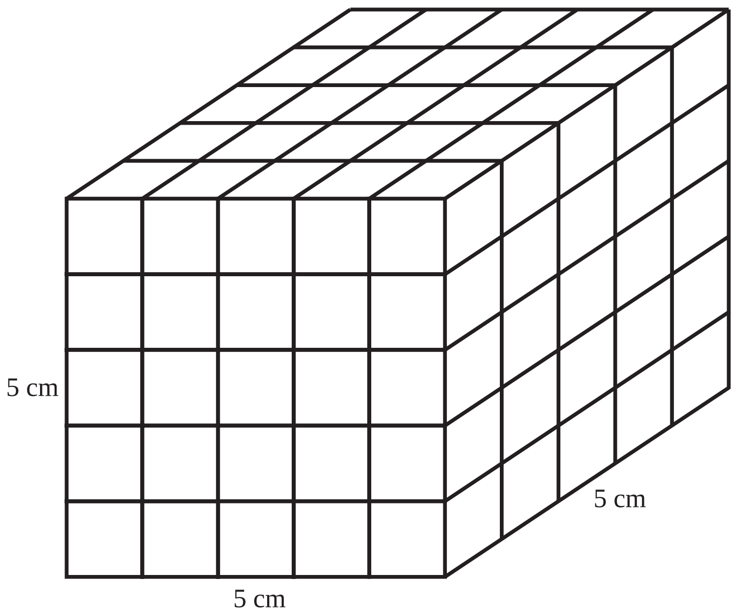
5. The diagram below shows a large cube made from 1 cm cubes.

   (a) How many small cubes are in each layer of the large cube?

   (b) What is the volume of the large cube?

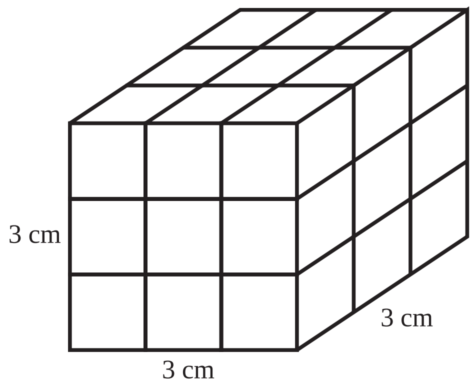


6. What is the volume of this cube:

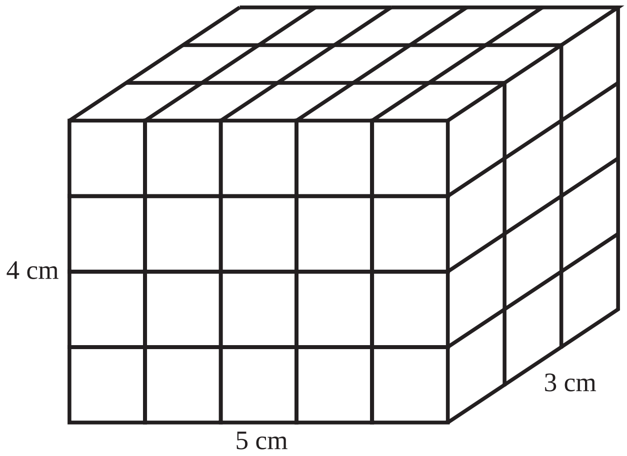


7. (a) What is the volume of the cube shown in the diagram opposite?

   (b) The top layer is cut off. What is the volume of the solid that remains?



8. Sally is asked to make a cuboid with sides of length 7 cm, 3 cm and 4 cm. She runs out of cubes when she has made the shape in the diagram:



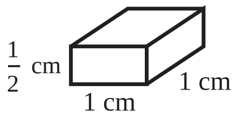
(a) What is the volume of the shape she has made?

(b) How many more cubes would she need to make the required shape?

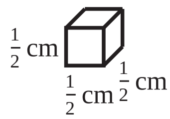
9. How many of each of these shapes would be needed to make a 1 cm cube?

What is the volume of each of these shapes:

(a)

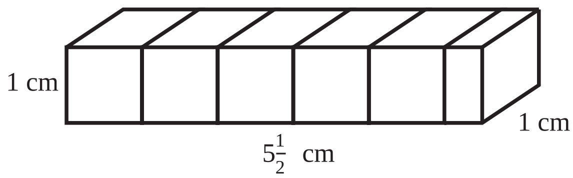


(b)

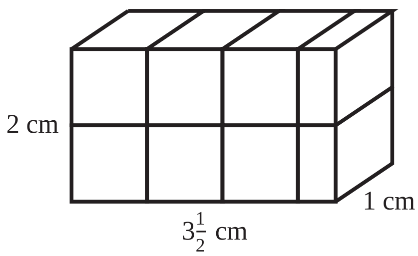


10. What is the volume of each of these shapes:

(a)



(b)



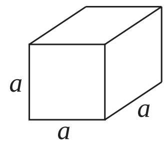
22.2 Volume of a Cube

In this section we consider the volume of a cube and the units of volume.

Volume of a cube $= a \times a \times a$

$= a^3$

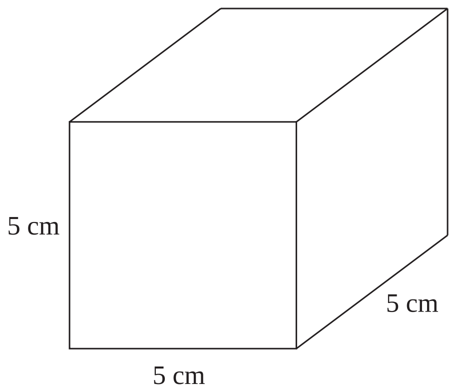
where a is the length of the each side of the cube



Note: If the sides of the cube are measured in cm, the volume will be measured in cm^3.

Example 1

What is the volume of this cube:



Solution

Volume $= 5^3$

$= 5 \times 5 \times 5$

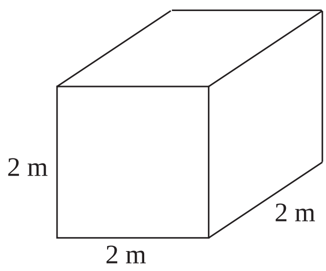
$= 125 \text{ cm}^3$

Example 2

What is the volume of this cube in:

(a) m^3,

(b) cm^3?



Solution

(a) Volume $= 2^3$

$= 2 \times 2 \times 2$

$= 8 \text{ m}^3$

(b) Remember that $1 \text{ m} = 100 \text{ cm}$, so $2 \text{ m} = 200 \text{ cm}$.

Volume $= 200^3$

$= 200 \times 200 \times 200$

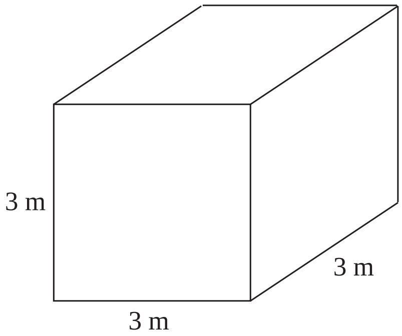
$= 8\,000\,000 \text{ cm}^3$

$$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$

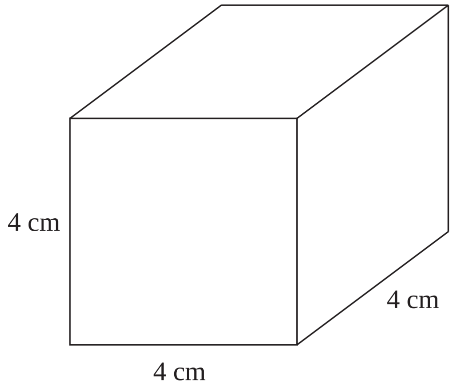
Exercises

1. What is the volume of each of these cubes:

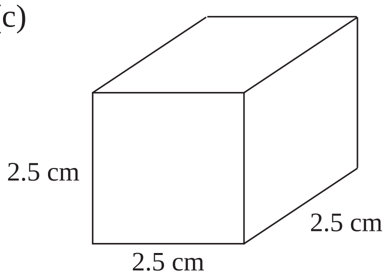
(a)



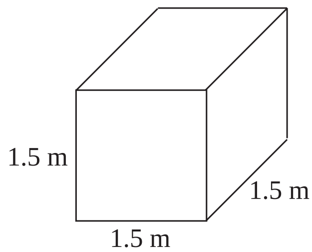
(b)



(c)



(d)



2. A cube has sides of length 30 cm. What is the volume of the cube in:

(a) cm^3,

(b) m^3?

3. A large box is a cube with sides of length 80 cm. Smaller boxes, which are also cubes, have sides of lengths 20 cm.

(a) What is the volume of the large box?

(b) What is the volume of a small box?

(c) How many small boxes will fit in the large box?

4. A cube has sides of length $\frac{1}{2}$ m.

Calculate the volume of the cube:

(a) in m^3, giving your answer as a fraction,

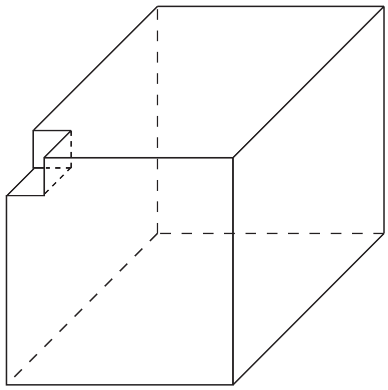
(b) in m^3, giving your answer as a decimal,

(c) in cm^3.

5. A cube has sides of length 10 cm. Calculate the volume of the cube in:

(a) cm^3, (b) m^3.

6. The diagram shows a cube with sides of length 30 cm. A smaller cube with sides of length 5 cm has been cut out of the larger cube.



(a) What is the volume of the large cube before the small cube is cut out?

(b) What is the volume of the small cube?

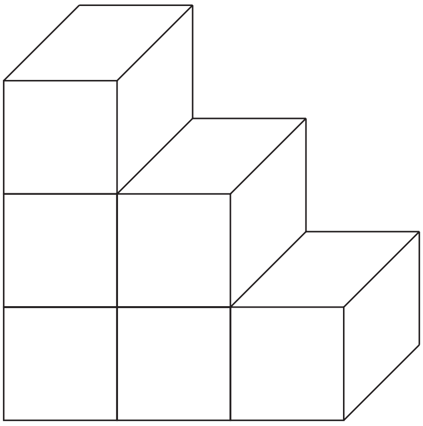
(c) What is the volume of the shape that is left?

7. Wooden building blocks are cubes with sides of length 4 cm. A child builds a tower 6 blocks high. What is the volume of the tower?

8. This 'staircase' is built from wooden cubes with sides of length 6 cm.

(a) What is the volume of the staircase?

(b) A similar staircase is 4 blocks high instead of 3. What is the volume of this staircase?



9. A cube has volume 343 cm^3.

(a) How long are the sides of the cube?

(b) What is the area of one face of the cube?

(c) What is the total area of the surface of the cube?

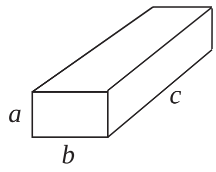
10. The area of one face of a cube is 81 cm^2.
What is the volume of the cube?

22.3 Volume of a Cuboid

We now consider the volume of any cuboid.

Volume $= a \times b \times c$

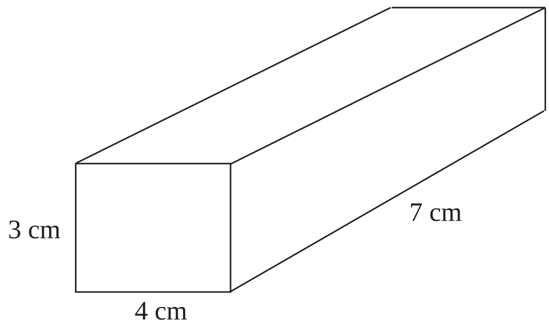
$= abc$



where a, b and c are the lengths of the sides of the cuboid

Example 1

Calculate the volume of this cuboid:



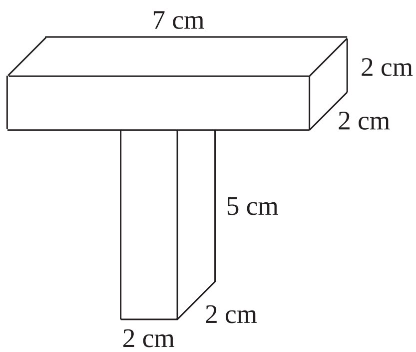
Solution

Volume $= 3 \times 4 \times 7$

$= 84 \text{ cm}^3$

Example 2

A letter 'T' shape is made by sticking together 2 cuboids as shown in the diagram. What is the total volume of the letter 'T' ?



Solution

First find the volume of the top cuboid:

Volume $= 7 \times 2 \times 2$

$= 28 \text{ cm}^3$

Then find the volume of the upright cuboid:

Volume $= 2 \times 2 \times 5$

$= 20 \text{ cm}^3$

These two volumes can then be added to give the total volume:

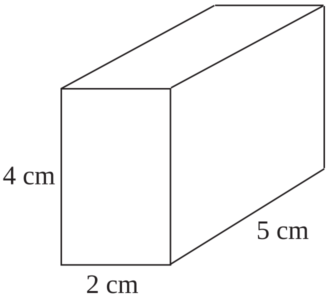
Total volume $= 28 + 20$

$= 48 \text{ cm}^3$

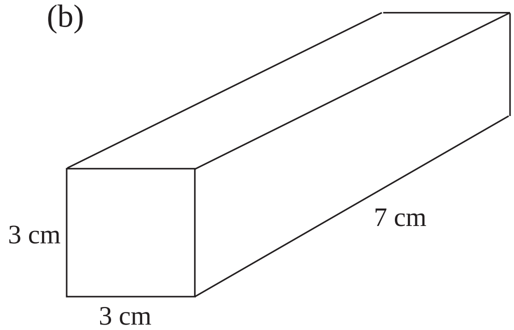
Exercises

1. Calculate the volume of each of these cuboids:

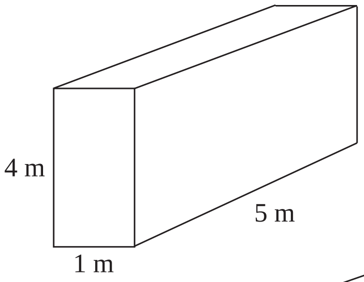
(a)



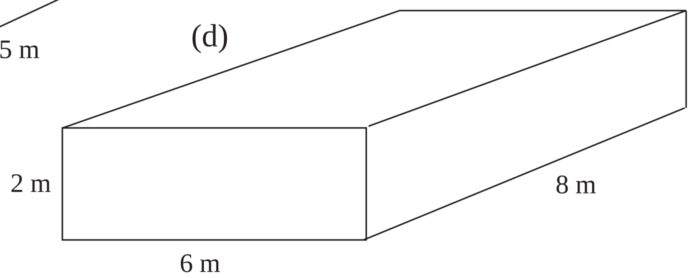
(b)



(c)



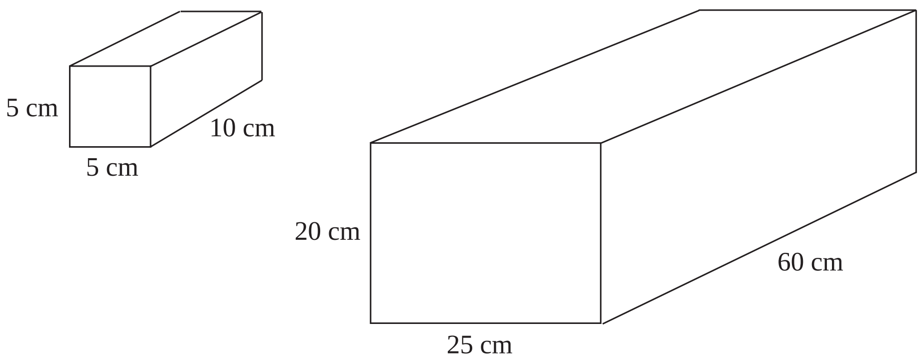
(d)



2. A cuboid has sides of length 5 m, 3 m and 1 m. What is the volume of the cuboid in:

   (a) m^3, (b) cm^3?

3. The diagram shows a large box and a small box, both of which are cuboids.



   (a) Calculate the volume of the large box.

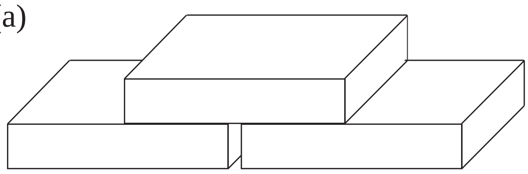
   (b) Calculate the volume of the small box.

   (c) How many of the small boxes would fit in the large box?

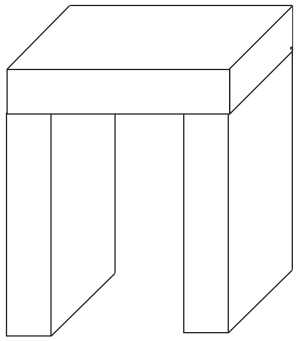
4. A set of wooden building blocks contains wooden blocks that are 10 cm by 2 cm by 4 cm. The blocks are used to make the shapes below.

   Calculate the volume of each shape.

   (a)



   (b)

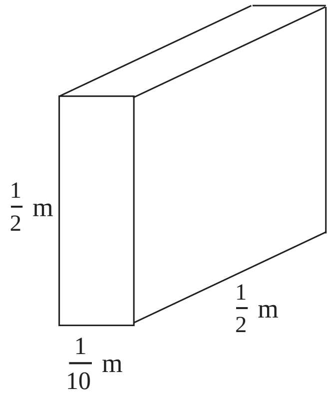


5. Calculate the volume of this cuboid, giving your answer:

   (a) in m^3, using fractions,

   (b) in m^3, using decimals,

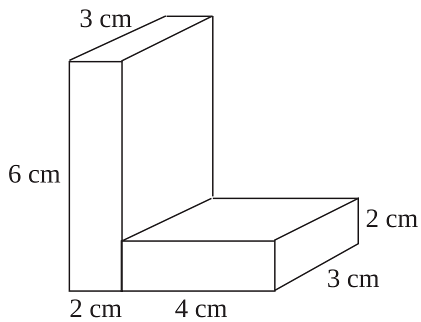
   (c) in cm^3.



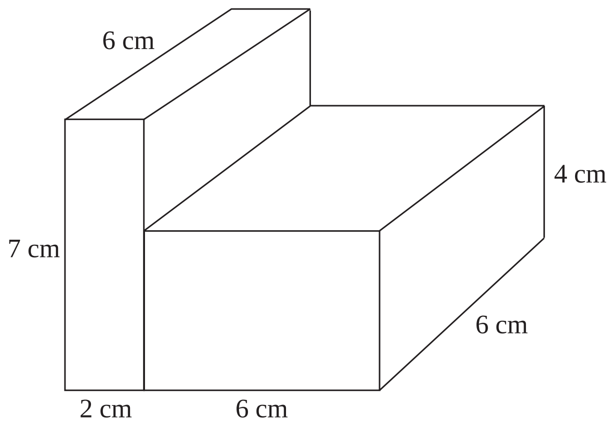
6. A letter 'L' shape is made from two cuboids.

   (a) Calculate the volume of each cuboid.

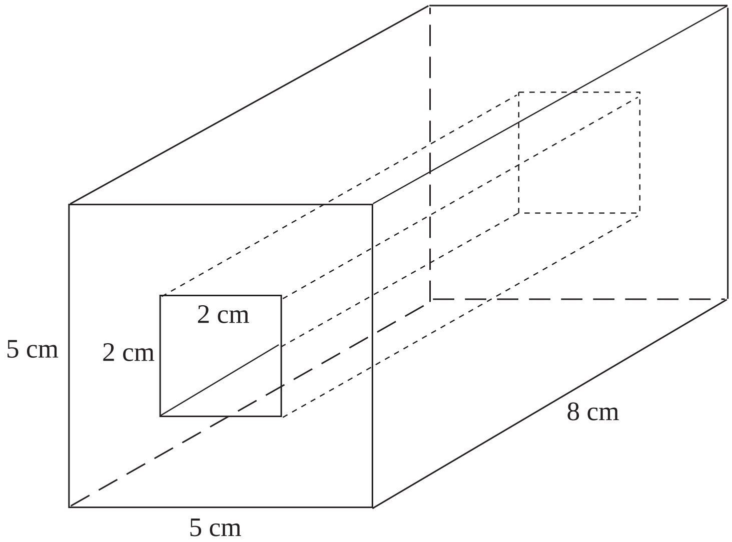
   (b) Calculate the volume of this letter 'L' shape.



7. Calculate the volume of this solid:



8. The diagram shows a wooden block that has had a square hole cut through it. Calculate the volume of wood in the block.



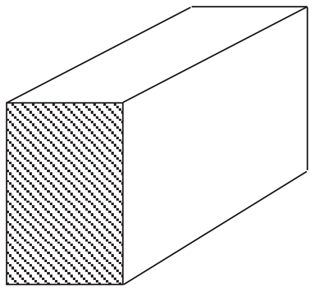
9. The diagram shows a cuboid. The area of the shaded end is 8 cm^2.

How long is the cuboid if its volume is:

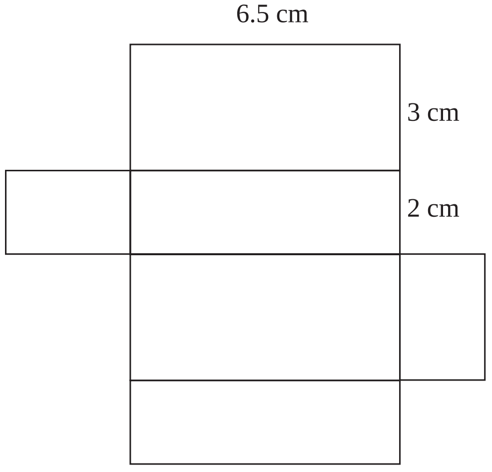
(a) 80 cm^3,

(b) 96 cm^3,

(c) 20 cm^3?



10. The shape in the diagram can be folded to form a cuboid. Calculate the volume of the cuboid.



22.4 Capacity

When we refer to how much liquid a tank or container can hold, we often talk about its *capacity* in litres. This is another way of describing its volume.

$$1000 \text{ cm}^3 = 1 \text{ litre}$$

Example 1

What is the capacity, in litres, of a tank with dimensions 1 m by 1 m by 1 m?

Solution

Working in centimetres,

$$\text{Volume} = 100 \times 100 \times 100$$

$$= 1000\,000 \text{ cm}^3$$

$$\text{Capacity (in litres)} = \frac{1000\,000}{1000}$$

$$= 1000 \text{ litres}$$

Note that the volume of this tank is also 1 m^3, so $1 \text{ m}^3 = 1000$ litres.

$$1 \text{ m}^3 = 1000 \text{ litres}$$

Example 2

A tank measures 3 m by 4 m by 2 m. What is the capacity of the tank in litres?

Solution

$$\text{Volume} = 3 \times 4 \times 2$$

$$= 24 \text{ m}^3$$

But $1 \text{ m}^3 = 1000$ litres

so capacity $= 24 \times 1000$

$= 24\,000$ litres

Example 3

Calculate the volume of a bottle of capacity $700\ \text{cm}^3$.

Solution

$$\text{Volume} = \frac{700}{1000}$$

$$= 0.7 \text{ litres}$$

Exercises

1. Convert these volumes into cm^3:

   (a) 2 litres (b) 5 litres (c) $\frac{1}{2}$ litre

   (d) 0.2 litres (e) 1.5 litres (f) 2.7 litres

2. Convert these volumes into litres:

   (a) $3000\ \text{cm}^3$ (b) $7000\ \text{cm}^3$ (c) $10\,000\ \text{cm}^3$

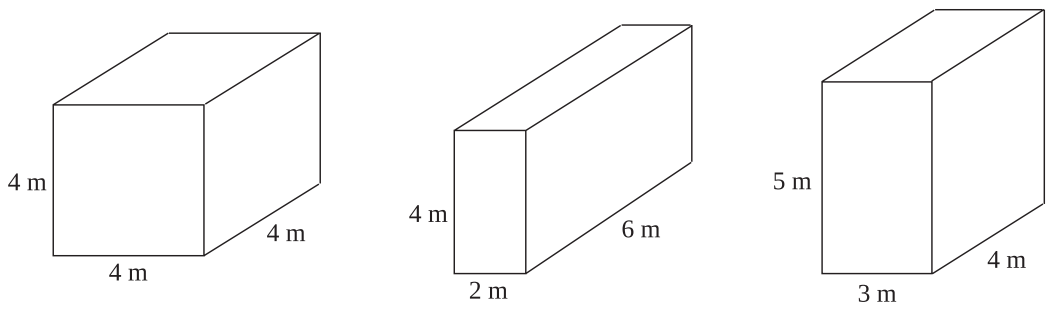
   (d) $750\ \text{cm}^3$ (e) $250\ \text{cm}^3$ (f) $4900\ \text{cm}^3$

3. A tank has dimensions 3 m by 3 m by 2 m.
   Calculate the capacity of the tank in:

   (a) m^3 (b) litres.

4. A tank holds 5000 litres. Calculate the volume of the tank in m^3.

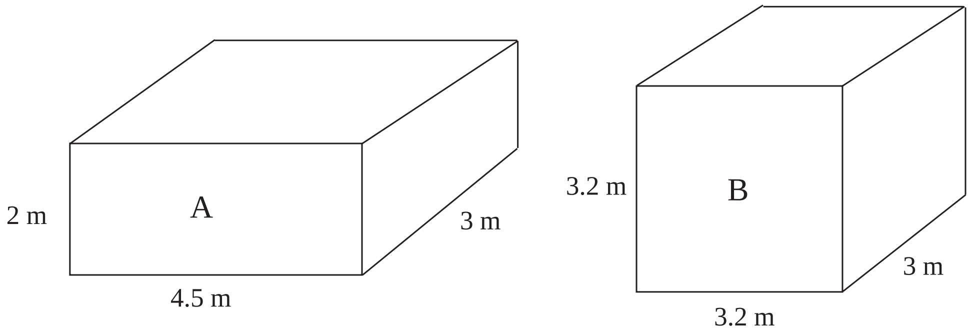
5. Work out which of these tanks has the greatest capacity:



A B C

6. A large bottle holds 2 litres of lemonade. The lemonade is poured out into glasses that each hold 25 cm^3. How many glasses can be filled?

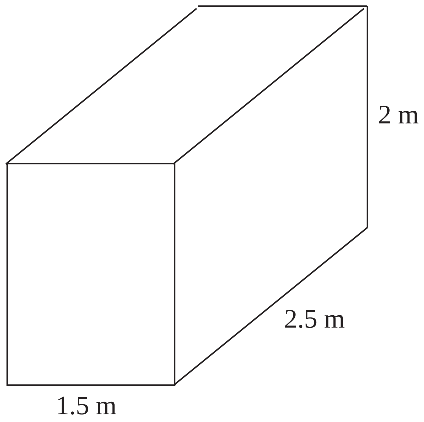
7. The diagram shows 2 different tanks:



(a) Which tank has the greater capacity?

(b) How many more litres does the larger tank hold than the smaller one?

8. The base of a tank is 1.5 m by 2.5 m and its height is 2 m. It is part full of water.



(a) What is the volume of water in the tank, in litres, if the water is 1.2 m deep?

(b) How many litres of water does the tank contain when it is $\frac{1}{4}$ full?

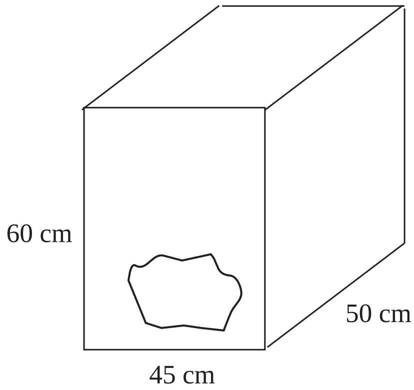
(c) How deep is the water when the tank contains 3000 litres?

9. A tank contains 12.5 litres of liquid. Cans of capacity 800 cm^3 are filled from the tank.

(a) How many cans can be filled from the tank?

(b) How much liquid is left over?

10. Ben puts a rock in the tank shown in the diagram and then fills the tank to the top with water. Then he takes the rock out and the water level drops by 5 cm.



(a) What is the capacity of the tank when it is full?

(b) What is the volume of water in the tank when the rock has been taken out?

(c) Calculate the volume of the rock, in cm^3.

(d) How many litres of water would be needed to fill the tank to the top again?

22.5 Density

If you were to fill boxes of the same capacity with different materials you would find some easier to lift than others. For example, a box of sand would be much heavier than a box of polystyrene beads. We say that sand is more *dense* than polystyrene. Density, mass and volume are connected by the relationships:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{Mass} = \text{density} \times \text{volume}$$

$$\text{Volume} = \frac{\text{mass}}{\text{density}}$$

Mercury (the only metal which is liquid at room temperature) has a density of 13 600 kg/m^3; air has density 1.4 kg/m^3 and water 1000 kg/m^3 or 1 gram/cm^3.

Example 1

Calculate the mass of 3 litres of water.

Solution

3 litres $= 3000 \ \text{cm}^3$

Mass $= \text{density} \times \text{volume}$

$= 1 \times 3000$

$= 3000$ grams

$= 3$ kg

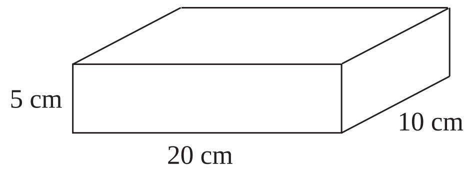
Example 2

This metal block has mass 2 kg.

Calculate the density of the metal in:

(a) grams/cm^3,

(b) kg/cm^3.



Solution

First find the volume of the block:

$$\text{Volume} = 5 \times 20 \times 10$$
$$= 1000 \ \text{cm}^3$$

(a) Note that $2 \text{ kg} = 2000$ grams.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{2000}{1000}$$
$$= 2 \text{ grams/cm}^3$$

(b)
$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{2}{1000}$$
$$= 0.002 \text{ kg/cm}^3$$

Example 3

A type of wood has density 0.7 grams/cm^3. A piece of this wood is 3 cm by 10 cm by 180 cm.

What is the mass of this piece of wood, in:

(a) grams (b) kg?

Solution

(a) First calculate the volume of the wood:

$$\text{Volume} = 3 \times 10 \times 180$$
$$= 5400 \text{ cm}^3$$

$$\text{Mass} = \text{density} \times \text{volume}$$
$$= 0.7 \times 5400$$
$$= 3780 \text{ grams}$$

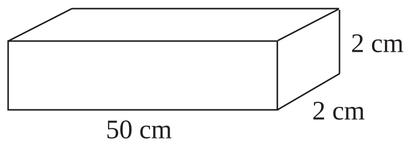
(b) 3780 grams = 3.78 kg

Exercises

1. Calculate the mass of the following volumes of water:

   (a) 100 cm^3 (b) 2 litres (c) 0.5 litres.

2. The mass of the metal block below is 3 kg.



   What is the density of the block, in:

   (a) kg/cm^3,

   (b) grams/cm^3?

3. A polystyrene block has dimensions 1 m by 2 m by 3 m. The mass of the block is 24 kg.

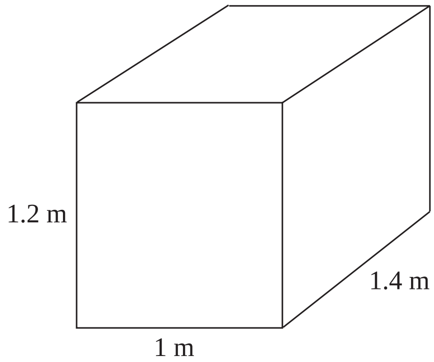
   (a) Calculate the density of the polystyrene in grams/cm^3.

   (b) A smaller block of polystyrene has dimensions 50 cm by 20 cm by 30 cm. What is its mass?

4. The diagram shows a tank that is to be filled with water. Calculate the mass of water, in kg, if the tank is to be:

   (a) full,

   (b) $\frac{1}{2}$ full.



5. A rectangular block of metal, 5 cm by 8 cm by 10 cm, has a mass of 500 grams. Calculate the density of the metal in

   (a) g/cm^3,

   (b) kg/cm^3.

6. A book has dimensions 1 cm by 24 cm by 30 cm. Its mass is 576 grams.

   (a) Calculate the density of the book.

   (b) What is the mass of a book with dimensions 1.5 cm by 20 cm by 15 cm?

7. The density of concrete is 4 $grams/cm^3$.

   (a) Calculate the mass of a concrete block with dimensions 10 cm by 45 cm by 22 cm.

   (b) Calculate the volume of a concrete block with a mass of 5 kg.

8. A box with dimensions 6 cm by 5 cm by 2 cm is full of soil. The mass of the soil in the box is 72 grams.

   (a) Calculate the density of the soil.

   (b) Calculate the mass of soil, in kg, needed to fill a window box which has dimensions 70 cm by 20 cm by 25 cm.

9. The density of sea water is *not* the same as the density of pure water.

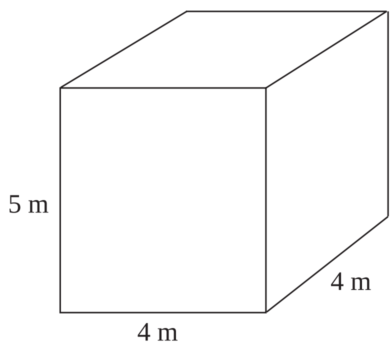
   When this tank is filled with sea water the mass of the water is 82 400 kg.

   (a) If the tank was filled with pure water, what would be the mass of the water?

   (b) Does pure water or sea water have the higher density?

   (c) What is the density of sea water in $grams/cm^3$?

   (d) What is the mass of 1 litre of sea water?



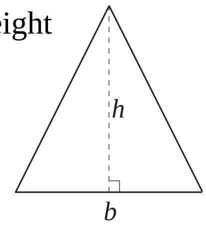
10. One type of metal has a density of 4 grams/ cm^3. Another type has a density of 5 grams/ cm^3. Weights are made from both types of metal.
Calculate the difference in volume of 500 gram weights made from the two types of metal.

22.6 Volume of a Triangular Prism

We now look at how to find the volume of a triangular prism. You will need to remember how to find the area of a triangle in order to do this:

$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

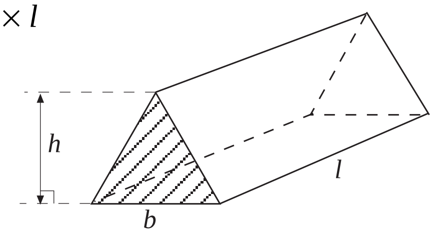
$$= \frac{1}{2} \times b \times h$$



A triangular prism has the same triangular cross-section throughout its length.

$$\text{Volume of triangular prism} = \text{area of cross-section} \times \text{length}$$

$$= \frac{1}{2} \times b \times h \times l$$

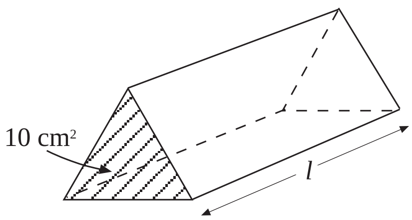


Example 1

The diagram opposite shows a triangular prism.
The area of the end of the prism is 10 cm^3.

Calculate the volume of the prism, if:

(a) $l = 5$ cm, (b) $l = 2$ m.



Solution

(a) Volume = area of cross-section × length

$$= 10 \times 5$$

$$= 50 \text{ cm}^3$$

(b) 2 m $= 200$ cm

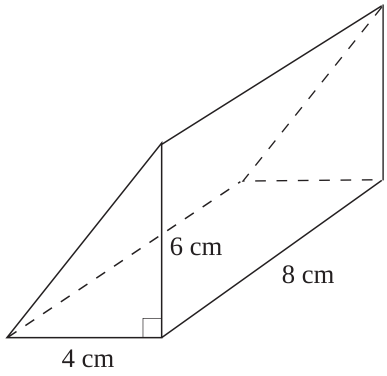
Volume $=$ area of cross-section $\times$ length

$= 10 \times 200$

$= 2000 \text{ cm}^3$

Example 2

Calculate the volume of this triangular prism:



Solution

Area of cross-section $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 4 \times 6$

$= 12 \text{ cm}^2$

Volume of prism $=$ area of cross-section $\times$ length

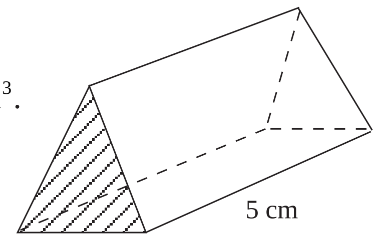
$= 12 \times 8$

$= 96 \text{ cm}^3$

Example 3

The triangular prism opposite has a volume of 82 cm^3.

Calculate the area of the shaded part of the prism.



Solution

Volume $=$ area of cross-section $\times$ length

$82 =$ shaded area $\times 5$

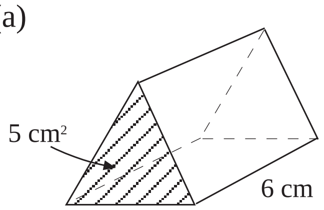
Shaded area $= \frac{82}{5}$

$= 16.4 \text{ cm}^2$

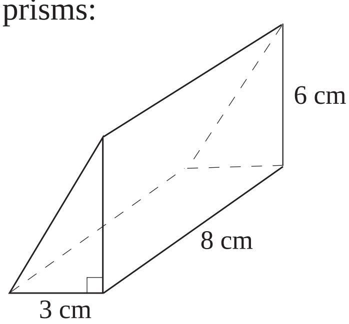
Exercises

1. Calculate the volume of each of these triangular prisms:

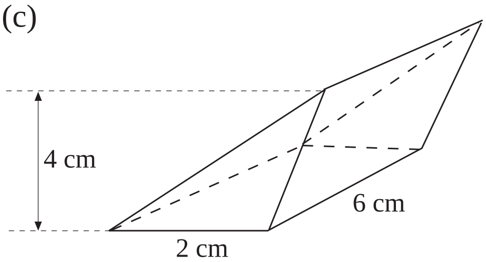
(a)



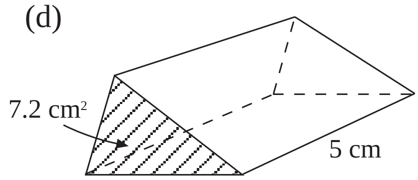
(b)



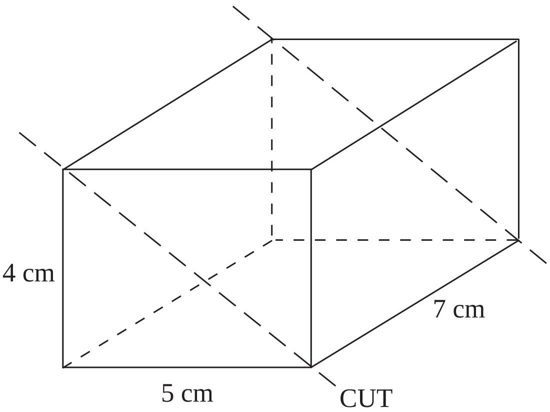
(c)



(d)



2. Two triangular prisms are formed by cutting the rectangular block below, as shown.

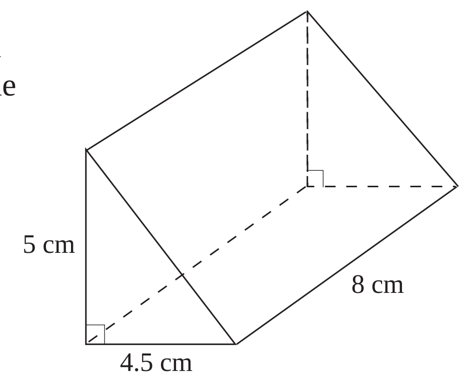


Calculate the volume of each of the triangular prisms formed.

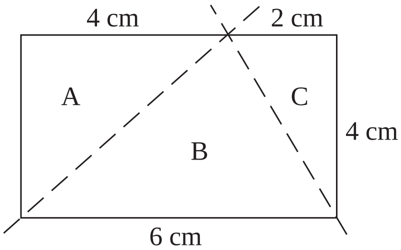
3. Two identical triangular prisms are stuck together to form a rectangular block. One prism is shown opposite.

(a) What is the volume of this prism?

(b) What is the volume of the rectangular block formed when the two prisms are stuck together?



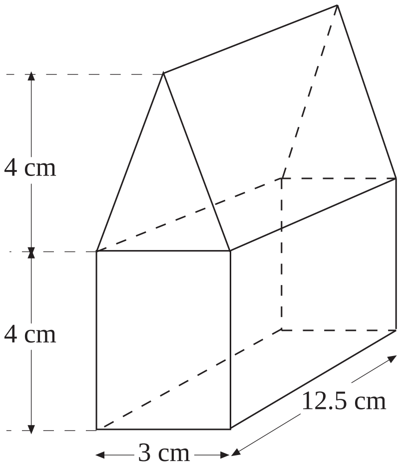
4. A rectangular block has dimension 4 cm by 6 cm by 10 cm. The block is cut to form 3 triangular prisms. The diagram shows how the end of the block is cut.



Calculate the volume of each of the triangular prisms.

5. A model house is made by sticking a triangular prism on top of a rectangular block, as shown in the diagram.

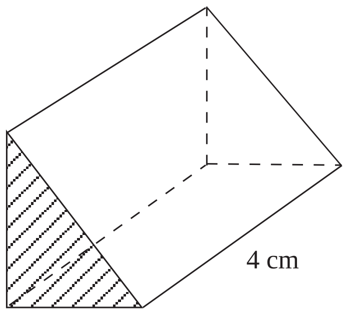
Calculate the volume of the model house.



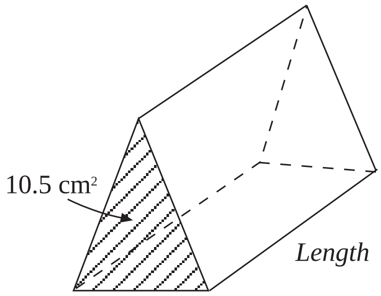
6. The triangular prism opposite has a volume of 84 cm^3.

(a) What is the area of the shaded end of the prism?

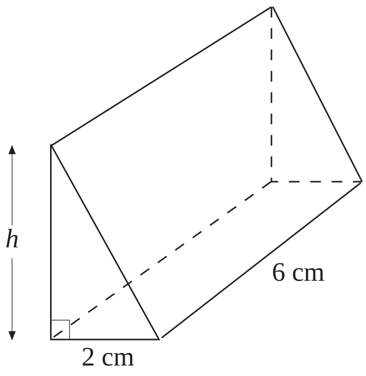
(b) If the length was increased to 6 cm, what would now be the volume of the prism?



7. The volume of this triangular prism is 63 cm^3. Find the length of the prism if the area of the shaded end is 10.5 cm^2.



8. The volume of this triangular prism is 42 cm^3.
What is the height of the prism?

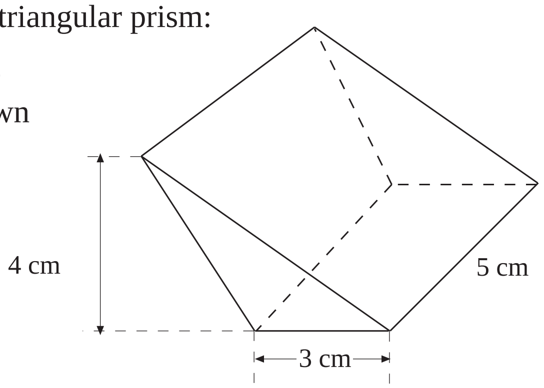


9. (a) Calculate the volume of this triangular prism:

(b) A similar prism has the same shape but all the lengths shown have been *doubled*.

What is the volume of this prism?

(c) How many times larger is your answer to (b) than your answer to (a)?

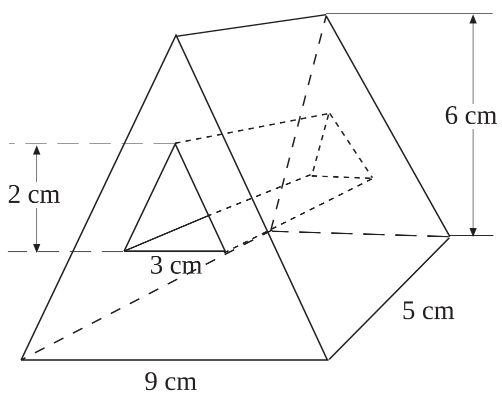


(d) Repeat (a), (b) and (c) for any other prism. Do you get the same answer to (c)?

(e) What happens if you *treble* the lengths of the original prism?

(f) What do you think would happen for other enlargements?

10. A triangular prism has a triangular hole cut in it.



Calculate the volume of the prism *after* the hole has been cut out.