3 Indices and Standard Form

3.1 Index Notation

Here we revise the use of index notation. You will already be familiar with the notation for squares and cubes

$$\boxed{\begin{aligned} a^2 &= a \times a \\ a^3 &= a \times a \times a \end{aligned}}$$, and

this is generalised by defining:

$$\boxed{a^n = \underbrace{a \times a \times \ldots \times a}_{n \text{ of these}}}$$

Example 1

Calculate the value of:

(a) 5^2 (b) 2^5 (c) 3^3 (d) 10^4

Solution

(a) $5^2 = 5 \times 5$
$= 25$

(b) $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

(c) $3^3 = 3 \times 3 \times 3$
$= 27$

(d) $10^4 = 10 \times 10 \times 10 \times 10$
$= 10\,000$

Example 2

Copy each of the following statements and fill in the missing number or numbers:

(a) $2^{\square} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(b) $9 = 3^{\square}$

(c) $1000 = 10^{\square}$

(d) $5^3 = \square \times \square \times \square$

Solution

(a) $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(b) $9 = 3 \times 3 = 3^2$

(c) $1000 = 10 \times 10 \times 10 = 10^3$

(d) $5^3 = 5 \times 5 \times 5$

Example 3

(a) Determine 2^5.

(b) Determine 2^3.

(c) Determine $2^5 \div 2^3$.

(d) Express your answer to (c) in index notation.

Solution

(a) $2^5 = 32$

(b) $2^3 = 8$

(c) $2^5 \div 2^3 = 32 \div 8$
$= 4$

(d) $4 = 2^2$

Exercises

1. Calculate:

(a)	2^3	(b)	10^2	(c)	3^2
(d)	10^3	(e)	9^2	(f)	3^3
(g)	2^4	(h)	3^4	(i)	7^2

2. Copy each of the following statements and fill in the missing numbers:

(a) $10 \times 10 \times 10 \times 10 \times 10 = 10^{\square}$

(b) $3 \times 3 \times 3 \times 3 = 3^{\square}$

(c) $7 \times 7 \times 7 \times 7 \times 7 = 7^{\square}$

(d) $8 \times 8 \times 8 \times 8 \times 8 = 8^{\square}$

(e) $5 \times 5 = 5^{\square}$

(f) $19 \times 19 \times 19 \times 19 = 19^{\square}$

(g) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{\square}$

(h) $11 \times 11 \times 11 \times 11 \times 11 \times 11 = 11^{\square}$

3. Copy each of the following statements and fill in the missing numbers:

(a) $8 = 2^{\square}$ (b) $81 = 3^{\square}$

(c) $100 = 10^{\square}$ (d) $81 = 9^{\square}$

(e) $125 = 5^{\square}$ (f) $1\,000\,000 = 10^{\square}$

(g) $216 = 6^{\square}$ (h) $625 = 5^{\square}$

4. Is 10^2 bigger than 2^{10} ?

5. Is 3^4 bigger than 4^3 ?

6. Is 5^2 bigger than 2^5 ?

7. Copy each of the following statements and fill in the missing numbers:

(a) $49 = \square^{2}$ (b) $64 = \square^{3}$

(c) $64 = \square^{6}$ (d) $64 = \square^{2}$

(e) $100\,000 = \square^{5}$ (f) $243 = \square^{5}$

8. Calculate:

(a) $2^2 + 2^3$ (b) $2^2 \times 2^3$

(c) $3^2 + 2^2$ (d) $3^2 \times 2^2$

(e) $2^3 \times 10^3$ (f) $10^3 + 2^5$

9. Calculate:

(a) $(3+2)^4$ (b) $(3-2)^4$

(c) $(7-4)^3$ (d) $(7+4)^3$

10. Writing your answers in index form, calculate:

(a) $10^2 \times 10^3$ (b) $2^3 \times 2^7$

(c) $3^4 \div 3^2$ (d) $2^5 \div 2^2$

(e) $10^6 \div 10^2$ (f) $5^4 \div 5^2$

11. (a) Without using a calculator, write down the values of k and m.

$$64 = 8^2 = 4^k = 2^m$$

(b) Complete the following:

$$2^{15} = 32\,768$$

$$2^{14} = \square$$

(KS3/99Ma/Tier 5-7/P1)

3.2 Laws of Indices

There are three rules that should be used when working with indices:

When m and n are positive integers,

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$ or $\dfrac{a^m}{a^n} = a^{m-n} \quad (m \geq n)$
3. $(a^m)^n = a^{m \times n}$

These three results are logical consequences of the definition of a^n, but really need a formal proof. You can 'verify' them with particular examples as below, but this is not a proof:

$$\begin{aligned} 2^7 \times 2^3 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^{10} \qquad \text{(here } m = 7,\ n = 3 \text{ and } m + n = 10\text{)} \end{aligned}$$

or,

$$2^7 \div 2^3 = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2}$$

$$= 2\times2\times2\times2$$

$$= 2^4 \qquad \text{(again } m=7,\ n=3 \text{ and } m-n=4\text{)}$$

Also, $\left(2^7\right)^3 = 2^7\times2^7\times2^7$

$= 2^{21}$ (using rule 1) (again $m=7,\ n=3$ and $m\times n=21$)

The proof of the first rule is given below:

Proof

$$a^m\times a^n = \underbrace{a\times a\times\ldots\times a}_{m \text{ of these}} \times \underbrace{a\times a\times\ldots\times a}_{n \text{ of these}}$$

$$= \underbrace{a\times a\times\ldots\times a\times a\times a\ldots\times a}_{(m+n) \text{ of these}}$$

$$= a^{m+n}$$

The second and third rules can be shown to be true for all positive integers m and n in a similar way.

We can see an important result using rule 2:

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

but $\frac{x^n}{x^n} = 1$, so

$$\boxed{x^0 = 1}$$

This is true for any non-zero value of x, so, for example, $3^0 = 1$, $27^0 = 1$ and $1001^0 = 1$.

Example 1

Fill in the missing numbers in each of the following expressions:

(a) $2^4 \times 2^6 = 2^{\square}$ (b) $3^7 \times 3^9 = 3^{\square}$

(c) $3^6 \div 3^2 = 3^{\square}$ (d) $\left(10^4\right)^3 = 10^{\square}$

Solution

(a) $2^4 \times 2^6 = 2^{4+6}$

$= 2^{10}$

(b) $3^7 \times 3^9 = 3^{7+9}$

$= 3^{16}$

(c) $3^6 \div 3^2 = 3^{6-2}$

$= 3^4$

(d) $\left(10^4\right)^3 = 10^{4\times 3}$

$= 10^{12}$

Example 2

Simplify each of the following expressions so that it is in the form a^n, where n is a number:

(a) $a^6 \times a^7$ (b) $\dfrac{a^4 \times a^2}{a^3}$ (c) $\left(a^4\right)^3$

Solution

(a) $a^6 \times a^7 = a^{6+7}$

$= a^{13}$

(b) $\dfrac{a^4 \times a^2}{a^3} = \dfrac{a^{4+2}}{a^3}$

$= \dfrac{a^6}{a^3}$

$= a^{6-3}$

$= a^3$

(c) $\left(a^4\right)^3 = a^{4\times 3}$

$= a^{12}$

Exercises

1. Copy each of the following statements and fill in the missing numbers:

(a) $2^3 \times 2^{7} = 2^{\square}$ (b) $3^6 \times 3^5 = 3^{\square}$

(c) $3^7 \div 3^4 = 3^{\square}$ (d) $8^3 \times 8^4 = 8^{\square}$

(e) $\left(3^2\right)^5 = 3^{\square}$ (f) $\left(2^3\right)^6 = 2^{\square}$

(g) $\dfrac{3^6}{3^2} = 3^{\square}$ (h) $\dfrac{4^7}{4^2} = 4^{\square}$

2. Copy each of the following statements and fill in the missing numbers:

(a) $a^3 \times a^2 = a^{\square}$ (b) $b^7 \div b^2 = b^{\square}$

(c) $\left(b^2\right)^5 = b^{\square}$ (d) $b^6 \times b^4 = b^{\square}$

(e) $\left(z^3\right)^9 = z^{\square}$ (f) $\dfrac{q^{16}}{q^7} = q^{\square}$

3. Explain why $9^4 = 3^8$.

4. Calculate:

(a) $3^0 + 4^0$ (b) $6^0 \times 7^0$

(c) $8^0 - 3^0$ (d) $6^0 + 2^0 - 4^0$

5. Copy each of the following statements and fill in the missing numbers:

(a) $3^6 \times 3^{\square} = 3^{17}$ (b) $4^6 \times 4^{\square} = 4^{11}$

(c) $\dfrac{a^6}{a^{\square}} = a^4$ (d) $\left(z^{\square}\right)^6 = z^{18}$

(e) $\left(a^{19}\right)^{\square} = a^{95}$ (f) $p^{16} \div p^{\square} = p^7$

(g) $\left(p^{\square}\right)^8 = p^{40}$ (h) $q^{13} \div q^{\square} = q$

6. Calculate:

(a) $\frac{2^3}{2^2} + 3^0$ (b) $\frac{3^4}{3^3} - 3^0$

(c) $\frac{5^4}{5^2} + \frac{6^2}{6}$ (d) $\frac{7^7}{7^5} - \frac{5^9}{5^7}$

(e) $\frac{10^8}{10^5} - \frac{5^6}{5^3}$ (f) $\frac{4^{17}}{4^{14}} - \frac{4^{13}}{4^{11}}$

7. Fill in the missing numbers in each of the following expressions:

(a) $8^2 = 2^{\square}$ (b) $81^3 = 9^{\square} = 3^{\square}$

(c) $25^6 = 5^{\square}$ (d) $4^7 = 2^{\square}$

(e) $125^4 = 5^{\square}$ (f) $1000^6 = 10^{\square}$

(g) $81 = \square^4$ (h) $256 = \square^4 = \square^8$

8. Fill in the missing numbers in each of the following expressions:

(a) $8 \times 4 = 2^{\square} \times 2^{\square}$
$= 2^{\square}$

(b) $25 \times 625 = 5^{\square} \times 5^{\square}$
$= 5^{\square}$

(c) $\frac{243}{9} = \frac{3^{\square}}{3^{\square}}$
$= 3^{\square}$

(d) $\frac{128}{16} = \frac{2^{\square}}{2^{\square}}$
$= 2^{\square}$

9. Is each of the following statements true or false?

(a) $3^2 \times 2^2 = 6^4$ (b) $5^4 \times 2^3 = 10^7$

(c) $\frac{6^8}{2^8} = 3^8$ (d) $\frac{10^8}{5^6} = 2^2$

10. Copy and complete each expression:

(a) $\left(2^{6}\times 2^{3}\right)^{4} = \left(2^{\square}\right)^{4} = 2^{\square}$ (b) $\left(\frac{3^{6}}{3^{2}}\right)^{5} = \left(3^{\square}\right)^{5} = 3^{\square}$

(c) $\left(\frac{2^{3}\times 2^{4}}{2^{7}}\right)^{4} = \left(2^{\square}\right)^{4} = 2^{\square}$ (d) $\left(\frac{3^{2}\times 9}{3^{3}}\right)^{4} = \left(3^{\square}\right)^{4} = 3^{\square}$

(e) $\left(\frac{6^{2}\times 6^{8}}{6^{3}}\right)^{4} = \left(6^{\square}\right)^{4} = 6^{\square}$ (f) $\left(\frac{7^{8}}{7^{2}\times 7^{3}}\right)^{5} = \left(7^{\square}\right)^{5} = 7^{\square}$

3.3 Negative Indices

Using negative indices produces fractions. In this section we practice working with negative indices. From our work in the last section, we see that

$$a^{2}\div a^{3} \quad = \quad a^{2-3} \quad = \quad a^{-1}$$

but we know that

$$a^{2}\div a^{3} \quad = \quad \frac{a\times a}{a\times a\times a} \quad = \quad \frac{1}{a}, \text{ a fraction.}$$

So clearly,

$$\boxed{a^{-1} \quad = \quad \frac{1}{a}}$$

In same way,

$$\boxed{\begin{aligned} a^{-2} &= \frac{1}{a^{2}} \\ &= \frac{1}{a\times a} \\ \\ a^{-3} &= \frac{1}{a^{3}} \\ &= \frac{1}{a\times a\times a} \end{aligned}}$$

and, in general,

$$a^{-n} = \frac{1}{a^n}$$

for positive integer values of n. The three rules at the start of section 3.2 can now be used for any integers m and n, not just for positive values.

Example 1

Calculate, leaving your answers as fractions:

(a) 3^{-2} (b) $2^{-1} - 4^{-1}$ (c) 5^{-3}

Solution

(a) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

(b) $2^{-1} - 4^{-1} = \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

(c) $5^{-3} = \frac{1}{5^3}$

$= \frac{1}{125}$

Example 2

Simplify:

(a) $\frac{6^7}{6^9}$ (b) $6^4 \times 6^{-3}$ (c) $\left(10^2\right)^{-3}$

Solution

(a) $\frac{6^7}{6^9} = 6^{7-9}$

$= 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

(b) $6^4 \times 6^{-3} = 6^{4+(-3)}$

$= 6^{4-3} = 6^1 = 6$

(c) $\left(10^{2}\right)^{-3} = 10^{-6}$

$= \dfrac{1}{10^{6}}$

$= \dfrac{1}{1000\,000}$

Exercises

1. Write the following numbers as fractions *without using any indices*:

(a) 4^{-1} (b) 2^{-3} (c) 10^{-3}

(d) 7^{-2} (e) 4^{-3} (f) 6^{-2}

2. Copy the following expressions and fill in the missing numbers:

(a) $\dfrac{1}{49} = \dfrac{1}{7^{\square}} = 7^{\square}$ (b) $\dfrac{1}{100} = \dfrac{1}{10^{\square}} = 10^{\square}$

(c) $\dfrac{1}{81} = \dfrac{1}{9^{\square}} = 9^{\square}$ (d) $\dfrac{1}{16} = \dfrac{1}{2^{\square}} = 2^{\square}$

(e) $\dfrac{1}{10\,000\,000} = \dfrac{1}{10^{\square}} = 10^{\square}$ (f) $\dfrac{1}{1024} = \dfrac{1}{2^{\square}} = 2^{\square}$

3. Calculate:

(a) $4^{-1} + 3^{-1}$ (b) $6^{-1} + 2^{-1}$

(c) $5^{-1} - 10^{-1}$ (d) $10^{-2} - 10^{-3}$

(e) $4^{-1} - 10^{-1}$ (f) $6^{-1} + 7^{-1}$

4. Simplify the following expressions giving your answers in the form of a number to a power:

(a) $4^{7} \times 4^{-6}$ (b) $5^{7} \times 5^{-3}$

(c) $\dfrac{7^{4}}{7^{-6}}$ (d) $\left(3^{2}\right)^{-4}$

(e) $\left(6^{-2}\right)^{-3}$ (f) $8^{4} \times 8^{-9}$

(g) $\dfrac{7^{2}}{7^{-2}}$ (h) $\dfrac{8^{9}}{8^{-9}}$

5. Copy each of the following expressions and fill in the missing numbers;

(a) $\frac{1}{9} = 3^{\square}$ (b) $\frac{1}{100} = 10^{\square}$

(c) $\frac{1}{125} = 5^{\square}$ (d) $\frac{5}{5^4} = 5^{\square}$

(e) $\frac{6^2}{6^3} = 6^{\square}$ (f) $\frac{2^2}{2^{10}} = 2^{\square}$

6. Simplify the following expressions:

(a) $\frac{x^8}{x^3}$ (b) $\frac{x^7}{x^9}$

(c) $\frac{x^4}{x^8}$ (d) $\left(x^6\right)^{-4}$

(e) $\left(\frac{1}{x^2}\right)^4$ (f) $\left(x^{-8}\right)^3$

7. Copy and complete the following statements:

(a) $0.1 = 10^{\square}$ (b) $0.25 = 2^{\square}$

(c) $0.0001 = 10^{\square}$ (d) $0.2 = 5^{\square}$

(e) $0.001 = 10^{\square}$ (f) $0.02 = 50^{\square}$

8. Copy the following expressions and fill in the missing numbers:

(a) $\frac{x^4}{x^{\square}} = x^2$ (b) $x^6 \times x^{\square} = x^2$

(c) $x^9 \times x^{\square} = x^2$ (d) $\frac{x^7}{x^{\square}} = x^{-2}$

(e) $\frac{x^3}{x^{\square}} = x^4$ (f) $\left(x^3\right)^{\square} = x^{-6}$

9. Copy the following expressions and fill in the missing numbers:

(a) $\frac{1}{8} = 2^{\square}$ (b) $\frac{1}{25} = 5^{\square}$

(c) $\frac{1}{81} = 9^{\square}$ (d) $\frac{1}{10\,000} = 10^{\square}$

10. If $a = b^3$ and $b = \frac{1}{c^2}$, express a as a power of c, without having any fractions in your final answer.

3.4 Standard Form

Standard form is a convenient way of writing very large or very small numbers. It is used on a scientific calculator when a number is too large or too small to be displayed on the screen.

Before using standard form, we revise multiplying and dividing by powers of 10.

Example 1

Calculate:

(a) 3×10^4 (b) 3.27×10^3

(c) $3 \div 10^2$ (d) $4.32 \div 10^4$

Solution

(a) $3 \times 10^4 = 3 \times 10\,000$

$= 30\,000$

(b) $3.27 \times 10^3 = 3.27 \times 1000$

$= 3270$

(c) $3 \div 10^2 = \frac{3}{100}$

$= 0.03$

(d) $4.32 \div 10^4 = \dfrac{4.32}{10\,000}$

$= \dfrac{432}{1\,000\,000}$

$= 0.000432$

These examples lead to the approach used for standard form, which is a reversal of the approach used in Example 1.

> In *standard form*, numbers are written as
>
> $$a \times 10^n$$
>
> where $1 \le a < 10$ and n is an integer.

Example 2

Write the following numbers in standard form:

(a) 5720 (b) 7.4

(c) 473 000 (d) 6 000 000

(e) 0.09 (f) 0.000621

Solution

(a) $5720 = 5.72 \times 1000$

$= 5.72 \times 10^3$

(b) $7.4 = 7.4 \times 1$

$= 7.4 \times 10^0$

(c) $473\,000 = 4.73 \times 100\,000$

$= 4.73 \times 10^5$

(d) $6\,000\,000 = 6 \times 1\,000\,000$

$= 6 \times 10^6$

(e) $0.09 = \dfrac{9}{100}$

$= 9 \div 10^2$

$= 9 \times 10^{-2}$

(f) $0.000621 = \dfrac{6.21}{10\,000}$

$= \dfrac{6.21}{10^4}$

$= 6.21 \times 10^{-4}$

Example 3

Calculate:

(a) $\left(3 \times 10^6\right) \times \left(4 \times 10^3\right)$

(b) $\left(6 \times 10^7\right) \div \left(5 \times 10^{-2}\right)$

(c) $\left(3 \times 10^4\right) + \left(2 \times 10^5\right)$

Solution

(a) $\left(3 \times 10^6\right) \times \left(4 \times 10^3\right) = (3 \times 4) \times \left(10^6 \times 10^3\right)$

$= 12 \times 10^9$

$= 1.2 \times 10^1 \times 10^9$

$= 1.2 \times 10^{10}$

(b) $\left(6 \times 10^7\right) \div \left(5 \times 10^{-2}\right) = (6 \div 5) \times \left(10^7 \div 10^{-2}\right)$

$= 1.2 \times 10^9$

(c) $\left(3 \times 10^4\right) + \left(2 \times 10^5\right) = 30\,000 + 200\,000$

$= 230\,000$

$= 2.3 \times 10^5$

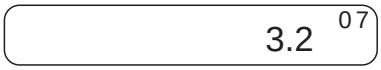
Note on Using Calculators

Your calculator will have a key EE or EXP for entering numbers in standard form.

For example, for 3.2×10^7, press

3 · 2 EXP 7

which will appear on your display like this:



Some calculators also display the $' \times 10 '$ part of the number, but not all do. You need to find out what your calculator displays. Remember, you must always write the $' \times 10 '$ part when you are asked to give an answer in standard form.

Exercises

1. Calculate:

(a) 6.21×1000 (b) 8×10^3 (c) 4.2×10^2

(d) $3 \div 1000$ (e) $6 \div 10^2$ (f) $3.2 \div 10^3$

(g) 6×10^{-3} (h) 9.2×10^{-1} (i) 3.6×10^{-2}

2. Write each of the following numbers in standard form:

(a) 200 (b) 8000

(c) 9 000 000 (d) 62 000

(e) 840 000 (f) 12 000 000 000

(g) 61 800 000 000 (h) 3 240 000

3. Convert each of the following numbers from standard form to the normal decimal notation:

(a) 3×10^4 (b) 3.6×10^4 (c) 8.2×10^3

(d) 3.1×10^2 (e) 1.6×10^4 (f) 1.72×10^5

(g) 6.83×10^4 (h) 1.25×10^6 (i) 9.17×10^3

4. Write each of the following numbers in standard form:

(a) 0.0004 (b) 0.008

(c) 0.142 (d) 0.0032

(e) 0.00199 (f) 0.000000062

(g) 0.0000097 (h) 0.00000000000021

5. Convert the following numbers from standard form to the normal decimal format:

(a) 6×10^{-2} (b) 7×10^{-1} (c) 1.8×10^{-3}

(d) 4×10^{-3} (e) 6.2×10^{-3} (f) 9.81×10^{-4}

(g) 6.67×10^{-1} (h) 3.86×10^{-5} (i) 9.27×10^{-7}

6. Without using a calculator, determine:

(a) $\left(4 \times 10^{4}\right) \times \left(2 \times 10^{5}\right)$ (b) $\left(2 \times 10^{6}\right) \times \left(3 \times 10^{5}\right)$

(c) $\left(6 \times 10^{4}\right) \times \left(8 \times 10^{-9}\right)$ (d) $\left(3 \times 10^{-8}\right) \times \left(7 \times 10^{-4}\right)$

(e) $\left(6.1 \times 10^{6}\right) \times \left(2 \times 10^{-5}\right)$ (f) $\left(3.2 \times 10^{-5}\right) \times \left(4 \times 10^{-9}\right)$

7. Without using a calculator, determine:

(a) $\left(9 \times 10^{7}\right) \div \left(3 \times 10^{4}\right)$ (b) $\left(8 \times 10^{5}\right) \div \left(2 \times 10^{-2}\right)$

(c) $\left(6 \times 10^{-2}\right) \div \left(2 \times 10^{-3}\right)$ (d) $\left(6 \times 10^{4}\right) \div \left(3 \times 10^{-6}\right)$

(e) $\left(4.8 \times 10^{12}\right) \div \left(1.2 \times 10^{3}\right)$ (f) $\left(3.6 \times 10^{8}\right) \div \left(9 \times 10^{3}\right)$

8. Without a calculator, determine the following, giving your answers in both normal and standard form::

(a) $\left(6 \times 10^{5}\right) + \left(3 \times 10^{6}\right)$ (b) $\left(6 \times 10^{2}\right) + \left(9 \times 10^{3}\right)$

(c) $6 \times 10^{5} - 1 \times 10^{4}$ (d) $8 \times 10^{-2} + 9 \times 10^{-3}$

(e) $6 \times 10^{-4} + 8 \times 10^{-3}$ (f) $6 \times 10^{-4} - 3 \times 10^{-5}$

9. Use a calculator to determine:

(a) $\left(3.4 \times 10^{6}\right) \times \left(2.1 \times 10^{4}\right)$ (b) $\left(6 \times 10^{21}\right) \times \left(8.2 \times 10^{-11}\right)$

(c) $\left(3.6 \times 10^{5}\right) \times \left(4.5 \times 10^{7}\right)$ (d) $\left(8.2 \times 10^{11}\right) \div \left(4 \times 10^{-8}\right)$

(e) $\left(1.92 \times 10^{6}\right) \times \left(3.2 \times 10^{-11}\right)$ (f) $\left(6.2 \times 10^{14}\right)^{3}$

10. The radius of the earth is 6.4×10^6 m. Giving your answers in standard form, correct to 3 significant figures, calculate the circumference of the earth in:

(a) m (b) cm (c) mm (d) km

11. Sir Isaac Newton (1642-1727) was a mathematician, physicist and astronomer.
In his work on the gravitational force between two bodies he found that he needed to multiply their masses together.

(a) Work out the value of the mass of the Earth multiplied by the mass of the Moon. Give your answer in standard form.

Mass of Earth $= 5.98 \times 10^{24}$ kg
Mass of Moon $= 7.35 \times 10^{22}$ kg

Newton also found that he needed to work out the square of the distance between the two bodies.

(b) Work out the square of the distance between the Earth and the Moon. Give your answer in standard form.

Distance between Earth and Moon $= 3.89 \times 10^5$ km

Newton's formula to calculate the gravitational force (F) between two bodies is $F = \dfrac{G m_1 m_2}{R^2}$ where

G is the gravitational constant, m_1 and m_2 are the masses of the two bodies, and R is the distance between them.

(c) Work out the gravitational force (F) between the Sun and the Earth using the formula $F = \dfrac{G m_1 m_2}{R^2}$ with information in the box below.
Give your answer in standard form.

$m_1 m_2 = 1.19 \times 10^{55}$ kg^2
$R^2 = 2.25 \times 10^{16}$ km^2
$G = 6.67 \times 10^{-20}$

(KS3/95/Ma/Levels 6-8/P1)

12. (a) Which of these statements is true?

(i) 4×10^3 is a larger number than 4^3.

(ii) 4×10^3 is the same size as 4^3.

(iii) 4×10^3 is a smaller number than 4^3.

Explain your answer.

(b) One of the numbers below has the same value as 3.6×10^4. Write down the number.

36^3 36^4 $(3.6 \times 10)^4$ 0.36×10^3 0.36×10^5

(c) One of the numbers below has the same value as 2.5×10^{-3}. Write down the number.

25×10^{-4} 2.5×10^3 -2.5×10^3 0.00025 2500

(d) $(2 \times 10^2) \times (2 \times 10^2)$ can be written more simply as 4×10^4.

Write the following values as simply as possible:

(i) $(3 \times 10^2) \times (2 \times 10^{-2})$

(ii) $\dfrac{6 \times 10^8}{2 \times 10^4}$

(KS3/98/Ma/Tier 6-8/P1)

3.5 Fractional Indices

Indices that are fractions are used to represent square roots, cube roots and other roots of numbers.

$a^{\frac{1}{2}}$	=	$\sqrt{a}$	for example,	$9^{\frac{1}{2}}$	=	3
$a^{\frac{1}{3}}$	=	$\sqrt[3]{a}$	for example,	$8^{\frac{1}{3}}$	=	2
$a^{\frac{1}{4}}$	=	$\sqrt[4]{a}$	for example,	$625^{\frac{1}{4}}$	=	5
$a^{\frac{1}{n}}$	=	$\sqrt[n]{a}$				

Example 1

Calculate:

(a) $81^{\frac{1}{2}}$ (b) $1000^{\frac{1}{3}}$ (c) $4^{-\frac{1}{2}}$

Solution

(a) $81^{\frac{1}{2}} = \sqrt{81}$

$= 9$

(b) $1000^{\frac{1}{3}} = \sqrt[3]{1000}$

$= 10$

(c) $4^{-\frac{1}{2}} = \dfrac{1}{4^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{4}}$

$= \dfrac{1}{2}$

Exercises

1. Calculate:

(a) $49^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $16^{\frac{1}{2}}$

(d) $81^{-\frac{1}{2}}$ (e) $100^{-\frac{1}{2}}$ (f) $25^{-\frac{1}{2}}$

(g) $9^{\frac{1}{2}}$ (h) $36^{-\frac{1}{2}}$ (i) $144^{\frac{1}{2}}$

2. Calculate:

(a) $8^{\frac{1}{3}}$ (b) $8^{-\frac{1}{3}}$ (c) $125^{\frac{1}{3}}$

(d) $64^{-\frac{1}{3}}$ (e) $216^{\frac{1}{3}}$ (f) $1000\,000^{-\frac{1}{3}}$

3. Calculate:

(a) $32^{\frac{1}{5}}$ (b) $64^{-\frac{1}{2}}$ (c) $10\,000^{\frac{1}{4}}$

(d) $81^{-\frac{1}{4}}$ (e) $625^{\frac{1}{4}}$ (f) $100\,000^{-\frac{1}{5}}$

4. Calculate:

(a) $\left(\frac{4\times 8}{2}\right)^{\frac{1}{2}}$ (b) $\left(\frac{9\times 27}{3}\right)^{\frac{1}{4}}$

(c) $\left(\frac{125\times 5}{25}\right)^{\frac{1}{2}}$ (d) $\left(\frac{625}{5}\right)^{-\frac{1}{3}}$

5. Is each of the following statements *true* or *false*:

(a) $16^{\frac{1}{2}}=8$ (b) $16^{\frac{1}{4}}=2$

(c) $81^{\frac{1}{3}}=9$ (d) $\left(\frac{1}{100}\right)^{-\frac{1}{2}}=10$

6. Simplify:

(a) $\left(x^{9}\right)^{\frac{1}{3}}$ (b) $\left(a^{10}\right)^{-\frac{1}{2}}$

(c) $\frac{a}{a^{\frac{1}{2}}}$ (d) $\frac{a^{\frac{1}{2}}}{a}$

7. Simplify:

(a) $\frac{x^{\frac{3}{2}}}{x}$ (b) $\frac{x}{x^{\frac{3}{2}}}$

(c) $\frac{a^{\frac{1}{3}}}{a}$ (d) $\frac{a^{\frac{1}{3}}}{a^{\frac{1}{2}}}$

8. Calculate:

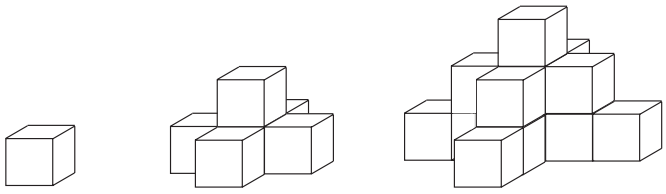
(a) $4^{-\frac{1}{2}}+4^{\frac{1}{2}}$ (b) $\left(9^{0}+9^{\frac{1}{2}}\right)^{\frac{1}{2}}$

(c) $\left(256^{\frac{1}{2}}\right)^{\frac{1}{2}}$ (d) $\left(9-9^{0}\right)^{\frac{1}{3}}$

UNIT 3 *Indices and Standard Form* Activities

Activities

ACTIVITY 3.1 *Towers*



How many cubes are needed to build a tower which has 100 steps?

At first sight, this might seem daunting but we will see ways of tackling this kind of problem by trying to find the formula which fits the data.

1. Complete the following table:

No. of Steps	1	2	3	4	5	6
No. of Cubes	1	6				

From your table you can see that the number of cubes needed increases much faster than the number of steps - but how much faster?

2. Compute the value of n^2 for $n = 1, 2, \ldots$, and also for $2n^2$ and $3n^2$ by completing the table below:

n	1	2	3	4	5	6
n^2	1	4	9	...	...	...
$2n^2$	2	8	...	...	...	...
$3n^2$	3	12	...	...	...	...

Which of the sequences given appears closest to the sequence found in question 1 ?

3. From questions 1 and 2, complete the table:

No. of Steps, n	1	2	3	4	5	6
No. of Cubes	1	6	...	...	...	...
$2n^2$	2	8				
No. of Cubes $- 2n^2$	-1	-2				

Deduce the formula for the number of cubes in the form

$$\text{number of cubes} = 2n^2 - kn$$

for some constant k.

4. Now solve the problem of finding the number of cubes needed for the 100-step tower.

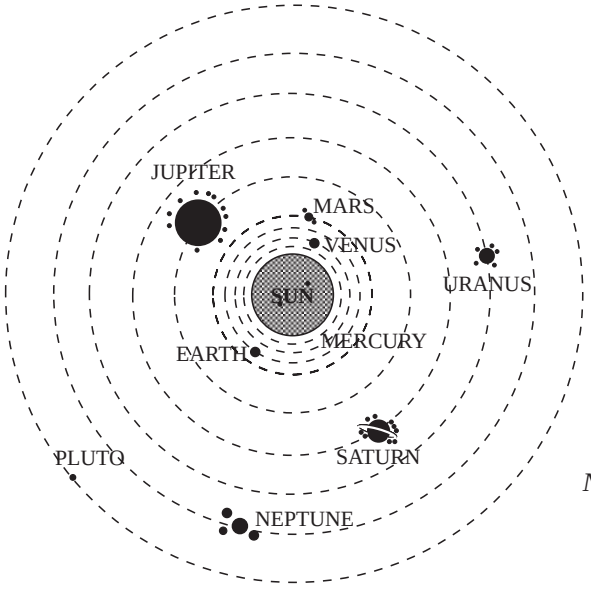
ACTIVITY 3.2 *Bode's Law*

In 1772, the German astronomer, *Johann Bode*, published his law which relates the distance ratio:

$$x_n = \frac{\text{distance of the planet from the Sun}}{\text{distance of the Earth from the Sun}}$$

to n, the number which Bode used to specify each planet, as shown in the following table.

$n = 1$	Venus
$n = 2$	Earth
$n = 3$	Mars
$n = 4$	
$n = 5$	Jupiter
$n = 6$	Saturn
$n = 7$	Uranus
$n = 8$	Neptune
$n = 9$	Pluto



Not to scale

He stated his law as:

$$x_n = 0.4 + 0.3 \times 2^{n-1}$$

1. Use this formula to determine $x_1, x_2, ..., x_9$.

2. Find the first and second differences of this sequence. What do you notice?

3. The actual distances are given in the table opposite. Find the actual values of

   $x_1 \left(= \frac{108.2}{149.6}\right)$, $x_2 \left(= \frac{149.6}{149.6}\right)$, $x_3 \left(= \frac{227.9}{149.6}\right)$, x_4, ... up to x_{10},

   and compare with predicted values from *Bode's Law*, ignoring the x_4 value.

Planet	*Distance from Sun* (in millions of km)
Mercury	57.9
Venus	108.2
Earth	149.6
Mars	227.9
Jupiter	778.3
Saturn	1427.0
Uranus	2870.0
Neptune	4497.0
Pluto	5907.0

4. A large number of asteroids are found at about 433.8×10^6 km from the Sun.
   Does *Bode's Law* provide confirmation that there was once a single planet at this distance from the sun? [*Hint*: consider x_4.]

5. Does the data support the view that *Neptune* and *Pluto* were once a single planet?

ACTIVITY 3.3 *Measuring and Standard Form*

We normally measure in units such as cm, m or km, as appropriate to a particular problem. Using different units can be a good opportunity to use standard form.

1. The distance between two towns is 108 km. Convert this distance to m, cm and mm, using standard form for your answers.

2. The area of a plot of land is 42 km^2. Convert this area to m^2, cm^2 and mm^2, using standard form for your answers.

3. The radius of a planet is 5000 km. Calculate the circumference of the planet in m, cm and mm.

4. Calculate the number of seconds in 1 year, giving your answer in standard form.

5. The volume of an adult human is estimated at 100 000 cm^3.
   Convert this volume to:

   (a) mm^3,

   (b) km^3,

   using standard form for your answers.

6. A spaceship travels at a speed of 2000 km/h. Convert this speed to mm/second.

ACTIVITY 3.4 *Standard Index Form*

The object of this game is to calculate numbers in standard index form to see which is the largest or smallest. It is suitable as a whole-class activity.

For each of the two numbers, a and b, given in standard form, determine which of

$a \times b$ $a \div b$ $b \div a$ $a + b$

is the largest number and which the smallest number.

For example:

A : $a = 4 \times 10^{3}$, $b = 2 \times 10^{-4}$

B : $a = 5 \times 10^{5}$, $b = 2 \times 10^{2}$

C : $a = 3 \times 10^{4}$, $b = 8 \times 10^{5}$

1. Working in pairs, one pupil chooses the values for a and b and sets the problem for their partner; the problem is then repeated the other way round.

2. Can you find any rules that determine which of the numbers is the largest?

ACTIVITIES 3.1 - 3.2 *Notes and Solutions*

Notes and solutions given only where appropriate.

3.1 1. 15, 28, 45, 66

2.

n	1	2	3	4	5	6
n^2	1	4	9	16	25	36
$2n^2$	2	8	18	32	50	72
$3n^2$	3	12	27	48	75	118

3.

No. of Steps, n	1	2	3	4	5	6
No. of Cubes	1	6	15	28	45	66
$2n^2$	2	8	18	32	50	72
No. of Cubes $- 2n^2$	-1	-2	-3	-4	-5	-6

4. $k = 1$

5. 19 900

3.2 1.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
0.7	1.0	1.6	2.8	5.2	10.0	19.6	38.8	77.2

2. 1st Difference 0.3 0.6 1.2 2.4 4.8 9.6 19.2 38.4

2nd Difference 0.3 0.6 1.2 2.4 4.8 9.6 19.2

1st and 2nd differences follow exactly the same pattern.

3. Values 0.72 1.0 1.52 2.89 5.20 9.54 19.18 30.06 39.48

Comparison good to x_8 inclusive.

4. x_4 in close agreement with Bode's Law, providing strong evidence that there was once a planet at this distance from the Sun.

5. Since Pluto (x_9) does not agree with Bode's Law, but is much closer to x_8, evidence exists that Pluto and Neptune were once a single planet.

ACTIVITIES 3.3 - 3.4 *Notes and Solutions*

3.3 1. 1.08×10^5 mm, 1.08×10^7 cm, 1.08×10^8 mm

2. 4.2×10^7 m^2, 4.2×10^{11} cm^2, 4.2×10^{13} mm^2

3. 3.14×10^7 m, 3.17×10^9 cm, 3.14×10^{10} mm

4. 3.154×10^7 seconds

5. (a) 1×10^8 mm^3 (b) 1×10^{-10} km^3

6. 5.56×10^5 mm/second

3.4

	Largest	*Smallest*
A :	$a \div b$	$b \div a$
B :	$a \times b$	$b \div a$
C :	$a \times b$	$a \div b$

UNIT 3 *Indices and Standard Form* Lesson Plans **St**

These are based on 45/50 minute lessons.

Lesson No.	*Suggested Plan*	*References*
1.	**Index Notation**	
	Introduction	OS 3.1
	Exercises - interactively	PB 3.1, Q1
	Exercises - interactively	PB 3.1, Q2
	Exercises	PB 3.1, Q3
	Review answers	
	Set homework	PB 3.1, Q7 and Q8 or Q11
2.	**Laws of Indices**	
	Discuss homework	
	Introduction	OS 3.2
	Exercises - interactively	PB 3.2, Q1
	Mental Test	M 3.1
	Review answers	
	Set homework	BP 3.2, Q3 or Activity 3.1
3.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 3.1
4.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 3 *Indices and Standard Form* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
1.	**Index Notation**	
	Introduction	OS 3.1
	Exercises - interactively	PB 3.1, Q1
	Exercises - interactively	PB 3.1, Q2
	Exercises	PB 3.1, Q3
	Review answers	
	Set homework	PB 3.1, Q7 and Q11
2.	**Laws of Indices 1**	
	Discuss homework	
	Introduction	OS 3.2 / 3.3
	Exercises - interactively	PB 3.2, Q1
	Exercises	PB 3.2, Q2
	Review answers	
	Activity	Activity 3.1
	Set homework	Complete Activity 3.1 or PB 3.2, Q5
3.	**Laws of Indices 2**	
	Discuss homework	
	Exercises	PB 3.2, Q6
	Review answers	
	Exercises - interactively	PB 3.2, Q7
	Mental Test	M 3.2
	Review answers	
	Set homework	PB 3.2, Q8 and Q9
4.	**Negative Indices**	
	Discuss homework	
	Introduction to concept	OS 3.4
	Exercises - interactively	PB 3.3, Q1
	Exercises	PB 3.3, Q2
	Review answers	
	Activity	Activity 3.2
	Set homework	Complete Activity 3.2 or PB 3.3, Q3

UNIT 3 *Indices and Standard Form* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
5.	**Standard Form**	
	Discuss homework	
	Introduction	OS 3.5 and 3.6
	Exercises	PB 3.4, Q1
	Review answers	
	Exercises	PB 3.4, Q2
	Review answers	
	Exercises	PB 3.4, Q3
	Review answers	
	Activity	Activity 3.3
	Set homework	Complete Activity 3.3 or PB 3.4, Q4 and Q5
6.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 3.2
7.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 3 *Indices and Standard Form* Lesson Plans **E**

Lesson No.	*Suggested Plan*	*References*
1.	**Laws of Indices 1**	
	Review index notation	OS 3.1
	Exercises	PB 3.1, Q11
	Review answers	
	Laws of indices	OS 3.2
	Exercises	PB 3.2, Q1 and Q2
	Review answers	
	Set homework	Activity 3.1
2.	**Laws of Indices 2**	
	Discuss homework	
	Laws of indices - recap	OS 3.3
	Exercises	PB 3.2, Q6
	Review answers	
	Exercises	PB 3.2, Q7
	Review answers	
	Activity	Activity 3.2
	Set homework	Complete Activity 3.2 or PB 3.2, Q9 and Q10
3.	**Negative Indices**	
	Discuss homework	
	Introduction	OS 3.4
	Exercises - interactively	PB 3.3, Q1
	Exercises - interactively	PB 3.3, Q2
	Mental Test	M 3.3
	Review answer	
	Exercises	PB 3.3, Q4
	Review answers	
	Set homework	PB 3.3, Q5, Q6 and Q7
4.	**Standard Form 1**	
	Discuss homework	
	Introduction	OS 3.5 / 3.6
	Exercises	PB 3.4, Q2
	Review answers	
	Exercises	PB 3.4, Q3
	Review answers	
	Activity	Activity 3.3
	Set homework	Complete Activity 3.3 or PB 3.4, Q4

UNIT 3 *Indices and Standard Form* Lesson Plans **E**

Lesson No.	*Suggested Plan*	*References*
5.	**Standard Form 2**	
	Discuss homework	
	Calculations in standard form	OS 3.7
	Exercises	PB 3.4, Q6
	Review answers	
	Exercises	PB 3.4, Q7
	Review answers	
	Activity	Activity 3.4
	Set homework	Complete Activity 3.4 or PB 3.4, Q11 or Q12
6.	**Fractional Indices**	
	Discuss homework	
	Definitions	OS 3.8
	Exercises - interactively	PB 3.5, Q1
	Exercises	PB 3.5, Q2
	Review answers	
	Exercises	PB 3.5, Q5
	Review answers	
	Set homework	PB 3.5, Q4 and Q6
7.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 3.3
8.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 3 Indices and Standard Form Mental Tests

M 3.1 Standard Route *(no calculator)*

1. Calculate 7^2. (49)
2. Calculate 10^2. (100)
3. Calculate 8^2. (64)
4. Calculate 10^3. (1000)
5. Calculate 2^3. (8)
6. Calculate 2^4. (16)
7. Calculate 3^3. (27)
8. Calculate 10^4. (10 000)
9. Calculate $2^2 + 3^2$. (13)
10. Calculate $5^2 + 10^2$. (125)

M 3.2 Academic Route *(no calculator)*

1. Calculate 9^2. (81)
2. Calculate 6^2. (36)
3. Calculate 5^3. (125)
4. Calculate 10^4. (10 000)
5. Calculate 2^5. (32)
6. Calculate $3^2 + 3^3$. (36)
7. Calculate $5^3 + 10^2$. (225)
8. Simplify $a^2 \times a^3$. $\left(a^5\right)$
9. Simplify $a^7 \div a^4$. $\left(a^3\right)$
10. Simplify $\left(a^4\right)^3$. $\left(a^{12}\right)$

UNIT 3 Indices and Standard Form Mental Tests

M 3.3 Express Route *(no calculator)*

1. Calculate 12^2 (144)

2. Calculate 2^5. (32)

3. Calculate 5^3. (125)

4. Calculate $10^4 + 10^5$. (110 000)

5. Calculate $4^3 + 2^4$. (80)

6. Simplify $a^4 \times a^5$. $\left(a^9\right)$

7. Simplify $a^{17} \div a^{14}$. $\left(a^3\right)$

8. Simplify $\left(a^6\right)^7$. $\left(a^{42}\right)$

9. Write 4^{-2} as a fraction. $\left(\frac{1}{16}\right)$

10. Write 10^{-3} as a decimal. (0.001)

UNIT 3 *Indices and Standard Form* Overhead Slides

Overhead Slides

OS 3.1 *Indices*

Complete the following calculations:

1. $2 \times 2 \times 2 \times 2 = 2^{\square}$

2. $7 \times 7 \times 7 = 7^{\square}$

3. $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^{\square}$

4. $1000 = 10 \times 10 \times 10$

   $= 10^{\square}$

5. $1\,000\,000 =$

   $= 10^{\square}$

6. $8 = 2^{\square}$

7. $27 = 3^{\square}$

OS 3.2 *Laws of Indices 1*

The Laws of Indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n} \quad (m \geq n)$$

$$\left(a^m\right)^n = \left(a^n\right)^m = a^{m \times n}$$

Complete the following statements:

1. $3^6 \times 3^7 = 3^{\square}$

2. $4^3 \times 4^5 = 4^{\square}$

3. $5^2 \times 5^7 = 5^{\square}$

4. $6^3 \times 6^2 = 6^{\square}$

5. $7^8 \div 7^2 = 7^{\square}$

6. $9^{10} \div 9^7 = 9^{\square}$

7. $8^4 \div 8^2 = 8^{\square}$

OS 3.3 *Laws of Indices 2*

Complete the following statements:

1. $\left(10^{2}\right)^{3} = 10^{\square}$

2. $\left(2^{4}\right)^{7} = 2^{\square}$

3. $\left(6^{3}\right)^{4} = 6^{\square}$

4. $a^{6} \times a^{7} = a^{\square}$

5. $a^{12} \div a^{5} = a^{\square}$

6. $\left(x^{4}\right)^{2} = x^{\square}$

7. $x^{3} \times x^{9} = x^{\square}$

8. $\left(x^{11}\right)^{2} = x^{\square}$

9. $z^{11} \div z^{7} = z^{\square}$

OS 3.4 *Negative Indices*

Complete each of the following statements:

1. $4^{-2} = \frac{1}{4^2} = \frac{1}{\quad}$

2. $1^{-1} = \frac{1}{\quad} =$

3. $10^{-3} = \frac{1}{\quad} = \frac{1}{\quad}$

4. $2^{-4} = \frac{1}{\quad} = \frac{1}{\quad}$

5. $2^{-1} + 4^{-1} = \frac{1}{\quad} + \frac{1}{\quad}$

$=$

OS 3.5 *Standard Form*

Complete each of the following statements:

1. $36\,200 = 3.62 \times 10^{\square}$

2. $4710 = 4.71 \times 10^{\square}$

3. $8\,400\,000 = \square \times 10^{\square}$

4. $92\,000 = \square \times 10^{\square}$

5. $0.0042 = 4.2 \times 10^{\square}$

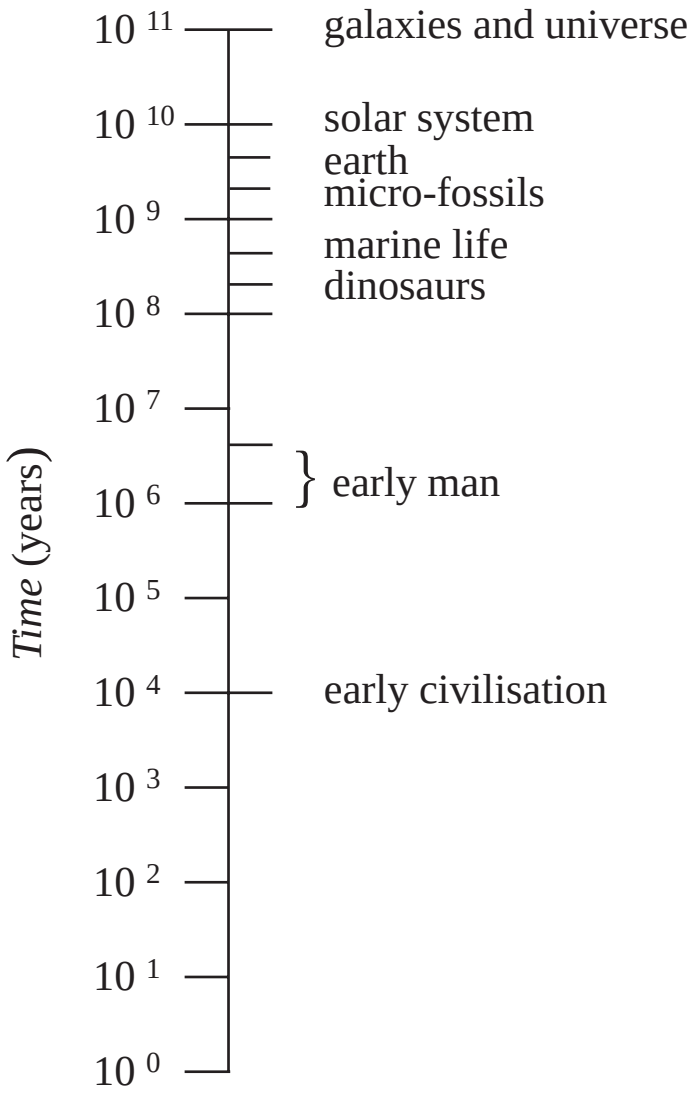
6. $0.0168 = 1.68 \times 10^{\square}$

7. $0.0000062 = \square \times 10^{\square}$

8. $0.00000000041 = \square \times 10^{\square}$

OS 3.6 *Time Scale*

Non-linear Scale



Galaxies and universe formed	10^5 million years ago
Solar system formed	10^4 million years ago
Mocro-fossils formed	3.2×10^3 million years ago
Marine evolution	6×10^2 million years ago
Early man evolved	4 to 1 million years ago
Early civilisation began	10^4 years ago

OS 3.7 *Calculations in Standard Form*

Complete each of the following statements:

1. $\left(6 \times 10^{4}\right) \times \left(2 \times 10^{5}\right) = (6 \times 2) \times \left(10^{4} \times 10^{5}\right)$

$= \quad \times 10^{\square}$

$= \quad \times 10^{\square}$

2. $\left(8 \times 10^{7}\right) \times \left(3 \times 10^{-2}\right) = (\quad \times \quad) \times (10^{\square} \times 10^{\square})$

$= \quad \times 10^{\square}$

$= \quad \times 10^{\square}$

3. $\left(8 \times 10^{16}\right) \div \left(2 \times 10^{5}\right) = (8 \div 2) \times \left(10^{16} \div 10^{5}\right)$

$= \quad \times 10^{\square}$

4. $\left(4.2 \times 10^{13}\right) \div \left(3 \times 10^{4}\right) = (\quad \div \quad) \times (10^{\square} \div 10^{\square})$

$= \quad \times 10^{\square}$

OS 3.8 *Fractional Indices*

Complete each of the following statements:

1. $4 = \sqrt{16} = 16^{\square}$

2. $3 = \sqrt{\square} = \square^{\frac{1}{2}}$

3. $\frac{1}{2} = \frac{1}{\sqrt[3]{8}} = 8^{\square}$

4. $3 = \sqrt{\square} = \square^{\frac{1}{3}}$

5. $\left(\frac{6 \times 8}{3}\right)^{\frac{1}{2}} =$

6. $\left(\frac{10 \times 15}{6}\right)^{\frac{1}{2}} =$

Practice Book *UNIT 3 Indices and Standard Form* Answers

3.1 Index Notation

1. (a) 8 (b) 100 (c) 9 (d) 1000 (e) 81
   (f) 27 (g) 16 (h) 81 (i) 49

2. (a) $10 \times 10 \times 10 \times 10 \times 10 = 10^{\boxed{5}}$ (b) $3 \times 3 \times 3 \times 3 = 3^{\boxed{4}}$
   (c) $7 \times 7 \times 7 \times 7 \times 7 = 7^{\boxed{5}}$ (d) $8 \times 8 \times 8 \times 8 \times 8 = 8^{\boxed{5}}$
   (e) $5 \times 5 = 5^{\boxed{2}}$ (f) $19 \times 19 \times 19 \times 19 = 19^{\boxed{4}}$
   (g) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{\boxed{7}}$ (h) $11 \times 11 \times 11 \times 11 \times 11 \times 11 = 11^{\boxed{6}}$

3. (a) $8 = 2^{\boxed{3}}$ (b) $81 = 3^{\boxed{4}}$ (c) $100 = 10^{\boxed{2}}$ (d) $81 = 9^{\boxed{2}}$
   (e) $125 = 5^{\boxed{3}}$ (f) $1000\,000 = 10^{\boxed{6}}$ (g) $216 = 6^{\boxed{3}}$ (h) $625 = 5^{\boxed{4}}$

4. No, because $10^{2} = 100$ and $2^{10} = 1024$.

5. Yes, because $3^{4} = 81$ and $4^{3} = 64$.

6. No, because $5^{2} = 25$ and $2^{5} = 32$.

7. (a) $49 = \boxed{7}^{2}$ (b) $64 = \boxed{4}^{3}$ (c) $64 = \boxed{2}^{6}$
   (d) $64 = \boxed{8}^{2}$ (e) $100\,000 = \boxed{10}^{5}$ (f) $243 = \boxed{3}^{5}$

8. (a) 12 (b) 32 (c) 13 (d) 36
   (e) 8000 (f) 1032

9. (a) 625 (b) 1 (c) 27 (d) 1331

10. (a) 10^{5} (b) 2^{10} (c) 3^{2} (d) 2^{3}
    (e) 10^{4} (f) 5^{2}

11. (a) $k = 3,\ m = 6$ (b) 16 384

3.2 Laws of Indices

1. (a) $2^{3} \times 2^{7} = 2^{\boxed{10}}$ (b) $3^{6} \times 3^{5} = 3^{\boxed{11}}$ (c) $3^{7} \div 3^{4} = 3^{\boxed{3}}$
   (d) $8^{3} \times 8^{4} = 8^{\boxed{7}}$ (e) $\left(3^{2}\right)^{5} = 3^{\boxed{10}}$ (f) $\left(2^{3}\right)^{6} = 2^{\boxed{18}}$
   (g) $\dfrac{3^{6}}{3^{2}} = 3^{\boxed{4}}$ (h) $\dfrac{4^{7}}{4^{2}} = 4^{\boxed{5}}$

3.2 Answers

2. (a) $a^3 \times a^2 = a^{\boxed{5}}$ (b) $b^7 \div b^2 = b^{\boxed{5}}$ (c) $\left(b^2\right)^5 = b^{\boxed{10}}$

(d) $b^6 \times b^4 = b^{\boxed{10}}$ (e) $\left(z^3\right)^9 = z^{\boxed{27}}$ (f) $\dfrac{q^{16}}{q^7} = q^{\boxed{9}}$

3. $9^4 = \left(3^2\right)^4 = 3^{2\times 4} = 3^8$

4. (a) 2 (b) 1 (c) 0 (d) 1

5. (a) $3^6 \times 3^{\boxed{11}} = 3^{17}$ (b) $4^6 \times 4^{\boxed{5}} = 4^{11}$ (c) $\dfrac{a^6}{a^{\boxed{2}}} = a^4$

(d) $\left(z^{\boxed{3}}\right)^6 = z^{18}$ (e) $\left(a^{19}\right)^{\boxed{5}} = a^{95}$ (f) $p^{16} \div p^{\boxed{9}} = p^7$

(g) $\left(p^{\boxed{5}}\right)^8 = p^{40}$ (h) $q^{13} \div q^{\boxed{12}} = q$

6. (a) 3 (b) 2 (c) 31 (d) 24
(e) 875 (f) 48

7. (a) $8^2 = 2^{\boxed{6}}$ (b) $81^3 = 9^{\boxed{6}} = 3^{12}$ (c) $25^6 = 5^{12}$

(d) $4^7 = 2^{\boxed{14}}$ (e) $125^4 = 5^{12}$ (f) $1000^6 = 10^{\boxed{18}}$

(g) $81 = \boxed{3}^4$ (h) $256 = \boxed{4}^4 = \boxed{2}^8$

8. (a) $8 \times 4 = 2^{\boxed{3}} \times 2^{\boxed{2}} = 2^{\boxed{5}}$ (b) $25 \times 625 = 5^{\boxed{2}} \times 5^{\boxed{4}} = 5^{\boxed{6}}$

(c) $\dfrac{243}{9} = \dfrac{3^{\boxed{5}}}{3^{\boxed{2}}} = 3^{\boxed{3}}$ (d) $\dfrac{128}{16} = \dfrac{2^{\boxed{7}}}{2^{\boxed{4}}} = 2^{\boxed{3}}$

9. (a) False, $3^2 \times 2^2 = 6^2$ (b) False, $5^4 \times 2^3$ cannot be simplified as a single power.

(c) True (d) False, $\dfrac{10^8}{5^6}$ cannot be simplified as a single power.

10. (a) $\left(2^6 \times 2^3\right)^4 = \left(2^9\right)^4 = 2^{36}$ (b) $\left(\dfrac{3^6}{3^2}\right)^5 = \left(3^4\right)^5 = 3^{20}$

(c) $\left(\dfrac{2^3 \times 2^4}{2^7}\right)^4 = \left(2^0\right)^4 = 2^0$ (d) $\left(\dfrac{3^2 \times 9}{3^3}\right)^4 = \left(3^1\right)^4 = 3^4$

(e) $\left(\dfrac{6^2 \times 6^8}{6^3}\right)^4 = \left(6^7\right)^4 = 6^{28}$ (f) $\left(\dfrac{7^8}{7^2 \times 7^3}\right)^5 = \left(7^3\right)^5 = 7^{15}$

3.3 Negative Indices

1. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{49}$

   (e) $\frac{1}{64}$ (f) $\frac{1}{36}$

2. (a) $\frac{1}{49} = \frac{1}{7^2} = 7^{-2}$ (b) $\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

   (c) $\frac{1}{81} = \frac{1}{9^2} = 9^{-2}$ (d) $\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

   (e) $\frac{1}{1000\,000} = \frac{1}{10^7} = 10^{-7}$ (f) $\frac{1}{1024} = \frac{1}{2^{10}} = 2^{-10}$

3. (a) $\frac{7}{12}$ (b) $\frac{2}{3}$ (c) $\frac{1}{10}$ (d) $\frac{9}{1000}$

   (e) $\frac{3}{20}$ (f) $\frac{13}{42}$

4. (a) 4^1 (b) 5^4 (c) 7^{10} (d) 3^{-8}

   (e) 6^6 (f) 8^{-5} (g) 7^4 (h) 8^{18}

5. (a) $\frac{1}{9} = 3^{\boxed{-2}}$ (b) $\frac{1}{100} = 10^{\boxed{-2}}$ (c) $\frac{1}{125} = 5^{\boxed{-3}}$

   (d) $\frac{5}{5^4} = 5^{\boxed{-3}}$ (e) $\frac{6^2}{6^3} = 6^{\boxed{-1}}$ (f) $\frac{2^2}{2^{10}} = 2^{\boxed{-8}}$

6. (a) x^5 (b) x^{-2} (c) x^{-4} (d) x^{-24}

   (e) x^{-8} (f) x^{-24}

7. (a) $0.1 = 10^{\boxed{-1}}$ (b) $0.25 = 2^{\boxed{-2}}$ (c) $0.0001 = 10^{\boxed{-4}}$

   (d) $0.2 = 5^{\boxed{-1}}$ (e) $0.001 = 10^{\boxed{-3}}$ (f) $0.02 = 50^{\boxed{-1}}$

8. (a) $\frac{x^4}{x^{\boxed{2}}} = x^2$ (b) $x^6 \times x^{\boxed{-4}} = x^2$ (c) $x^9 \times x^{\boxed{-7}} = x^2$

   (d) $\frac{x^7}{x^{\boxed{9}}} = x^{-2}$ (e) $\frac{x^3}{x^{\boxed{-1}}} = x^4$ (f) $\left(x^3\right)^{\boxed{-2}} = x^{-6}$

3.3 Answers

9. (a) $\frac{1}{8} = 2^{\boxed{-3}}$ (b) $\frac{1}{25} = 5^{\boxed{-2}}$ (c) $\frac{1}{81} = 9^{\boxed{-2}}$ (d) $\frac{1}{10\,000} = 10^{\boxed{-4}}$

10. $a = b^3 = \left(\frac{1}{c^2}\right)^3 = \left(c^{-2}\right)^3 = c^{-6}$

3.4 Standard Form

1. (a) 6210 (b) 8000 (c) 420 (d) 0.003 (e) 0.06
   (f) 0.0032 (g) 0.006 (h) 0.92 (i) 0.036

2. (a) 2×10^2 (b) 8×10^3 (c) 9×10^6 (d) 6.2×10^4
   (e) 8.4×10^5 (f) 1.2×10^{10} (g) 6.18×10^{10} (h) 3.24×10^6

3. (a) 30 000 (b) 36 000 (c) 8200
   (d) 310 (e) 16 000 (f) 172 000
   (g) 68 300 (h) 1 250 000 (i) 9170

4. (a) 4×10^{-4} (b) 8×10^{-3} (c) 1.42×10^{-1} (d) 3.2×10^{-3}
   (e) 1.99×10^{-3} (f) 6.2×10^{-8} (g) 9.7×10^{-6} (h) 2.1×10^{-13}

5. (a) 0.06 (b) 0.7 (c) 0.0018 (d) 0.004 (e) 0.0062
   (f) 0.000981 (g) 0.667 (h) 0.0000386 (i) 0.000000927

6. (a) 8×10^9 (b) 6×10^{11} (c) 4.8×10^{-4}
   (d) 2.1×10^{-11} (e) 1.22×10^2 (f) 1.28×10^{-13}

7. (a) 3×10^3 (b) 4×10^7 (c) 3×10^1
   (d) 2×10^{10} (e) 4×10^9 (f) 4×10^4

8. (a) $3600\,000 = 3.6 \times 10^6$ (b) $9600 = 9.6 \times 10^3$
   (c) $590\,000 = 5.9 \times 10^5$ (d) $0.089 = 8.9 \times 10^{-2}$
   (e) $0.0086 = 8.6 \times 10^{-3}$ (f) $0.00057 = 5.7 \times 10^{-4}$

9. (a) 7.14×10^{10} (b) 4.92×10^{11} (c) 1.62×10^{13} (d) 2.05×10^{19}
   (e) 6.144×10^{-5} (f) 2.38328×10^{44}

3.4 Answers

10. (a) 4.02×10^{7} m (b) 4.02×10^{9} cm (c) 4.02×10^{10} mm (d) 4.02×10^{4} km

11. (a) $4.3953 \times 10^{47}\ \text{kg}^2$ (b) $1.51321 \times 10^{11}\ \text{km}^2$ (c) 3.53×10^{19} N

12. (a) Statement (i) is the true one because $4 \times 10^{3} = 4000$ and $4^{3} = 64$.
(b) 0.36×10^{5} (c) 25×10^{-4} (d) (i) 6×10^{0} (ii) 3×10^{4}

3.5 Fractional Indices

1. (a) 7 (b) 8 (c) 4 (d) $\frac{1}{9}$ (e) $\frac{1}{10}$
(f) $\frac{1}{5}$ (g) 3 (h) $\frac{1}{6}$ (i) 12

2. (a) 2 (b) $\frac{1}{2}$ (c) 5
(d) $\frac{1}{4}$ (e) 6 (f) $\frac{1}{100}$

3. (a) 2 (b) $\frac{1}{8}$ (c) 10
(d) $\frac{1}{3}$ (e) 5 (f) $\frac{1}{10}$

4. (a) 4 (b) 3 (c) 5 (d) $\frac{1}{5}$

5. (a) False, because $16^{\frac{1}{2}} = 4$ (b) True
(c) False, because $9 = 81^{\frac{1}{2}}$ (d) True

6. (a) x^{3} (b) a^{-5} (c) $a^{\frac{1}{2}}$ (d) $a^{-\frac{1}{2}}$

7. (a) $x^{\frac{1}{2}}$ (b) $x^{-\frac{1}{2}}$ (c) $a^{-\frac{2}{3}}$ (d) $a^{-\frac{1}{6}}$

8. (a) $2\frac{1}{2}$ (b) 2 (c) 4 (d) 2

UNIT 3 *Indices and Standard Form* Revision Test 3.1 (Standard)

1. Calculate:

   (a) 4^2 (b) 10^3 (c) 2^4

   (d) 3^3 (e) 5^2 (f) 11^2

   (10 marks)

2. Copy each of the following statements and fill in the missing numbers:

   (a) $36 = 6^{\square}$ (b) $10\,000 = 10^{\square}$

   (c) $81 = 9^{\square}$ (d) $81 = 3^{\square}$

   (e) $3^2 \times 3^6 = 3^{\square}$ (f) $7^4 \times 7^6 = 7^{\square}$

   (g) $7^9 \div 7^3 = 7^{\square}$ (h) $9^7 \div 9^2 = 9^{\square}$

   (14 marks)

3. Calculate:

   (a) $10^2 + 10^4$

   (b) $2^3 + 3^3$

   (c) $7^2 + 2^7$

   (6 marks)

UNIT 3 *Indices and Standard Form* Revision Test 3.2 (Academic)

1. Calculate:

   (a) 7^3 (b) 3^4 (c) 10^6

   (5 marks)

2. Copy each of the following statements and fill in the missing numbers:

   (a) $64 = 8^{\square}$ (b) $64 = 2^{\square}$

   (c) $100\,000 = 10^{\square}$ (d) $3^2 \times 3^9 = 3^{\square}$

   (e) $7^{14} \div 7^3 = 7^{\square}$ (f) $\left(2^6\right)^3 = 2^{\square}$

   (10 marks)

3. Write each of the following as a fraction:

   (a) 4^{-1} (b) 6^{-2} (c) 9^{-2}

   (d) 5^{-3} (e) 10^{-4} (f) 4^{-3}

   (9 marks)

4. Copy and complete each of the following statements:

   (a) $x^7 \times x^4 = x^{\square}$

   (b) $\left(x^3\right)^5 = x^{\square}$

   (c) $x^7 \div x^3 = x^{\square}$

   (6 marks)

UNIT 3 *Indices and Standard Form* Revision Test 3.3 (Express)

1. Calculate:

   (a) 8^3 (b) 10^5

   (2 marks)

2. Write each of the following as a fraction:

   (a) 4^{-1} (b) 8^{-2} (c) 7^{-3}

   (4 marks)

3. Calculate:

   (a) $169^{\frac{1}{2}}$ (b) $400^{\frac{1}{2}}$ (c) $81^{\frac{1}{4}}$

   (4 marks)

4. Write each of the following numbers in standard form:

   (a) 3 740 000 (b) 0.000629

   (4 marks)

5. Change each of the following numbers from standard form to the normal notation:

   (a) 6.72×10^8 (b) 3.421×10^{-2}

   (2 marks)

6. Simplify:

   (a) $x^6 \times x^5$ (b) $\left(p^7\right)^8$ (c) $q^4 \times q^{-9}$

   (d) $p^2 \div p^5$ (e) $\dfrac{p^6 \times p^7}{p^2}$ (f) $\left(\dfrac{p^2 \times p^5}{p}\right)^{-3}$

   (10 marks)

7. Calculate:

   (a) $\left(3 \times 10^{15}\right) \times \left(6.2 \times 10^{-8}\right)$

   (b) $\left(4.2 \times 10^{17}\right) \div \left(7 \times 10^4\right)$

   (4 marks)

Revision Test 3.1 (Standard) Answers

1.	(a)	16	B1	
	(b)	1000	B2	
	(c)	16	B2	
	(d)	27	B2	
	(e)	25	B1	
	(f)	121	B2	*(10 marks)*

2.	(a)	$36 = 6^{\boxed{2}}$	B1	
	(b)	$10\,000 = 10^{\boxed{4}}$	B2	
	(c)	$81 = 9^{\boxed{2}}$	B1	
	(d)	$81 = 3^{\boxed{4}}$	B2	
	(e)	$3^2 \times 3^6 = 3^{\boxed{8}}$	B2	
	(f)	$7^4 \times 7^6 = 7^{\boxed{10}}$	B2	
	(g)	$7^9 \div 7^3 = 7^{\boxed{6}}$	B2	
	(h)	$9^7 \div 9^2 = 9^{\boxed{5}}$	B2	*(14 marks)*

3.	(a)	$10^2 + 10^4 = 100 + 10\,000$		
		$= 10100$	B2	
	(b)	$2^3 + 3^3 = 8 + 27$		
		$= 35$	B2	
	(c)	$7^2 + 2^7 = 49 + 128$		
		$= 177$	B2	*(6 marks)*

(TOTAL MARKS 30)

Revision Test 3.2 (Academic) Answers

1.	(a)	$7^3 = 343$	B2	
	(b)	$3^4 = 81$	B2	
	(c)	$10^6 = 1000000$	B1	*(5 marks)*
2.	(a)	$64 = 8^{\boxed{2}}$	B1	
	(b)	$64 = 2^{\boxed{6}}$	B2	
	(c)	$100000 = 10^{\boxed{5}}$	B1	
	(d)	$3^2 \times 3^9 = 3^{\boxed{11}}$	B2	
	(e)	$7^{14} \div 7^3 = 7^{\boxed{11}}$	B2	
	(f)	$\left(2^6\right)^3 = 2^{\boxed{18}}$	B2	*(10 marks)*
3.	(a)	$\frac{1}{4}$	B1	
	(b)	$\frac{1}{36}$	B1	
	(c)	$\frac{1}{81}$	B1	
	(d)	$\frac{1}{125}$	B2	
	(e)	$\frac{1}{10000}$	B2	
	(f)	$\frac{1}{64}$	B2	*(9 marks)*
4.	(a)	$x^7 \times x^4 = x^{\boxed{11}}$	B2	
	(b)	$\left(x^3\right)^5 = x^{\boxed{15}}$	B2	
	(c)	$x^7 \div x^3 = x^{\boxed{4}}$	B2	*(6 marks)*

(TOTAL MARKS 30)

Revision Test 3.3 (Express) Answers

1.	(a)	512	B1	
	(b)	100 000	B1	*(2 marks)*
2.	(a)	$\frac{1}{4}$	B1	
	(b)	$\frac{1}{64}$	B1	
	(c)	$\frac{1}{343}$	B2	*(4 marks)*
3.	(a)	13	B1	
	(b)	20	B1	
	(c)	3	B2	*(4 marks)*
4.	(a)	3.74×10^{6}	B2	
	(b)	6.29×10^{-4}	B2	*(4 marks)*
5.	(a)	672 000 000	B1	
	(b)	0 03421	B1	*(2 marks)*
6.	(a)	x^{11}	B1	
	(b)	p^{56}	B1	
	(c)	q^{-5}	B2	
	(d)	p^{-3}	B2	
	(e)	p^{11}	B2	
	(f)	p^{-18}	B2	*(10 marks)*
7.	(a)	$18.6 \times 10^{7} = 1.86 \times 10^{8}$	B2	
	(b)	$0.6 \times 10^{13} = 6 \times 10^{12}$	B2	*(4 marks)*

(TOTAL MARKS 30)

UNIT 3 *Indices and Standard Form* Teaching Notes

Historical Background and Introduction

General use of decimal notation for whole numbers and decimal fractions dates from 1585 when *Simon Stevin* (1548–1620) published his book, *Die Thiende*. Stevin used powers of 10 to introduce place value and showed how the algebra of powers (the *index laws*) led to relatively simple ways of doing arithmetic. We write a number such as

three hundred and sixteen and a quarter

in terms of powers of 10 as

$$3\times10^{2}+1\times10^{1}+6\times10^{0}+2\times10^{-1}+5\times10^{-2}$$

and shorten this to 316.25.

Here $10^{n}=\underbrace{10\times10\times...\times10}_{(n\text{ times})}$, when $n>0$, $10^{0}=1$, and $10^{-n}=\dfrac{1}{10^{n}}$.

When we multiply 316.25 by 10 we use the index law,

$$10^{n}\times10=10^{n+1}$$

(and the distributive law) to obtain the quick answer 3162.5.

The two basic *index laws*,

$$10^{a}\times10^{b}=10^{a+b}\quad\text{and}\quad\left(10^{a}\right)^{b}=10^{ab},$$

can be easily checked from the definitions when *a* and *b* are positive integers. A little more thought is needed when *a* and/or *b* are negative integers (or fractions!) The definitions of $10^{0}\left(=1\right)$ and $10^{-n}\left(=\dfrac{1}{10^{n}}\right)$, and later of $10^{\frac{1}{2}}\left(=\sqrt{10}\right)$, are chosen to ensure that the basic index laws

$$x^{a}\times x^{b}=x^{a+b},\quad\left(x^{a}\right)^{b}=x^{ab}\quad\text{and}\quad x^{a}.y^{a}=(xy)^{a}$$

remain true.

[Note: some care is needed when $x\le0$, 0^{a} is not defined when $a\le0$, and x^{a} may have no meaning when $x<0$ and *a* is fractional.]

The index laws allow us to write very large numbers in a compact and manageable form. For example, the number of atoms in the universe is frighteningly large but elementary arguments show that this number is approximately 10^{50}. Scientific notation provides an agreed way of giving in standard form the approximate value of very large numbers which occur in science, e.g.

$$2^{10}=1024=1.024\times10^{3}\approx1\times10^{3}$$
$$2^{20}=1\,048\,576\approx1.05\times10^{6}.$$

Writing numbers in this form makes it easy to do rough calculations. For example,

$$2^{40}=\left(2^{20}\right)^{2}\approx\left(1.05\times10^{6}\right)^{2}$$
$$\left(1.05\times10^{6}\right)^{2}=(1.05)^{2}\times\left(10^{6}\right)^{2}=1.1025\times10^{12}$$

$$2^{-20}=\frac{1}{2^{20}}\approx\frac{1}{1.05\times10^{6}}\approx0.95\times10^{-6}$$
$$0.95\times10^{-6}=9.5\times10^{-7}$$

UNIT 3 *Indices and Standard Form* Teaching Notes

In this unit we extend *index notation*, already encountered in Years 7 and 8, to non-positive integer values, e.g. a^{-1}, $a^{\frac{1}{2}}$, $a^{\frac{1}{4}}$, noting that the rules of indices still hold and we also introduce *scientific notation*, i.e. standard form, which many pupils will already have experienced with very large or very small numbers, when using their calculators.

Routes		**Standard**	**Academic**	**Express**
3.1	Index Notation	✓	✓	(✓)
3.2	Laws of Indices	(✓)	✓	✓
3.3	Negative Indices	×	(✓)	✓
3.4	Standard Form	×	(✓)	✓
3.5	Fractional Indices	×	×	✓

Language	**Standard**	**Academic**	**Express**
Negative indices	×	(✓)	✓
Standard form (i.e. scientific notation, standard index notation)	×	(✓)	✓
Fractional indices	×	×	✓

Misconceptions

- pupils need to realise that $a^m \times a^n \neq a^{mn}$ (in general); this is easy to show with a simple example, e.g. $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
  $= 2^5 \neq 2^6$

- there may well be problems with $a^{\frac{1}{2}} = \sqrt{a}$ and pupils might confuse $(a^n)^m$ with $a^n \times a^m$. For these later manipulations, always convince your students by writing out examples fully, e.g.
  $$\begin{aligned}(a^4)^3 &= a^4 \times a^4 \times a^4 \\ &= (a \times a \times a \times a) \times (a \times a \times a \times a) \times (a \times a \times a \times a) \\ &= a^{12}\end{aligned}$$

UNIT 3 *Indices and Standard Form* Teaching Notes

- confusion can arise with the inverse of a fractional number; for example:

$$\left(\frac{1}{10}\right)^{-2} \neq \frac{1}{100}$$

as $\left(\frac{1}{10}\right)^{-2} = \left[\left(\frac{1}{10}\right)^{-1}\right]^2 = (10)^2 = 100$

- $a^0 \neq 0$, but 1; this is often problematic, but you can help pupils to understand why we define $a^0 = 1$ by looking at the sequence, for example, $5^{\frac{1}{n}}$, as n gets larger (on their calculators).

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9A	3.1	11	40
" "	3.2	9	44
" "	3.3	10	49
" "	3.4	8, 11, 12	53,55
" "	3.5	7	57

UNIT 4 *Fractions and Percentages* Extra Exercises 4.1

1. For each of the following statements, complete the fraction with the missing number:

(a) $\frac{4}{5} = \frac{\square}{40}$ (b) $\frac{1}{3} = \frac{\square}{12}$ (c) $\frac{3}{7} = \frac{12}{\square}$

(d) $\frac{88}{100} = \frac{\square}{25}$ (e) $\frac{28}{64} = \frac{\square}{16}$ (f) $\frac{11}{44} = \frac{1}{\square}$

2. Write each of the following fractions in its simplest form:

(a) $\frac{9}{30}$ (b) $\frac{11}{55}$ (c) $\frac{44}{99}$

(d) $\frac{21}{28}$ (e) $\frac{56}{64}$ (f) $\frac{25}{30}$

(g) $\frac{15}{45}$ (h) $\frac{24}{32}$ (i) $\frac{84}{100}$

3. (a) For each of the following statements, complete the fraction with the missing number:

$\frac{3}{5} = \frac{\square}{35}$ and $\frac{4}{7} = \frac{\square}{35}$

(b) Which fraction is *larger,* $\frac{3}{5}$ or $\frac{4}{7}$?

4. Write the following fractions in order of size, with the smallest first:

$$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}$$

UNIT 4 *Fractions and Percentages* Extra Exercises 4.2

1. Calculate:

   (a) $\frac{1}{2}$ of 88 (b) $\frac{1}{3}$ of 39 (c) $\frac{1}{4}$ of 500

   (d) $\frac{1}{7}$ of 42 (e) $\frac{1}{9}$ of 54 (f) $\frac{1}{8}$ of 1000

   (g) $\frac{2}{3}$ of 90 (h) $\frac{2}{7}$ of 210 (i) $\frac{3}{5}$ of 145

   (j) $\frac{4}{5}$ of 80 (k) $\frac{7}{11}$ of 2200 (l) $\frac{5}{8}$ of 3000

2. A crate holds 120 eggs. In transit, $\frac{1}{10}$ of the eggs are cracked.
   (a) How many eggs are cracked?
   (b) How many eggs are *not* cracked?

3. A footballer expects to score 1 goal in $\frac{2}{3}$ of the matches he plays. How many goals would he expect to score in:

   (a) 12 matches, (b) 30 matches, (c) 141 matches ?

4. Nazim has to drive 320 miles. He decides to stop when he has completed $\frac{3}{5}$ of the journey. How far will he travel before he stops?

5. A small company decides to give $\frac{2}{15}$ of its profits to a charity. How much would it give away if it made a profit of £66 000 ?

UNIT 4 *Fractions and Percentages* Extra Exercises 4.3

1. Calculate:

(a) $\frac{3}{11}+\frac{4}{11}$ (b) $\frac{3}{10}+\frac{5}{10}$ (c) $\frac{4}{9}-\frac{1}{9}$

(d) $\frac{3}{5}+\frac{2}{7}$ (e) $\frac{4}{9}+\frac{3}{4}$ (f) $\frac{5}{7}+\frac{2}{3}$

(g) $\frac{3}{11}+\frac{1}{4}$ (h) $\frac{5}{8}-\frac{2}{5}$ (i) $\frac{1}{2}-\frac{2}{9}$

2. Calculate:

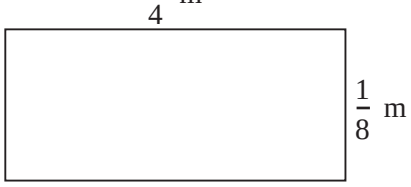
(a) $\frac{1}{2}\times\frac{3}{4}$ (b) $\frac{5}{8}\times\frac{2}{3}$ (c) $\frac{3}{5}\times\frac{2}{9}$

(d) $\frac{6}{7}\times\frac{14}{15}$ (e) $\frac{11}{12}\times\frac{6}{7}$ (f) $\frac{3}{8}\div\frac{2}{3}$

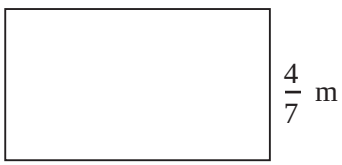
(g) $\frac{4}{5}\div\frac{1}{2}$ (h) $1\frac{1}{2}\times\frac{3}{4}$ (i) $4\frac{1}{5}\times1\frac{1}{2}$

3. Calculate the area and perimeter of each of the following rectangles:

(a)

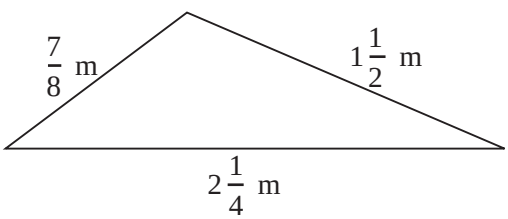


(b)

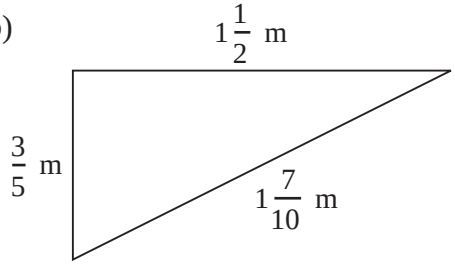


4. Calculate the perimeter of each of the following triangles:

(a)



(b)



UNIT 4 *Fractions and Percentages* Extra Exercises 4.4

1. Write each of the following percentages as fractions in their simplest form:

   (a) 75% (b) 10% (c) 95%

   (d) 34% (e) 80% (f) 4%

2. Write each of the following decimals as a percentage and as a fraction in its simplest form:

   (a) 0.18 (b) 0.95 (c) 0.37

   (d) 0.08 (e) 0.11 (f) 0.74

3. Write each of the following fractions as a decimal and as a percentage:

   (a) $\frac{3}{20}$ (b) $\frac{11}{25}$ (c) $\frac{7}{10}$

   (d) $\frac{1}{4}$ (e) $\frac{3}{25}$ (f) $\frac{5}{40}$

4. In a class of 25 pupils, 8 cannot swim.

   (a) What percentage of the class *can* swim?

   (b) What percentage of the class *cannot* swim?

5. At a factory, 30 of the 1000 workers voted to go on strike. What percentage was this?

UNIT 4 *Fractions and Percentages* Extra Exercises 4.5

1. (a) *Increase* £200 by 50%.
   (b) *Increase* £450 by 20%.
   (c) *Increase* £80 by 30%.
   (d) *Increase* £40 by 5%.
   (e) *Decrease* £20 by 10%.
   (f) *Decrease* £90 by 5%.
   (g) *Decrease* £120 by 11%.

2. In a sale, all the prices in a shop are reduced by 30%. Calculate the sale price of each of the following items:
   (a) a bike that cost £250,
   (b) a helmet that cost £30,
   (c) a pump that cost £5,
   (d) a pair of gloves that cost £3.20.

3. Anna earns £210 per week. She is given a 3% pay rise. How much does she now earn per week?

4. A keyboard originally cost £240. Its price was increased by 8%. Calculate the new price of the keyboard.

5. Ben puts £200 in a building society. Every year 6% interest is added to his money. How much does he have after:
   (a) 1 year, (b) 2 years ?

Extra Exercises 4.1 Answers

1. (a) $\frac{32}{40}$ (b) $\frac{4}{12}$ (c) $\frac{12}{28}$
   (d) $\frac{22}{25}$ (e) $\frac{7}{16}$ (f) $\frac{1}{4}$
2. (a) $\frac{3}{10}$ (b) $\frac{1}{5}$ (c) $\frac{4}{9}$
   (d) $\frac{3}{4}$ (e) $\frac{7}{8}$ (f) $\frac{5}{6}$
   (g) $\frac{1}{3}$ (h) $\frac{3}{4}$ (i) $\frac{21}{25}$
3. (a) $\frac{21}{35}, \frac{20}{35}$ (b) $\frac{3}{5}$
4. $\frac{3}{8}, \frac{1}{2}, \frac{7}{12}, \frac{3}{4}, \frac{5}{6},$

Extra Exercises 4.2 Answers

1. (a) 44 (b) 13 (c) 125
   (d) 6 (e) 6 (f) 125
   (g) 60 (h) 60 (i) 87
   (j) 64 (k) 1400 (l) 1875
2. (a) 12 (b) 108
3. (a) 8 (b) 20 (c) 94
4. 192 miles
5. £8 800

Extra Exercises 4.3 Answers

1. (a) $\frac{7}{11}$ (b) $\frac{8}{10} = \frac{4}{5}$ (c) $\frac{3}{9} = \frac{1}{3}$
   (d) $\frac{31}{35}$ (e) $\frac{43}{36} = 1\frac{7}{36}$ (f) $\frac{29}{21} = 1\frac{8}{21}$
   (g) $\frac{23}{44}$ (h) $\frac{9}{40}$ (i) $\frac{5}{18}$
2. (a) $\frac{3}{8}$ (b) $\frac{5}{12}$ (c) $\frac{2}{15}$
   (d) $\frac{4}{5}$ (e) $\frac{11}{14}$ (f) $\frac{9}{16}$
   (g) $\frac{8}{5} = 1\frac{3}{5}$ (h) $\frac{9}{8} = 1\frac{1}{8}$ (i) $\frac{63}{10} = 6\frac{3}{10}$
3. (a) $\frac{3}{32}$ m^2, $1\frac{3}{4}$ m (b) $\frac{18}{35}$ m^2, $\frac{103}{35} = 2\frac{33}{35}$ m
4. (a) $4\frac{5}{8}$ m (b) $3\frac{4}{5}$ m

Extra Exercises 4.4 Answers

1. (a) $\frac{3}{4}$ (b) $\frac{1}{10}$ (c) $\frac{19}{20}$
   (d) $\frac{17}{50}$ (e) $\frac{4}{5}$ (f) $\frac{1}{25}$
2. (a) 18%, $\frac{9}{50}$ (b) 95%, $\frac{19}{20}$ (c) 37%, $\frac{37}{100}$
   (d) 8%, $\frac{2}{25}$ (e) 11%, $\frac{11}{100}$ (f) 74%, $\frac{37}{50}$
3. (a) 0.15, 15% (b) 0.44, 44% (c) 0.7, 70%
   (d) 0.25, 25% (e) 0.12, 12% (f) 0.125, 12.5%
4. (a) 68% (b) 32%
5. 3%

Extra Exercises 4.5 Answers

1. (a) £300 (b) £540 (c) £104 (d) £42
   (e) £18 (f) £85.50 (g) £106.80
2. (a) £175 (b) £21 (c) £3.50 (d) £2.24
3. (a) £216.30
4. (a) £259.20
5. (a) £212.00 (b) £224.72

UNIT 4 *Fractions and Percentages* Lesson Plans **St**

These are based on 45/50 minute lessons.

Lesson No.	*Suggested Plan*	*References*
1.	**Equivalent Fractions**	
	Revise concept	OS 4.1
	Exercises	PB 4.1, Q1
	Review answers	
	Finding equivalent fractions	OS 4.2
	Exercises	PB 4.1, Q2
	Review answers	
	Activity	Activity 4.1 (or Activity 10.5 from Y7A)
	Set homework	PB 4.1, Q3 and Q6
2.	**Fractions of Quantities**	
	Discuss homework	
	Examples - interactively	OS 4.3
	Exercises	PB 4.2, Q1
	Review answers	
	Exercises	PB 4.2, Q2
	Review answers	
	Examples in context	PB 4.2, Q4, Q5 or Q6
	Set homework	PB 4.2, Q7 and Q11
3.	**Operations with Fractions**	
	Discuss homework	
	Addition of fractions	OS 4.4
	Exercises	PB 4.3, Q1
	Review answers	
	Subtraction of fractions	OS 4.4
	Exercises	PB 4.3, Q2
	Review answers	
	Set homework	PB 4.3, Q11
4.	**Fractions and Decimals**	
	Discuss homework	
	Fractions and decimal equivalents	OS 4.7
	Exercises	PB 4.4, Q5
	Review answers	
	Exercises	PB 4.4, Q4
	Review answers	
	Set homework	PB 4.4, Q7 (part of)

UNIT 4 *Fractions and Percentages* Lesson Plans **St**

Lesson No.	*Suggested Plan*	*References*
5.	**Decimals and Percentages**	
	Discuss homework	
	Decimal and percentage equivalents	OS 4.7
	Exercises	PB 4.4, Q1
	Review answers	
	Exercises	PB 4.4, Q3
	Review answers	
	Set homework	PB 4.4, Q7 (parts of)
6.	**Percentages and Fractions**	
	Discuss homework	
	Percentage and fraction equivalents	OS 4.7
	Exercises	PB 4.4, Q2
	Review answers	
	Exercises	PB 4.4, Q6
	Review answers	
	Activity	Activity 4.2 or Activity 4.3
	Set homework	PB 4.4, Q11 and Q12
7.	**Percentage Increase/Decrease**	
	Discuss homework	
	Introduction	OS 4.8
	Exercises	PB 4.5, parts of Q1
	Review answers	
	Exercises	PB 4.5, parts of Q2
	Review answers	
	Mental Test	M 4.1
	Review answers	
	Set homework	PB 4.5, Q11
8.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 4.1
9.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 4 *Fractions and Percentages* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
1.	**Equivalent Fractions**	
	Revise concept	OS 4.1
	Exercises	PB 4.1, Q1
	Review answers	
	Finding equivalent fractions	OS 4.2
	Exercises	PB 4.1, Q2
	Review answers	
	Activity	Activity 4.1 (or Activity 10.5 from Y7A)
	Set homework	PB 4.1, Q3 and Q6
2.	**Fractions of Quantities**	
	Discuss homework	
	Examples - interactively	OS 4.3
	Exercises	PB 4.2, Q1
	Review answers	
	Exercises	PB 4.2, Q2
	Review answers	
	Examples in context	PB 4.2, Q4, Q5 or Q6
	Set homework	PB 4.2, Q7 and Q11
3.	**Operations with Fractions: Addition and Subtraction**	
	Discuss homework	
	Addition of fractions	OS 4.4
	Exercises	PB 4.3, Q1
	Review answers	
	Subtraction of fractions	OS 4.4
	Exercises	PB 4.3, Q2
	Review answers	
	Set homework	PB 4.3, Q7 and Q11
4.	**Operations with Fractions: Multiplication and Division**	
	Discuss homework	
	Multiplication of fractions	OS 4.5
	Exercises	PB 4.3, parts of Q3
	Review answers	
	Division of fractions	OS 4.6
	Exercises	PB 4.3, parts of Q4
	Review answers	
	Questions in context	PB 4.3, Q10
	Review answers	
	Set homework	PB 4.3, Q11 and Q12

UNIT 4 *Fractions and Percentages* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
5.	**Fraction, Decimal and Percentage Equivalents**	
	Discuss homework	
	Introduction	OS 4.7
	Exercises - interactively	PB 4.4, choose from Q1-Q6
	Activity	Activity 4.2 or Activity 4.3 or Activity 9.1 from Y8A
	Set homework	PB 4.4, Q11 and Q12
6.	**Percentage Increase/Decrease**	
	Discuss homework	
	Introduction	OS 4.8
	Exercises	PB 4.5, Q11
	Review answers	
	Exercises	PB 4.5, Q12
	Review answers	
	Mental test	M 4.2
	Review answers	
	Set homework	PB 4.5, Q13 and Q14
7.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 4.2
8.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 4 *Fractions and Percentages* Lesson Plans **E**

Lesson No.	*Suggested Plan*	*References*
1.	**Operations with Fractions**	
	Addition and subtraction	OS 4.4
	Exercises	PB 4.3, Q12
	Review answers	
	Multiplication and division of fractions	OS 4.5 and OS 4.6
	Exercises	PB 4.3, Q8
	Review answers	
	Set homework	PB 4.3, Q10 or Activity 9.1 from Y8A
2.	**Fraction, Decimal and Percentage Equivalents**	
	Discuss homework	
	Introduction	OS 4.7
	Exercises - interactively	PB 4.4, choose from Q1-Q6
	Activity	Activity 4.2 or Activity 4.3 or Activity 9.1 from Y8A
	Set homework	PB 4.4, Q11 and Q12
3.	**Percentage Increase and Decrease**	
	Discuss homework	
	Introduction	OS 4.8
	Exercises	PB 4.5, Q12
	Review answers	
	Exercises	PB 4.5, Q13
	Review answers	
	Mental Test	M 4.3
	Review answers	
	Set homework	PB 4.5, Q14 and Q15
4.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 4.3
5.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 4 Fractions and Percentages Mental Tests

M 4.1 Standard Route *(no calculator)*

1. What is $\frac{1}{2}$ of 90 ? (45)
2. What is $\frac{3}{4}$ of 20 ? (15)
3. What is $\frac{1}{10}$ of 90 ? (9)
4. What is $\frac{3}{10}$ of 20 ? (6)
5. Write $\frac{4}{8}$ in its simplest form. $(\frac{1}{2})$
6. Write $\frac{6}{9}$ in its simplest form. $(\frac{2}{3})$
7. Write $\frac{2}{10}$ in its simplest form. $(\frac{1}{5})$
8. Calculate $\frac{1}{5}+\frac{2}{5}$. $(\frac{3}{5})$
9. Write 75% as a fraction in its simplest form. $(\frac{3}{4})$
10. Write 0.2 as a fraction in its simplest form. $(\frac{1}{5})$

UNIT 4 Fractions and Percentages Mental Tests

M 4.2 Academic Route *(no calculator)*

1. What is $\frac{1}{5}$ of 40 ? (8)

2. What is $\frac{3}{4}$ of 80 ? (60)

3. Write $\frac{5}{100}$ in its simplest form. $(\frac{1}{20})$

4. Write $\frac{3}{12}$ in its simplest form. $(\frac{1}{4})$

5. Calculate $\frac{2}{7} + \frac{3}{7}$. $(\frac{5}{7})$

6. Calculate $\frac{1}{2} + \frac{1}{4}$. $(\frac{3}{4})$

7. Calculate $\frac{1}{2} - \frac{1}{4}$. $(\frac{1}{4})$

8. Write 60% as a fraction in its simplest form. $(\frac{3}{5})$

9. Write 0.4 as a fraction in its simplest form. $(\frac{2}{5})$

10. Increase £300 by 20%. (£360)

UNIT 4 Fractions and Percentages Mental Tests

M 4.3 Express Route *(no calculator)*

1. What is $\frac{2}{5}$ of 90 ? (36)

2. What is $\frac{3}{4}$ of $\frac{1}{2}$? $(\frac{3}{8})$

3. Write $\frac{14}{49}$ in its simplest form. $(\frac{2}{7})$

4. Write $\frac{80}{500}$ in its simplest form. $(\frac{4}{25})$

5. Calculate $\frac{7}{9}+\frac{2}{3}$. $\left(\frac{13}{9}=1\frac{4}{9}\right)$

6. Calculate $\frac{1}{3}-\frac{1}{12}$. $\left(\frac{3}{12}=\frac{1}{4}\right)$

7. Calculate $\frac{1}{3}\times\frac{2}{5}$. $(\frac{2}{15})$

8. Write 64% as a fraction in its simplest form. $(\frac{16}{25})$

9. Write 0.45 as a fraction in its simplest form. $(\frac{9}{20})$

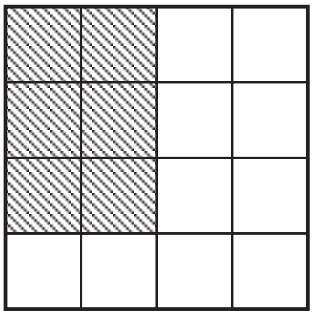
10. Decrease £300 by 15%. (£255)

UNIT 4 *Fractions and Percentages* Overhead Slides

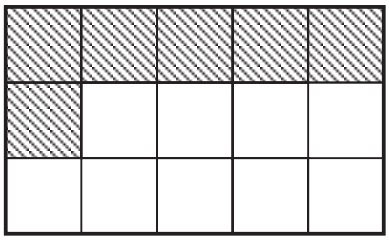
Overhead Slides

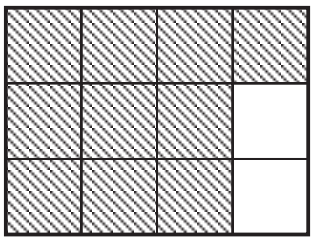
OS 4.1 *Equivalent Fractions 1*

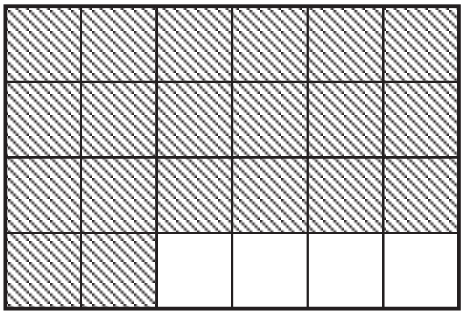
Write down, in two equivalent ways, the fraction of each shape that has been shaded:



$\frac{\quad}{16} = \frac{\quad}{8}$







OS 4.2 *Equivalent Fractions 2*

Complete the following statements:

1. $\frac{3}{4} = \frac{\square}{20} = \frac{\square}{40} = \frac{\square}{100}$

2. $\frac{5}{6} = \frac{\square}{24} = \frac{\square}{120} = \frac{\square}{600}$

3. $\frac{7}{8} = \frac{\square}{16} = \frac{\square}{80} = \frac{\square}{400}$

4. $\frac{2}{3} = \frac{\square}{21} = \frac{\square}{300} = \frac{\square}{3000}$

5. $\frac{11}{20} = \frac{\square}{100} = \frac{\square}{140} = \frac{\square}{160}$

6. $\frac{4}{5} = \frac{12}{\square} = \frac{28}{\square} = \frac{44}{\square}$

OS 4.3 *Fractions of Quantities*

Complete the following calculations:

1. $\frac{1}{5}$ of 20 =

   $\frac{3}{5}$ of 20 =

2. $\frac{1}{8}$ of £32 =

   $\frac{5}{8}$ of £32 =

3. $\frac{1}{5}$ of 85 kg =

   $\frac{4}{5}$ of 85 kg =

4. $\frac{1}{7}$ of 42 m =

   $\frac{5}{7}$ of 42 m =

OS 4.4 *Addition and Subtraction of Fractions*

Complete the following calculations:

1. $\frac{1}{2} + \frac{3}{5} = \frac{\square}{10} + \frac{\square}{10}$

$= \frac{\square}{10}$

---

2. $\frac{3}{5} + \frac{2}{3} = \frac{\square}{15} + \frac{\square}{\square}$

$= \frac{\square}{\square}$

---

3. $\frac{4}{7} + \frac{3}{4} = \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square}$

---

4. $\frac{5}{8} - \frac{2}{5} = \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square}$

OS 4.5 *Multiplication of Fractions*

Complete each of the following calculations:

1. $\frac{3}{5} \times \frac{1}{2} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

2. $\frac{4}{5} \times \frac{3}{7} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

3. $\frac{3}{8} \times \frac{2}{9} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

4. $1\frac{1}{2} \times 1\frac{1}{3} = \frac{\square}{2} \times \frac{\square}{3} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

OS 4.6 *Division of Fractions*

Complete each of the following calculations:

1. $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{\square}{\square}$

$= \frac{\square \times \square}{\square \times \square}$

$= \frac{\square}{\square}$

2. $\frac{4}{5} \div \frac{3}{7} = \frac{\square \times \square}{\square \times \square}$

$= \frac{\square \times \square}{\square \times \square}$

$= \frac{\square}{\square}$

3. $2\frac{1}{2} \div 1\frac{1}{2} = \frac{\square}{2} \div \frac{\square}{2}$

$= \frac{\square}{2} \times \frac{\square}{\square}$

$= \frac{\square \times \square}{\square \times \square}$

$= \frac{\square}{\square}$

OS 4.7 *Fractions, Decimals and Percentages*

Complete this table:

Fraction	*Decimal*	*Percentage*
	0.5	
		5%
$\frac{1}{4}$		
	0.18	
		20%
	0.34	
$\frac{6}{25}$		
	0.04	
$\frac{1}{8}$		

OS 4.8 *Percentage Increases and Decreases*

1. *Increase* 80 by 15%.

15% of 80 = $\square$ × 80

= $\square$

80 + $\square$ = $\square$

2. *Increase* 200 by 30%.

30% of 200 = $\square$ × 200

= $\square$

200 + $\square$ = $\square$

3. *Decrease* 350 by 8%.

8% of 350 = $\square$ × 350

= $\square$

350 − $\square$ = $\square$

Practice Book *UNIT 4 Fractions and Percentages* Answers

4.1 Equivalent Fractions

1. (a) $\frac{9}{12} = \frac{3}{4}$ (b) $\frac{4}{10} = \frac{2}{5}$ (c) $\frac{6}{8} = \frac{3}{4}$ (d) $\frac{9}{15} = \frac{3}{5}$

2. (a) $\frac{12}{20}$ (b) $\frac{9}{12}$ (c) $\frac{20}{35}$ (d) $\frac{10}{18}$ (e) $\frac{12}{28}$
   (f) $\frac{15}{40}$ (g) $\frac{24}{30}$ (h) $\frac{8}{36}$ (i) $\frac{54}{60}$ (j) $\frac{16}{28}$
   (k) $\frac{42}{66}$ (l) $\frac{40}{64}$

3. (a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{5}{8}$ (e) $\frac{5}{6}$
   (f) $\frac{1}{5}$ (g) $\frac{1}{4}$ (h) $\frac{3}{4}$ (i) $\frac{7}{50}$ (j) $\frac{3}{7}$

4. (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{1}{6}$ (e) $\frac{1}{5}$
   (f) $\frac{2}{5}$ (g) $\frac{2}{5}$ (h) $\frac{4}{5}$ (i) $\frac{3}{4}$ (j) $\frac{2}{3}$
   (k) $\frac{5}{7}$ (l) $\frac{4}{5}$

5. (a) Any two acceptable answers, e.g. $\frac{4}{14}$ and $\frac{6}{21}$
   (b) Any two acceptable answers, e.g. $\frac{6}{16}$ and $\frac{9}{24}$
   (c) Any two acceptable answers, e.g. $\frac{10}{18}$ and $\frac{15}{27}$

6. (a) False, because $\frac{4}{7} = \frac{12}{21}$ or because $\frac{4}{7} = \frac{16}{28}$
   (b) True (c) True (d) True

7. (a) $\frac{4}{5} = \frac{32}{40}$ $\frac{5}{8} = \frac{25}{40}$ (b) $\frac{4}{5}$ is larger than $\frac{5}{8}$

8. (a) $\frac{5}{7} = \frac{15}{21}$ $\frac{2}{3} = \frac{14}{21}$ (b) $\frac{2}{3}$ is smaller than $\frac{5}{7}$

9. $\frac{1}{2} = \frac{35}{70}$ $\frac{3}{5} = \frac{42}{70}$ $\frac{4}{7} = \frac{40}{70}$ so $\frac{3}{5}$ is the largest.

10. $\frac{1}{5} = \frac{36}{180}$ $\frac{1}{4} = \frac{45}{180}$ $\frac{2}{9} = \frac{40}{180}$ $\frac{1}{2} = \frac{90}{180}$ $\frac{5}{9} = \frac{100}{180}$ so the correct order is $\frac{1}{5}, \frac{2}{9}, \frac{1}{4}, \frac{1}{2}, \frac{5}{9}$

4.2 Answers

4.2 Fractions of Quantities

1. (a) 2 (b) 4 (c) 5 (d) 4 (e) 4
   (f) 6 (g) 2 (h) 8 (i) 4

2. (a) 15 (b) 6 (c) 9 (d) 16 (e) 12
   (f) 24 (g) 20 (h) 24 (i) 20

3. 14

4. (a) 110 ml (b) 220 ml

5. (a) 120 (b) 200

6. (a) 72 miles (b) 48 miles

7. (a) 180 (b) 120

8. (a) £12 (b) £5 (c) £13

9. 108 hectares

10. 6 disks

11. (a) 3 biscuits (b) $\frac{1}{2}$ (c) 15

4.3 Operations with Fractions

1. (a) $\frac{5}{7}$ (b) 1 (c) $\frac{4}{10} = \frac{2}{5}$
   (d) $\frac{4}{5}$ (e) $\frac{6}{9} = \frac{2}{3}$ (f) 1

2. (a) $\frac{5}{6}$ (b) $\frac{9}{20}$ (c) $\frac{10}{21}$ (d) $1\frac{3}{20}$
   (e) $\frac{29}{56}$ (f) $\frac{5}{6}$ (g) $1\frac{5}{12}$ (h) $1\frac{4}{15}$
   (i) $\frac{34}{35}$ (j) $1\frac{1}{2}$ (k) $\frac{19}{24}$ (l) $1\frac{19}{30}$

4.3 Answers

3. (a) $\frac{1}{6}$ (b) $\frac{8}{15}$ (c) $\frac{1}{12}$ (d) $\frac{5}{8}$

(e) $\frac{4}{7}$ (f) $\frac{3}{32}$ (g) $\frac{2}{5}$ (h) $\frac{1}{2}$

(i) $\frac{5}{12}$ (j) $\frac{2}{7}$ (k) $\frac{3}{8}$ (l) $\frac{7}{12}$

4. (a) $1\frac{1}{2}$ (b) $1\frac{1}{8}$ (c) $1\frac{1}{5}$ (d) $1\frac{2}{3}$

(e) $\frac{4}{7}$ (f) $\frac{5}{6}$ (g) $\frac{2}{5}$ (h) $\frac{14}{15}$

(i) $\frac{5}{7}$ (j) $\frac{2}{3}$ (k) $\frac{7}{16}$ (l) $1\frac{1}{6}$

5. (a) $3\frac{3}{8}$ (b) $3\frac{1}{3}$ (c) $4\frac{1}{12}$ (d) $4\frac{1}{3}$

(e) $3\frac{3}{4}$ (f) $1\frac{4}{5}$

6. Area $= \frac{3}{10}$ m^2 Perimeter $= 2\frac{3}{10}$ m^2

7. $\frac{1}{6}$ acre

8. $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$ is larger than $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

9. (a) $x = \frac{2}{3}$ (b) $x = 3\frac{3}{4}$

10. $2\frac{1}{4}$ m

11. (a) $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{4}{16} + \frac{2}{16} + \frac{1}{16} = \frac{7}{16}$ (b) £60

12. (a) He was wrong because $\frac{1}{3}$ of the square is smaller than $\frac{1}{3}$ of the rectangle. He actually added on $\frac{1}{2}$ of the square.

(b) $\frac{1}{3}$ (c) $\frac{1}{7}$ (d) $\frac{1}{n-1}$

4.4 Answers

4.4 Fraction, Decimal and Percentage Equivalents

1. (a) 0.6 (b) 0.7 (c) 0.2 (d) 0.45
   (e) 0.31 (f) 0.82 (g) 0.14 (h) 0.04
   (i) 0.63 (j) 0.02 (k) 0.01 (l) 0.19

2. (a) $\frac{4}{5}$ (b) $\frac{1}{4}$ (c) $\frac{2}{5}$ (d) $\frac{7}{20}$
   (e) $\frac{13}{20}$ (f) $\frac{1}{25}$ (g) $\frac{16}{25}$ (h) $\frac{41}{50}$
   (i) $\frac{7}{25}$ (j) $\frac{3}{50}$ (k) $\frac{7}{100}$ (l) $\frac{23}{25}$

3. (a) 74% (b) 99% (c) 50% (d) 6% (e) 26%
   (f) 2% (g) 30% (h) 0.2% (i) 4.2%

4. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{2}{5}$ (d) $\frac{7}{10}$
   (e) $\frac{31}{50}$ (f) $\frac{11}{25}$ (g) $\frac{37}{100}$ (h) $\frac{1}{25}$
   (i) $\frac{1}{20}$ (j) $\frac{6}{25}$ (k) $\frac{1}{10}$ (l) $\frac{37}{50}$

5. (a) 0.5 (b) 0.75 (c) 0.8 (d) 0.45
   (e) 0.7 (f) 0.03 (g) 0.19 (h) 0.46
   (i) 0.28 (j) 0.32 (k) 0.125 (l) 0.625

6. (a) 90% (b) 17% (c) 56% (d) 15%
   (e) 40% (f) 60% (g) 45% (h) 9%
   (i) 1% (j) 6% (k) 87.5% (l) 3.5%

4.4 Answers

7.

Fraction	*Decimal*	*Percentage*
$\frac{4}{5}$	0.8	80%
$\frac{17}{25}$	0.68	68%
$\frac{17}{20}$	0.85	85%
$\frac{19}{25}$	0.76	76%
$\frac{8}{25}$	0.32	32%
$\frac{3}{100}$	0.03	3%
$\frac{1}{200}$	0.005	0.5%

8. Labour 55%
Conservative 40%
Other 5%

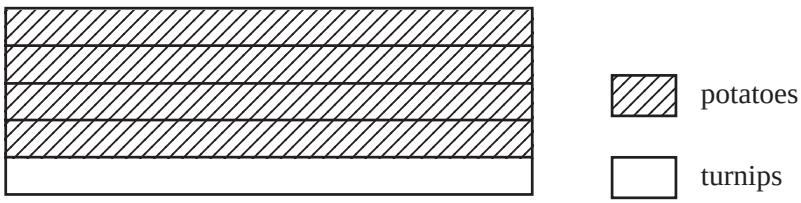
9. (a) 2.5% (b) 97.5%

10. 82.5%

11. (a) (i) 30% (ii) 20%

(b) (i) $\frac{1}{2}$ (ii) $\frac{3}{8}$

(c)



About $\frac{1}{5}$ of the patch is for turnips.

12. (a) $\frac{3}{10}$ is shaded. 30% is shaded.

(b) Any 4 triangles shaded (or the equivalent)

40% is shaded.

4.5 Percentage Increases and Decreases

1. (a) £120 (b) £520 (c) £100 (d) £53
   (e) 41.2 kg (f) 267.5 m

2. (a) £42 (b) 7.6 m (c) 72 kg (d) £35.20
   (e) 88.2 m (f) 315 kg

3. (a) £63 (b) £15.75 (c) £25.20

4. £30.90

5. John Smith £24 960
   Alice Holland £23 400
   Graham Hall £14 560
   Emma Graham £8 840

6. £52.50

7. (a) £235 (b) £82.25 (c) £49.35

8. (a) 7.84 m (b) 7.44 m (c) 7.04 m

9. Coat £18.20 Jeans £27.30
   Trainers £23.40 Shirt £9.10

10. £155.52

11. (a) 10% of 240 is 24
    5% of 240 is 12
    $2\frac{1}{2}\%$ of 240 is 6
    $17\frac{1}{2}\%$ of £240 is £42

    (b) 10% of 250 is 25
    10% of 250 is 25
    10% of 250 is 25
    5% of 250 is 12.5
    35% of £250 is £87.50

12. (a) $100\% - 26.1\% = 73.9\%$

    73.9% of $17.6 = \frac{73.9}{100} \times 17.6 = 13.0064$

    (b) Fall $= 17.0 - 12.2 = 4.8$

    Percentage fall $= \frac{4.8}{17.0} \times 100 = 28.2\%$ (to 1 decimal place)

4.5 Answers

13. (a) South America

(b) $\frac{13209}{148355} \times 100 = 8.9\%$ (to 1 decimal place)

(c)
$$\begin{aligned} 30\% &= 150 \text{ million km}^2 \\ 1\% &= 5 \text{ million km}^2 \\ \text{Total area } 100\% &= 500 \text{ million km}^2 \end{aligned}$$

14. (a) In 1996 they employed 10% more people, and 10% of 4000 = 400, so Tony was correct about 1996.

The workforce increased to 4400 in 1996, so in 1997 they employed an extra 10% of 4400, i.e. an extra 440, bringing the workforce to 4840. So Tony was wrong about 1997.

(b) Calculation (v) 4000×1.1^2

(c) Number of employees in 1997 $= n \times 0.8 \times 1.1 = 0.88n$

15. (a) £19.95 × 0.85 × 0.85 = £14.413875 = £14.41 (to nearest penny)

(b) £41.48 ÷ 0.85 = £48.80

(c) $£49.95 \times 0.85 \times 0.85 \times 0.85 \times 0.85 = £49.95 \times 0.85^4$

= £26.07 (to nearest penny)

(d) Each day the price is 88% if the previous day's price.

For 2 days, the price is $0.88^2 = 77.44\%$ of the original price

For 3 days, the price is $0.88^3 = 68.15\%$ of the original price

For 4 days, the price is $0.88^4 = 59.97\%$ of the original price

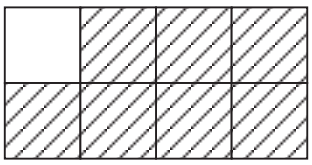
For 5 days, the price is $0.88^5 = 52.77\%$ of the original price

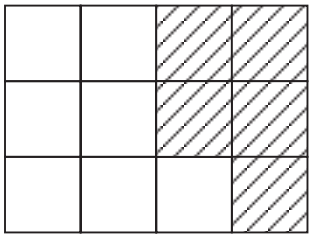
For 6 days, the price is $0.88^6 = 46.44\%$ of the original price

So it takes 6 days for the original prices to be reduced by more than 50%.

UNIT 4 *Fractions and Percentages* Revision Test 4.1 (Standard)

1. What fraction of each of the following rectangles is shaded:

(a) 

(b) 

(2 marks)

2. Copy each of the following statements and fill in the missing numbers:

(a) $\frac{1}{2} = \frac{\square}{8}$ (b) $\frac{2}{3} = \frac{\square}{9}$ (c) $\frac{4}{5} = \frac{\square}{25}$

(3 marks)

3. Write each of the following fractions in its simplest form:

(a) $\frac{6}{10}$ (b) $\frac{10}{16}$ (c) $\frac{6}{24}$

(6 marks)

4. Calculate the following, in each case giving your answer in its simplest form:

(a) $\frac{2}{7} + \frac{4}{7}$ (b) $\frac{7}{10} + \frac{1}{10}$ (c) $\frac{5}{8} + \frac{1}{8}$

(d) $\frac{1}{2} + \frac{1}{4}$ (e) $\frac{1}{5} + \frac{3}{10}$

(10 marks)

5. Calculate:

(a) $\frac{1}{2} \times 38$ (b) $\frac{2}{3} \times 60$ (c) $\frac{2}{5} \times 40$

(6 marks)

6. Convert the following decimals to fractions in their simplest form:

(a) 0.5 (b) 0.64

(3 marks)

UNIT 4 *Fractions and Percentages* Revision Test 4.2 (Academic)

1. Write each of the following fractions in its simplest form:

   (a) $\frac{4}{12}$ (b) $\frac{18}{72}$ (c) $\frac{42}{98}$

   (6 marks)

2. Calculate the following, giving each of your answers in its simplest form and as a mixed number if possible:

   (a) $\frac{3}{8}+\frac{1}{4}$ (b) $\frac{2}{3}+\frac{4}{5}$ (c) $\frac{7}{9}+\frac{3}{4}$

   (d) $\frac{2}{5}\times\frac{15}{11}$ (e) $\frac{4}{5}\times 85$ (f) $\frac{1}{2}\div\frac{3}{8}$

   (12 marks)

3. Write each of the following percentages as a fraction in its simplest form:

   (a) 70% (b) 38%

   (2 marks)

4. Write each of the following fractions as a percentage:

   (a) $\frac{3}{20}$ (b) $\frac{17}{25}$

   (4 marks)

5. Increase 420 by 30%.

   (3 marks)

6. A coat is normally sold for £40. In a sale its price is reduced by 15%. Calculate the sale price.

   (3 marks)

UNIT 4 *Fractions and Percentages* Revision Test 4.3 (Express)

1. Write each of the following fractions in its simplest form:

   (a) $\frac{42}{98}$ (b) $\frac{256}{400}$

   (4 marks)

2. Calculate the following, giving each of your answers in its simplest form and as a mixed number if possible:

   (a) $\frac{7}{9}+\frac{3}{4}$ (b) $\frac{2}{7}+\frac{3}{8}$ (c) $\frac{4}{5}\times 85$

   (d) $\frac{2}{5}\times\frac{9}{10}$ (e) $1\frac{1}{2}\times 1\frac{3}{4}$ (f) $2\frac{1}{2}\div 1\frac{2}{7}$

   (12 marks)

3. Write each of the following percentages as a fraction in its simplest form:

   (a) 38% (b) 8.25%

   (4 marks)

4. In a sale the prices of all items are decreased by 15%. Calculate the sale price of a coat that cost £52 before the sale.

   (3 marks)

5. Add $17\frac{1}{2}$% VAT to a price of £44.

   (3 marks)

6. Hannah invests £60 in a bank. She is paid 6% interest each year. How much does she have in the bank after 2 years?

   (4 marks)

Revision Test 4.1 (Standard) Answers

1.	(a)	$\frac{7}{8}$	B1	
	(b)	$\frac{5}{12}$	B1	*(2 marks)*
2.	(a)	$\frac{1}{2} = \frac{4}{8}$	B1	
	(b)	$\frac{2}{3} = \frac{6}{9}$	B1	
	(c)	$\frac{4}{5} = \frac{20}{25}$	B1	*(3 marks)*
3.	(a)	$\frac{3}{5}$	B2	
	(b)	$\frac{5}{8}$	B2	
	(c)	$\frac{1}{4}$	B2	*(6 marks)*
4.	(a)	$\frac{6}{7}$	B2	
	(b)	$\frac{8}{10} = \frac{4}{5}$	M1 A1	
	(c)	$\frac{6}{8} = \frac{3}{4}$	M1 A1	
	(d)	$\frac{3}{4}$	B2	
	(e)	$\frac{5}{10} = \frac{1}{2}$	M1 A1	*(10 marks)*
5.	(a)	19	B2	
	(b)	40	B2	
	(c)	16	B2	*(6 marks)*
6.	(a)	$\frac{1}{2}$	B1	
	(b)	$\frac{64}{100} = \frac{16}{25}$	B2	*(3 marks)*

(TOTAL MARKS 30)

Revision Test 4.2 (Academic) Answers

1.	(a)	$\frac{1}{3}$	B2	
	(b)	$\frac{1}{4}$	B2	
	(c)	$\frac{3}{7}$	B2	*(6 marks)*
2.	(a)	$\frac{5}{8}$	B2	
	(b)	$\frac{22}{15} = 1\frac{7}{15}$	B2	
	(c)	$\frac{55}{36} = 1\frac{19}{36}$	B2	
	(d)	$\frac{6}{11}$	B2	
	(e)	68	B2	
	(f)	$\frac{4}{3} = 1\frac{1}{3}$	B2	*(12 marks)*
3.	(a)	$\frac{7}{10}$	B1	
	(b)	$\frac{19}{50}$	B1	*(2 marks)*
4.	(a)	15%	B2	
	(b)	68%	B2	*(4 marks)*
5.	$420 + 126 = 546$		M1 A1 A1	*(3 marks)*
6.	£40 − £6 = £34		M1 A1 A1	*(3 marks)*

(TOTAL MARKS 30)

Revision Test 4.3 (Express) Answers

1.	(a)	$\frac{3}{7}$	B2	
	(b)	$\frac{16}{25}$	B2	*(4 marks)*
2.	(a)	$1\frac{19}{36}$	B2	
	(b)	$\frac{37}{56}$	B2	
	(c)	68	B2	
	(d)	$\frac{9}{25}$	B2	
	(e)	$2\frac{5}{8}$	B2	
	(f)	$1\frac{17}{18}$	B2	*(12 marks)*
3.	(a)	$\frac{19}{50}$	B2	
	(b)	$\frac{33}{400}$	B2	*(4 marks)*
4.	15% of £52 = £7.80		B2	
	£52.00 − £7.80 = £44.20		B1	*(3 marks)*
5.	$17\frac{1}{2}$% of £44 = £7.70		B2	
	£44.00 + £7.70 = £51.70		B1	*(3 marks)*
6.	Interest after Year 1 = £3.60		B1	
	Interest after Year 2 = £3.82		B1	
	Total after 2 years = £60 + £3.60 + £3.82		M1	
		= £67.42 to nearest pence	A1	*(4 marks)*
				(TOTAL MARKS 30)

UNIT 4 *Fractions and Percentages* Teaching Notes

Historical Background and Introduction

In this unit we revise the work on *fractions* and *percentages* that was covered in Y7 and Y8. It is important that you ensure that the concepts are understood and well practised, focusing particularly on the typical KS3 questions in the Exercises.

Routes		**Standard**	**Academic**	**Express**
4.1	Equivalent Fractions	✓	(✓)	×
4.2	Fractions of Quantities	✓	✓	×
4.3	Operations with Fractions	(✓)	✓	(✓)
4.4	Fraction, Decimal and Percentage Equivalents	✓	✓	(✓)
4.5	Percentage Increases and Decreases	(✓)	✓	(✓)

Language

No new language is introduced in this unit.

Misconceptions

- that 5% is 0.5 (not, as is correct, 0.05)
- that 5% as a fraction is $\frac{1}{5}$ (not, as is correct, $\frac{1}{20}$)
- in general, $\frac{1}{a}+\frac{1}{b}\neq\frac{1}{a+b}$ or $\frac{2}{a+b}$, but $=\left(\frac{a+b}{ab}\right)$
- pupils often have difficulty understanding that the fraction $\frac{1}{8}$ as a decimal, is 0.125, and as a percentage is 12.5%
- pupils must understand that if the price of a product is decreased by 10% and then increased by 10%, the final price will *not* be the same as the original price

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9A	4.1	9, 10	61
" "	4.3	11, 12	68-69
" "	4.5	14, 15	79-80

UNITS 1-3 Diagnostic Test 9A (Standard)

You have ONE HOUR to complete this test.
You must NOT use a calculator.

1. Convert each of the following binary numbers to base 10:

(a) 101 = ☐ (b) 1100 = ☐ (c) 10011 = ☐

(5 marks)

2. Convert each of the following base 10 numbers to binary numbers:

(a) 6 = ☐ (b) 17 = ☐ (c) 27 = ☐

(5 marks)

3. (a) Here is one way to start with 15 and end with 90 on a calculator.

15	+	75	=	90

Fill in the boxes to show a way to start with 90 and end with 15 on a calculator.

90	☐	☐	=	15

(b) Here is another way to start with 15 and end with 90 on a calculator.

15	×	6	=	90

Fill in the boxes to show another way to start with 90 and end with 15 on a calculator.

90	☐	☐	=	15

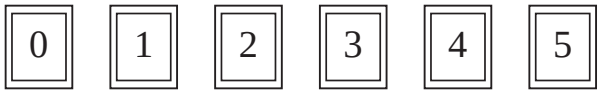
[KS3/95/Ma/Levels 3-5/P1]

(4 marks)

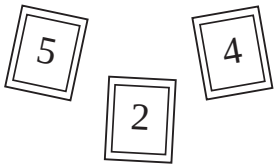
UNITS 1-3

Diagnostic Test 9A (Standard)

4. Here are some number cards:



Samira picked these three cards:



She made the number 425 with her cards.

(a) Make a *smaller* number with Samira's three cards.

(b) Make the *biggest* number you can with Samira's cards.

(c) Samira made the number 425 with her cards.

What extra card should she pick to make her number 10 times as big?

What number is 10 times as big as 425 ?

[KS3/94/Ma/3-5/P2]

(4 marks)

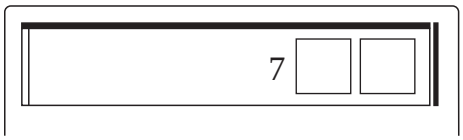
UNITS 1-3

Diagnostic Test 9A (Standard)

5. (a) Claire puts a 2 digit whole number into her calculator.

She *multiplies* the number by 10.

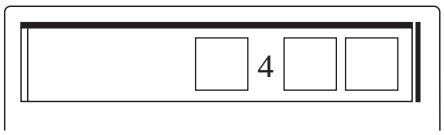
Fill in *one* digit which you *know* must be on the calculator display.



(b) Claire starts again with the *same* 2 digit whole number.

This time she *multiplies* it by 100.

Fill in *all* the digits that must be on the calculator display.



[KS3/95/Ma/Levels 4-6/P2]

(2 marks)

6. Lee has these cards:

He made the number 32.4 with four of his cards.

(a) Use some of Lee's cards to show the number *10 times* as big as 32.4.

(b) Use some of Lee's cards to show the number *100 times* as big as 32.4.

[KS3/94/Ma/3-5/P2]

(2 marks)

UNITS 1-3

Diagnostic Test 9A (Standard)

7. Here are the ingredients for 1 fruit cake:

1 Fruit Cake	
200 g	self-raising flour
100 g	castor sugar
150 g	margarine
125 g	mixed fruit
3	eggs

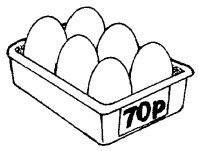
(a) Complete the table to show how much of each ingredient you need to make 10 fruit cakes.

Give your answers in grams *and* in kilograms.

10 Fruit Cakes

2000	g =	2	kg	self-raising flour
	g =		kg	castor sugr
	g =		kg	margarine
	g =		kg	mixed fruit
	30	eggs		

(b) 6 eggs cost 70p.



How much will 30 eggs cost? £

[KS3/98/Ma/Tier3-5/P1]

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Standard)

8. Tom has baked 50 small cakes for the school fair.

He wants to put them into bags of 4.

Tom thinks he will need 200 bags.

(a) You can tell that Tom must be wrong *before* you work out the right answer.
Explain why Tom must be wrong.

(b) Work out how many bags of 4 cakes Tom will have.

☐ bags

(c) How many cakes will Tom have left over?

☐ cakes

(d) Lela needs 27 cakes for a party.
Tom sells cakes in bags of 4.
How many bags must Lela buy?

☐ bags

[KS3/94/Ma/3-5/P2]

(4 marks)

9. Calculate

(a) 5^2 = ☐ (b) 3^4 = ☐ (c) 10^4 = ☐

(6 marks)

UNITS 1-3

Diagnostic Test 9A (Standard)

10. Complete each of the following statements, filling in the missing numbers:

(a) $49 = 7^{\square}$

(b) $1000 = 10^{\square}$

(c) $2^3 \times 2^2 = 2^{\square}$

(d) $4^3 \times 4^2 = 4^{\square}$

(e) $64 = \square^3$

(f) $5^4 \div 5^2 = 5^{\square}$

(10 marks)

11. Calculate:

(a) $2^3 + 3^2 = \square$

(b) $10^3 + 10^2 = \square$

(4 marks)

UNITS 1-3 Diagnostic Test 9A (Standard) Answers

1.	(a)	5	B1	
	(b)	$8 + 4 = 12$	M1 A1	
	(c)	$16 + 2 + 1 = 19$	M1 A1	*(5 marks)*
2.	(a)	110	B1	
	(b)	10001	B2	
	(c)	11011	B2	*(5 marks)*
3.	(a)	[−] [75] or [÷] [6]	B1 B1	
	(b)	As (a)	B1 B1	*(4 marks)*
4.	(a)	[2] [4] [5] or [2] [5] [4]	B1	
	(b)	[5] [4] [2]	B1	
	(c)	[0] ; 4250	B1 B1	*(4 marks)*
5.	(a)	7 [ ] [0]	B1	
	(b)	[7] 4 [0] [0]	B1	*(2 marks)*
6.	(a)	[3] [2] [4]	B1	
	(b)	[3] [2] [4] [0]	B1	*(2 marks)*
7.		1000 g = 1 kg castor sugar	B1	
		1500 g = 1.5 kg margarine	B1	
		1250 g = 1.25 kg mixed fruit	B1	
		£3.50	B1	*(4 marks)*
8.	(a)	200 is greater than 50	B1	
	(b)	12	B1	
	(c)	2	B1	
	(d)	7	B1	*(4 marks)*
9.	(a)	25 (b) 81 (c) 10 000	B2 B2 B2	*(6 marks)*

UNITS 1-3 Diagnostic Test 9A (Standard) Answers

10.	(a)	2	B1	
	(b)	3	B1	
	(c)	5	B2	
	(d)	5	B2	
	(e)	4	B2	
	(f)	2	B2	*(10 marks)*
11.	(a)	$8 + 9 = 17$	M1 A1	
	(b)	$1000 + 100 = 1100$	M1 A1	*(4 marks)*

(TOTAL MARKS 50)

UNITS 1-3 Diagnostic Test 9A (Standard) Answers

Marks

Unit	**1**	**2**	**3**	
Question	1 - 2	3 - 8	9 - 11	
Total marks available	10	20	20	*Final total*
Total				

Asessment

45 +	Excellent – should be on *Academic Route*
40 - 44	Very good progress
30 - 39	Good progress: look carefully at mistakes
20 - 29	Steady progress, but you will need to work more carefully and/or make more effort
- 19	Struggling, so look carefully at your weak points to see where to target extra work

UNITS 1-3 Diagnostic Test 9A (Academic)

You have ONE HOUR to complete this test.
You must NOT use a calculator.

1. Convert each of the following binary numbers to base 10:

   (a) 10011 = ☐ (b) 1100010 = ☐

   (4 marks)

2. Convert each of the following base 10 numbers to binary numbers:

   (a) 55 = ☐ (b) 138 = ☐

   (4 marks)

3. In binary, calculate 1011×11, showing your working.

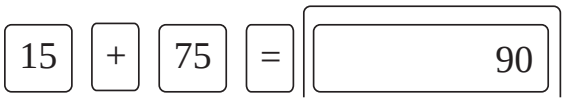
   = ☐

   (2 marks)

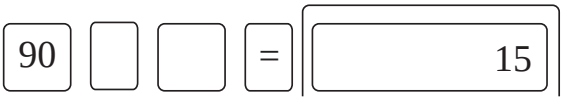
UNITS 1-3

Diagnostic Test 9A (Academic)

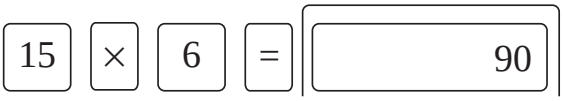
4. (a) Here is one way to start with 15 and end with 90 on a calculator.



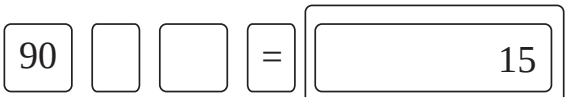
Fill in the boxes to show a way to start with 90 and end with 15 on a calculator.



(b) Here is another way to start with 15 and end with 90 on a calculator.



Fill in the boxes to show another way to start with 90 and end with 15 on a calculator.



[KS3/95/Ma/Levels 3-5/P1]

(4 marks)

5. Lee has these cards:

He made the number 32.4 with four of his cards.

(a) Use some of Lee's cards to show the number *10 times* as big as 32.4.

(b) Use some of Lee's cards to show the number *100 times* as big as 32.4.

[KS3/94/Ma/3-5/P2]

(2 marks)

UNITS 1-3

Diagnostic Test 9A (Academic)

6. Here are the ingredients for 1 fruit cake:

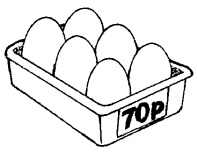
1 Fruit Cake	
200 g	self-raising flour
100 g	castor sugar
150 g	margarine
125 g	mixed fruit
3	eggs

(a) Complete the table to show how much of each ingredient you need to make 10 fruit cakes.

Give your answers in grams *and* in kilograms.

10 Fruit Cakes			
2000	g =	2	kg self-raising flour
	g =		kg castor sugar
	g =		kg margarine
	g =		kg mixed fruit
	30	eggs	

(b) 6 eggs cost 70p.



How much will 30 eggs cost? £ ☐

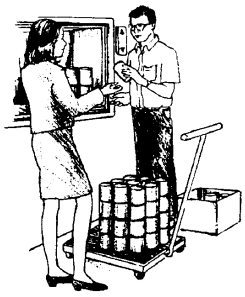
[KS3/98/Ma/Tier3-5/P1]

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Academic)

7. (a) Robert and Gwen must put 63 tins of food into a lift.



Each tin weighs 840 g.

Work out the total weight of the 63 tins, in grams.

Working:

☐ grams

(b) In the lift there is a sign.

It shows the greatest load that the lift can carry safely.

Look at the total weight of the 63 tins, which you worked out in part (a).
Is it safe to carry the 63 tins together in the lift?
Give a reason for your answer.

(c) Robert and Gwen must put the tins into a cupboard.

All the tins are the same size.

Gwen has measured the height of a tin. It is 14 cm.

Robert has measured the height of the cupboard. It is 1.24 m.

How many layers of tins can they keep in the cupboard?

Working:

☐ layers of tins

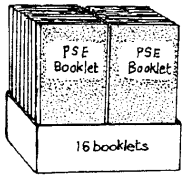
[KS3/95/Ma/Levels 4-6/P2]

(6 marks)

UNITS 1-3

Diagnostic Test 9A (Academic)

8. (a) A teacher needs 220 booklets.
The booklets are in *packs* of 16.
How many packs must the teacher order?
Show your working.



☐ packs

(b) Each booklet weighs 48 g.
How much do the 220 booklets weigh *altogether*?
Show your working. Give your answer in kg.

☐ kg

[KS3/98/Ma/Tier 3-5/P1]

(4 marks)

9. Calculate

(a) 6^3 = ☐ (b) 3^5 = ☐

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Academic)

10. Complete each of the following statements, filling in the missing numbers:

(a) $49 = 7^{\square}$

(b) $1\,000\,000 = 10^{\square}$

(c) $4^2 \times 4^5 = 4^{\square}$

(d) $\left(2^3\right)^2 = 2^{\square}$

(e) $81 = \square^4$

(f) $5^4 \div 5^2 = 5^{\square}$

(10 marks)

11. Write each of the following as a fraction:

(a) $3^{-1} = \square$

(b) $2^{-2} = \square$

(c) $5^{-2} = \square$

(6 marks)

UNITS 1-3 Diagnostic Test 9A (Academic) Answers

1.	(a)	$16 + 2 + 1 = 19$	M1 A1	
	(b)	$64 + 32 + 2 = 98$	M1 A1	*(4 marks)*
2.	(a)	110111	B2	
	(b)	10001010	B2	*(4 marks)*
3.		$\begin{array}{r} 1\,0\,1\,1\,0 \\ +\quad 1\,0\,1\,1 \\ \hline 1\,0\,0\,0\,0\,1 \end{array}$	M1 A1	*(2 marks)*
4.	(a)	[–] [75] or [÷] [6]	B1 B1	
	(b)	As (a)	B1 B1	*(4 marks)*
5.	(a)	[3] [2] [4]	B1	
	(b)	[3] [2] [4] [0]	B1	*(2 marks)*
6.		1000 g = 1 kg castor sugar	B1	
		1500 g = 1.5 kg margarine	B1	
		1250 g = 1.25 kg mixed fruit	B1	
		£3.50	B1	*(4 marks)*
7.	(a)	$\begin{array}{r} 840 \\ \times\ 63 \\ \hline 2520 \\ 50400 \\ \hline 52920 \end{array}$	M1 A1	
	(b)	No ; weight > 50 kg	B1	
	(c)	$124 \div 14 = 8$ (remainder 12) ; i.e. 8	M1 A1 A1	*(6 marks)*
8.		$220 \div 16 = 13$ remainder 12; 14 needed	M1 A1	
		$48 \times 220 = 10560$ gm; i.e. 10.56 kg	M1 A1	*(4 marks)*
9.	(a)	216	B2	
	(b)	243	B2	*(4 marks)*
10.	(a)	2	B1	
	(b)	6	B1	
	(c)	7	B2	
	(d)	6	B2	
	(e)	3	B2	
	(f)	2	B2	*(10 marks)*

UNITS 1-3 Diagnostic Test 9A (Academic) Answers

11. (a) $\frac{1}{3}$ B2

(b) $\frac{1}{4}$ B2

(c) $\frac{1}{25}$ B2 *(6 marks)*

(TOTAL MARKS 50)

UNITS 1-3 Diagnostic Test 9A (Academic) Answers

Marks

Unit	**1**	**2**	**3**	
Question	1 - 3	4 - 8	9 - 11	
Total marks available	10	20	20	*Final total*
Total				

Asessment

45 +	Excellent – should be on *Express Route*
40 - 44	Very good progress
30 - 39	Good progress: look carefully at mistakes
20 - 29	Steady progress, but you will need to work more carefully and/or make more effort
- 19	Struggling, so look carefully at the mistakes in your work; you might be better advised to transfer to the *Standard Route*

UNITS 1-3 Diagnostic Test 9A (Express)

You have ONE HOUR to complete this test.
You must NOT use a calculator.

1. Convert the binary number 1100010 to base 10.

1100010 = ☐

(2 marks)

2. Convert the base 10 number 138 to binary numbers.

138 = ☐

(2 marks)

3. In binary, calculate 1011×11, showing your working.

= ☐

(2 marks)

4. Convert the base 8 number 137 to base 10.

137 = ☐

(2 marks)

5. Convert the base 10 number 242 to base 5.

242 = ☐

(2 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

6. Lee has these cards:

He made the number 32.4 with four of his cards.

(a) Use some of Lee's cards to show the number *10 times* as big as 32.4.

(b) Use some of Lee's cards to show the number *100 times* as big as 32.4.

[KS3/94/Ma/3-5/P2]

(2 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

7. Here are the ingredients for 1 fruit cake:

1 Fruit Cake	
200 g	self-raising flour
100 g	castor sugar
150 g	margarine
125 g	mixed fruit
3	eggs

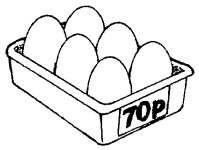
(a) Complete the table to show how much of each ingredient you need to make 10 fruit cakes.

Give your answers in grams *and* in kilograms.

10 Fruit Cakes

2000	g =	2	kg self-raising flour
	g =		kg castor sugar
	g =		kg margarine
	g =		kg mixed fruit
	30	eggs	

(b) 6 eggs cost 70p.



How much will 30 eggs cost? £ ☐

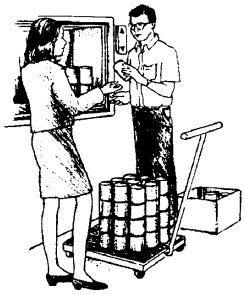
[KS3/98/Ma/Tier3-5/P1]

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

8. (a) Robert and Gwen must put 63 tins of food into a lift.



Each tin weighs 840 g.

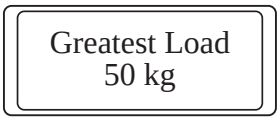
Work out the total weight of the 63 tins, in grams.

Working:

☐ grams

(b) In the lift there is a sign.

It shows the greatest load that the lift can carry safely.



Look at the total weight of the 63 tins, which you worked out in part (a).
Is it safe to carry the 63 tins together in the lift?
Give a reason for your answer.

(c) Robert and Gwen must put the tins into a cupboard.

All the tins are the same size.

Gwen has measured the height of a tin. It is 14 cm.

Robert has measured the height of the cupboard. It is 1.24 m.

How many layers of tins can they keep in the cupboard?

Working:

☐ layers of tins

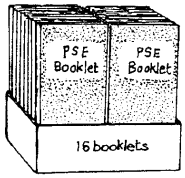
[KS3/95/Ma/Levels 4-6/P2]

(6 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

9. (a) A teacher needs 220 booklets.
The booklets are in *packs* of 16.
How many packs must the teacher order?
Show your working.



☐ packs

(b) Each booklet weighs 48 g.
How much do the 220 booklets weigh *altogether*?
Show your working. Give your answer in kg.

☐ kg

[KS3/98/Ma/Tier 3-5/P1]

(4 marks)

10. Pears are sold at 95p per kg. Showing your working,

(a) calculate the cost of 3.6 kg of these pears,

Cost = £ ☐

(b) calculate the mass of a bag of pears that is sold for £2.47.

Mass = ☐ kg

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

11. Complete each of the following statements, filling in the missing numbers:

(a) $49 = 7^{\square}$

(b) $1\,000\,000 = 10^{\square}$

(c) $4^2 \times 4^5 = 4^{\square}$

(d) $\left(2^3\right)^2 = 2^{\square}$

(e) $81 = \square^4$

(f) $5^4 \div 5^2 = 5^{\square}$

(10 marks)

12. Write each of the following as a fraction:

(a) $5^{-2} = \square$

(b) $2^{-4} = \square$

(4 marks)

13. The number 10^{100} is called a googol.

(a) Write the number, 50 googols, in standard index form. ☐

A nanometer is 10^{-9} metres.

(b) Write 50 nanometres in metres.

Give your answer in standard index form. ☐

(4 marks)

UNITS 1-3

Diagnostic Test 9A (Express)

14. Calculate:

(a) $16^{\frac{1}{4}}$ ☐

(b) $900^{\frac{1}{2}}$ ☐

(2 marks)

UNITS 1-3 Diagnostic Test 9A (Express) Answers

1.	$64 + 32 + 2 = 98$	M1 A1	*(2 marks)*
2.	10001010	B2	*(2 marks)*
3.	$\begin{array}{r} 1\,0\,1\,1\,0 \\ 1\,0\,1\,1 \\ \hline 1\,0\,0\,0\,0\,1 \end{array}$	M1 A1	*(2 marks)*
4.	$1 \times 64 + 3 \times 8 + 7 \times 1 = 95$	M1 A1	*(2 marks)*
5.	1432	B2	*(2 marks)*
6.	(a) 3 2 4	B1	
	(b) 3 2 4 0	B1	*(2 marks)*
7.	1000 g = 1 kg castor sugar	B1	
	1500 g = 1.5 kg margarine	B1	
	1250 g = 1.25 kg mixed fruit	B1	
	£3.50	B1	*(4 marks)*
8.	(a) $\begin{array}{r} 840 \\ \times\ 63 \\ \hline 2520 \\ 50400 \\ \hline 52920 \end{array}$	M1 A1	
	(b) No ; weight > 50 kg	B1	
	(c) $124 \div 14 = 8$ (remainder 12) ; i.e. 8	M1 A1 A1	*(6 marks)*
9.	$220 \div 16 = 13$ remainder 12; 14 needed	M1 A1	
	$48 \times 220 = 10560$ gm; i.e. 10.56 kg	M1 A1	*(4 marks)*
10.	(a) $95\text{p} \times 3.6 = £3.42$	M1 A1	
	(b) $247 \div 95 = 2.6$ Mass = 2.6 kg	M1 A1	*(4 marks)*
11.	(a) 2	B1	
	(b) 6	B1	
	(c) 7	B2	
	(d) 6	B2	
	(e) 3	B2	
	(f) 2	B2	*(10 marks)*

UNITS 1-3 Diagnostic Test 9A (Express) Answers

12. (a) $\frac{1}{25}$ B2

(b) $\frac{1}{16}$ B2 *(4 marks)*

13. (a) $50 \times 10^{100} = 5 \times 10^{101}$ M1 A1

(b) $50 \times 10^{-9} = 5 \times 10^{-8}$ m M1 A1 *(4 marks)*

14. (a) 2 (b) 30 B1 B1 *(2 marks)*

(TOTAL MARKS 50)

UNITS 1-3 Diagnostic Test 9A (Express) Answers

Marks

Unit	**1**	**2**	**3**	
Question	1 - 5	6 - 10	11 - 14	
Total marks available	10	20	20	*Final total*
Total				

Assessment

45 +	Excellent
40 - 44	Very good progress
35 - 39	Good progress: look carefully at mistakes
30 - 34	Steady progress, but you will need to work more carefully and/or make more effort
- 29	Struggling, so look carefully at the mistakes in your work; you might be better advised to transfer to the *Academic Route*