6 Probability

6.1 The Probability Scale

Probabilities are given on a scale of 0 to 1, as decimals or as fractions; sometimes probabilities are expressed as percentages using a scale of 0% to 100%, particularly on weather forecasts.

0	This is the probability of something that is *impossible.*
1	This is the probability of something that is *certain.*
$\frac{1}{2}$	This is the probability of something that is as *likely to happen* as it is *not to happen*.

Example 1

Decide whether or not each of the statements below is reasonable.

(a) The probability that it will snow on Christmas Day in London is 0.9.

(b) The probability that you will win a raffle prize is 0.5.

(c) The probability that you will go to bed before midnight tonight is 0.99.

(d) The probability that your pocket money is doubled tomorrow is 0.01.

Solution

(a) This is *not reasonable* as the probability given is much too high. It very rarely snows in London in late December, so the probability should be close to 0.

(b) This probability is far too high. You would need to have bought half of all the tickets sold to obtain this probability, so this statement is *not reasonable*.

(c) This is a *reasonable* statement as it is very likely that you will go to bed before midnight, but not certain that you will.

(d) This is a *reasonable* statement, as it is very unlikely that your pocket money will be doubled tomorrow, but not totally impossible.

Example 2

On a probability scale, mark and estimate the probability that:

(a) it will rain tomorrow,

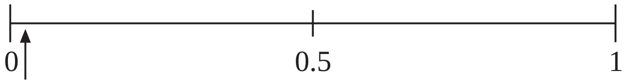
(b) England will win their next football match,

(c) someone in your class has a birthday tomorrow.

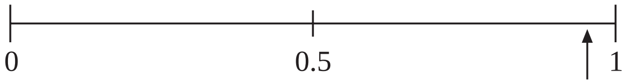
Solution

(a) This will depend on the time of year and the prevailing weather conditions.

During a dry spell in summer,

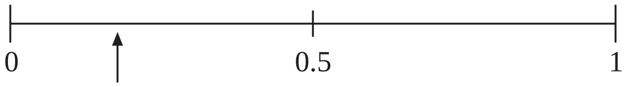


During a wet spell in winter,



(b) Based on their recent form, it is reasonable to say that England are slightly more likely to draw or lose their next match than to win it, so an estimate would be a little less than 0.5.

(c) The probability of this will be fairly small, as you can expect there to be about 2 or 3 birthdays per month for pupils in a class of about 30 pupils.



Exercises

1. Describe something that is:

(a) *very unlikely,* (b) *unlikely,*

(c) *likely,* (d) *very likely.*

2. State whether or not each of the statements below is reasonable.

(a) The probability that there will be a General Election next year is 0.2.

(b) The probability that England will win the next football World Cup is 0.8.

(c) The probability that it will not rain tomorrow is 0.9.

(d) The probability that your school will be hit by lightning in the next week is 0.1.

3. (a) List the things described, in order, with the most likely first.

A You travel on a bus that breaks down on the way home from school.

B Your pocket money is increased during the next two weeks.

C You enjoy your school lunch tomorrow.

D You have already had a birthday this year.

(b) Mark estimates of the probabilities of each of these on a copy of the probability scale similar to the one below:

4. Explain why the probability that you will be the first person to walk on the moon is zero.

5. Describe something that has a probability of zero.

6. (a) Do you agree that the probability that you will not be abducted by aliens in the next 24 hours is 1 ?

(b) Explain why.

7. Describe something that has a probability of 1.

8. When you toss a fair coin, the probability of obtaining a head is $\frac{1}{2}$ and the probability of obtaining a tail is $\frac{1}{2}$.

Describe something else that has a probability that is equal to or close to $\frac{1}{2}$.

9. A packet of sweets contains mostly *red* sweets, a few *green* sweets and only one *yellow* sweet. You take a sweet at random from the packet.

The events A, B, C and D are listed below.

A You take a *yellow* sweet.

B You take a *green* sweet.

C You take a *red* sweet.

D You take a *blue* sweet.

(a) Write these outcomes in order of probability, with the most likely first.

(b) Mark the probability of each outcome on a scale similar to the one below.

10. The probability that a train is late is 0.1. Which of the following statements is the most reasonable:

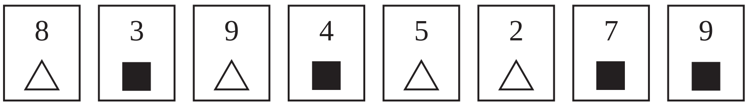
    A The train is *unlikely* to be late.

    B The train is *very unlikely* to be late.

    C The train is *likely* to be late.

    Explain why you have chosen your answer.

11. (a) Joe has these cards:



Sara takes a card without looking.

Joe says: "*On Sara's card,* ■ *is more likely than* △ ."

Explain why Joe is wrong.

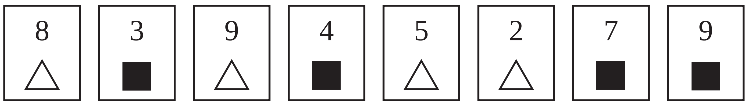
Choose one of the following words and phrases to fill in the gaps in the sentences below:

Impossible *Not Likely* *Certain* *Likely*

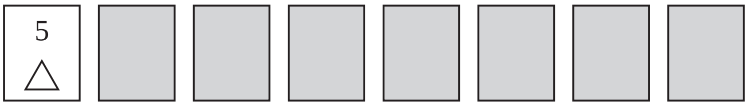
It is .... that the number on Sara's card will be *smaller than 10.*

It is .... that the number on Sara's card will be an *odd number.*

(b) Joe still has these cards:



He mixes them up and puts them face down on the table. Then he turns the first card over, like this:



Joe is going to turn the next card over.

Copy and complete this sentence:

On the next card, ... is *less likely* than ....

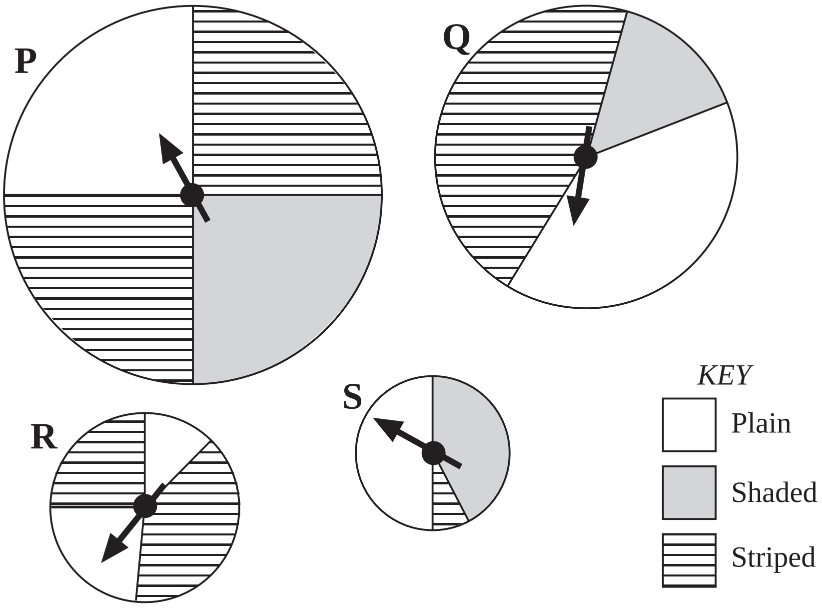
The number on the next card could be higher than 5 or lower than 5. Which of the following possibilities is *more likely*?

Higher than 5 *Lower than 5* *Cannot tell*

Explain your answer.

(KS3/97/Ma/Tier 3-5/P2)

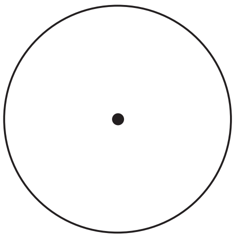
12. Here are four spinners, labelled P, Q, R and S.



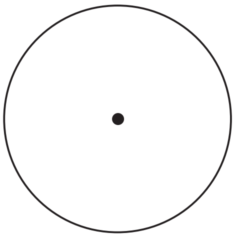
(a) Which spinner gives the *greatest* chance that the arrow will land on *plain*?

(b) Which spinner gives the *smallest* chance that the arrow will land on *shaded*?

(c) Shade a copy of the spinner shown so that it is *certain* that the arrow will land on *shaded*.



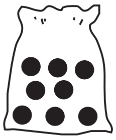
(d) Shade a copy of this spinner so that there is a 50% chance that the arrow will land on *shaded.*

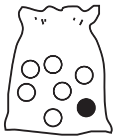


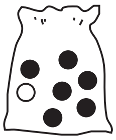
(KS2/98/Ma/Tier 4-6/P2)

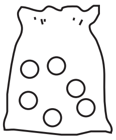
13. Bryn has some bags with some black beads and some white beads. He is going to take a bead from each bag without looking.

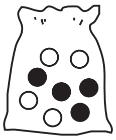
(a) Match the pictures to the statements. The first is done for you.


A


B


C


D


E

(i) It is *impossible* that Bryn will take a black bead from bag D.

(ii) It is *unlikely* that Bryn will take a black bead from bag ..... .

(iii) It is *equally likely* that Bryn will take a black bead or a white bead from bag ..... .

(iv) It is *likely* that Bryn will take a black bead from bag ..... .

(v) It is *certain* that Bryn will take a black bead from bag ..... .

(b) Bryn has 5 white beads in a bag.

He wants to make it *more likely* that he will take a *black* bead than a *white* bead out of the bag.

How many *black* beads should Bryn put into the bag?

(c) There are 20 beads altogether in another bag. All the beads are either black or white.

It is *equally likely* that Bryn will take a black bead or a white bead from the bag.

How many black beads and how many white beads are there in the bag?

(KS3/99/Ma/Tier 3-5/P2)

6.2 The Probability of a Single Event

In this section we consider the probabilities of equally likely events. When you roll a fair dice, each of the numbers 1 to 6 is equally likely to be on the uppermost face of the dice.

For equally likely events:

$$p(\text{a particular outcome}) = \frac{\text{number of ways of obtaining outcome}}{\text{total number of outcomes}}$$

Example 1

A card is taken at random from a full pack of 52 playing cards. What is the probability that it is:

(a) a red card,

(b) a 'Queen',

(c) a red 'Ace',

(d) the 'Seven of Hearts',

(e) an even number?

Solution

As each card is equally likely to be drawn from the pack there are 52 equally likely outcomes.

(a) There are 26 red cards in the pack, so:

$$\begin{aligned} p(\text{red}) &= \frac{26}{52} \\ &= \frac{1}{2} \end{aligned}$$

(b) There are 4 Queens in the pack, so:

$$\begin{aligned} p(\text{Queen}) &= \frac{4}{52} \\ &= \frac{1}{13} \end{aligned}$$

(c) There are 2 red Aces in the pack, so:

$$\begin{aligned} p(\text{red Ace}) &= \frac{2}{52} \\ &= \frac{1}{26} \end{aligned}$$

(d) There is only one 7 of Hearts in the pack, so:

$$p(\text{7 of Hearts}) = \frac{1}{52}$$

(e) There are 20 cards that have even numbers in the pack, so:

$$\begin{aligned} p(\text{even number}) &= \frac{20}{52} \\ &= \frac{5}{13} \end{aligned}$$

Example 2

A packet of sweets contains 18 *red* sweets, 12 *green* sweets and 10 *yellow* sweets. A sweet is taken at random from the packet. What is the probability that the sweet is:

(a) *red*,

(b) *not green*,

(c) *green* or *yellow* ?

Solution

The total number of sweets in the packet is 40, so there are 40 equally likely outcomes when one is taken at random.

(a) There are 18 *red* sweets in the packet, so:

$$p(\text{red}) = \frac{18}{40} = \frac{9}{20}$$

(b) There are 28 sweets that are *not green* in the packet, so:

$$p(\text{not green}) = \frac{28}{40} = \frac{7}{10}$$

(c) There are 22 sweets that are *green* or *yellow* in the packet, so:

$$p(\text{green or yellow}) = \frac{22}{40} = \frac{11}{20}$$

Example 3

You roll a fair dice 120 times. How many times would you expect to obtain:

(a) a 6, (b) an *even* score, (c) a score of *less than* 5 ?

Solution

(a) $p(6) = \frac{1}{6}$

$$\text{Expected number of 6s} = \frac{1}{6} \times 120 = 20$$

(b) $p(\text{even score}) = \frac{3}{6} = \frac{1}{2}$

$$\text{Expected number of even scores} = \frac{1}{2} \times 120 = 60$$

(c) $$p(\text{score less than } 5) = \frac{4}{6} = \frac{2}{3}$$

$$\text{Expected number of scores less than } 5 = \frac{2}{3} \times 120 = 80$$

Exercises

1. You roll a fair dice. What is the probability that you obtain:
   (a) a *five,* (b) a *three,* (c) an *even* number,
   (d) a multiple of 3, (e) a number *less than* 6 ?

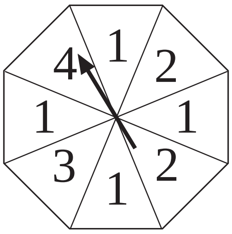
2. A jar contains 9 *red* counters and 21 *blue* counters. A counter is taken at random from the jar. What is the probability that it is:
   (a) *red,* (b) *blue,* (c) *green* ?

3. You take a card at random from a pack of 52 playing cards. What is the probability that the card is:
   (a) a red King, (b) a Queen or a King, (c) a 5, 6 or 7,
   (d) a Diamond, (e) *not* a Club ?

4. A jar contains 4 *red* balls, 3 *green* balls and 5 *yellow* balls. One ball is taken at random from the jar. What is the probability that it is:
   (a) *green,* (b) *red,* (c) *yellow,*
   (d) *not red,* (e) *yellow* or *red ?*

5. The faces of a regular tetrahedron are numbered 1 to 4. When it is rolled it lands face down on one of these numbers. What is the probability that this number is:
   (a) 2, (b) 3,
   (c) 1, 2 or 3, (d) an *even* number ?

6. A spinner is numbered as shown in the diagram. Each score is equally likely to occur.
   What is the probability of scoring:
   (a) 1, (b) 2,
   (c) 3, (d) 4,
   (e) 5, (f) a number less than 6 ?



7. You toss a fair coin 360 times.

   (a) How many times would you expect to obtain a head?

   (b) If you obtained 170 heads, would you think that the coin was biased? Explain why.

8. A spinner has numbers 1 to 5, so that each number is equally likely to be scored. How many times would you expect to get a score of 5, if the spinner is spun:

   (a) 10 times, (b) 250 times, (c) 400 times ?

9. A card is drawn at random from a pack of 52 playing cards, and then replaced. The process is repeated a total of 260 times. How many times would you expect the card drawn to be:

   (a) a 7, (b) a *red Queen,* (c) a *red* card,

   (d) a *Heart,* (e) a card with an *even* number ?

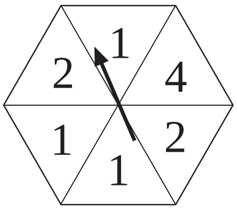
10. A six-sided spinner is shown in the diagram. It is spun 180 times.

    How many times would you expect to obtain:

    (a) a score of 1,

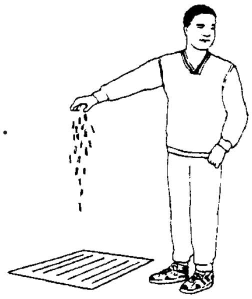
    (b) a score less than 4,

    (c) a score that is a *prime* number, (d) a score of 4 ?



11. Barry is doing an experiment. He drops 20 matchsticks at random onto a grid of parallel lines.

    Barry does the experiment 10 times and records his results. He wants to work out an estimate of probability.



Number of the 20 matchsticks that have fallen across a line

5	7	6	4	6	8	5	3	5	7

   (a) Use Barry's data to work out the probability that a *single matchstick* when dropped will fall across one of the lines. Show your working.

   (b) Barry continues the experiment until he has dropped the 20 matchsticks 60 times.

   About how many matchsticks *in total* would you expect to fall across one of the lines? Show your working.

(KS3/96/Ma/Tier 5-7/P2)

12. Les, Tom, Nia and Ann are in a singing competition. To decide the order in which they will sing all four names are put into a bag. Each name is taken out of the bag, one at a time, without looking.

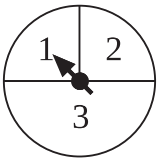
(a) Write down *all* the possible orders with *Tom* singing *second.*

(b) In a different competition there are 8 singers. The probability that Tom sings second is $\frac{1}{8}$.

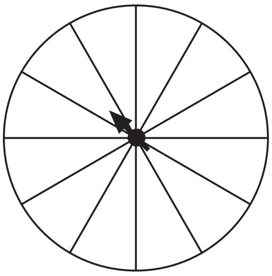
Work out the probability that Tom does *not* sing second.

(KS3/96/Ma/Tier 4-6/P1)

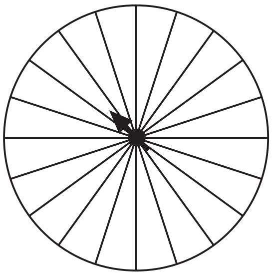
13. (a) What is the probability of getting a 3 on this spinner?



(b) Shade a copy of the following spinner so that the chance of getting a shaded section is double the chance of getting a white section.



(c) Shade a copy of the following spinner so that there is a 40% chance of getting a shaded section.



(KS3/95/Ma/Levels 4-6/P1)

14. Pat has 5 white beads and 1 black bead in her bag. She asks two friends about the probability of picking a black bead without looking in the bag.

Owen says: "*It is* $\frac{1}{5}$ *because there are 5 white beads and 1 black bead.*"

Maria says: "*It is* $\frac{1}{6}$ *because there are 6 beads and 1 is black.*"

(a) Which of Pat's friends is *correct*? Explain why the other friend is *wrong.*

(b) Tracy has a different bag of black beads and white beads.

The probability of picking a black bead from Tracy's bag is $\frac{7}{13}$.

What is the probability of picking a white bead from Tracy's bag?

(c) How many black beads and how many white beads could be in Tracy's bag?

(d) Peter has a different bag of black beads and white beads.

Peter has more beads in total than Tracy.

The probability of picking a black bead from Peter's bag is also $\frac{7}{13}$.

How many black beads and how many white beads could be in Peter's bag?

(KS3/94/Ma/4-6/P1)

15. Brightlite company makes light bulbs. The state of the company's machines can be:

available for use and being used
or available for use but not needed
or broken down.

(a) The table shows the probabilities of the state of the machines in July 1994. What is the missing probability?

State of machines: July 1994	*Probability*
Available for use, being used	
Available for use, not needed	0.09
Broken down	0.03

(b) During another month the probability of a machine being available for use was 0.92. What was the probability of a machine being broken down?

(c) Brightlite calculated the probabilities of a bulb failing within 1000 hours and within 2000 hours.

Copy and complete the table below to show the probabilities of a bulb still working at 1000 hours and at 2000 hours.

Time	*Failed*	*Still working*
At 1000 hours	0.07	
At 2000 hours	0.57	

(KS3/95/Ma/Levels 5-7/P1)

16. A machine sells sweets in *five* different colours:

red, green, orange, yellow, purple.

You *cannot choose* which colour you get.

There are the *same number* of each colour in the machine.

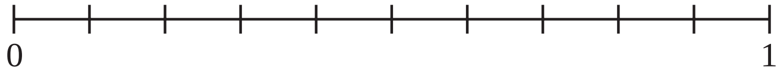
Two boys want to buy a sweet each.

Ken does not like orange sweets or yellow sweets. Colin likes them all.

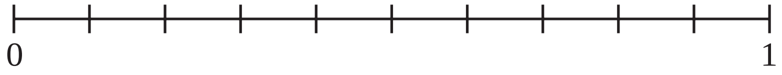
(a) What is the probability that Ken will get a sweet that he likes?

(b) What is the probability that Colin will get a sweet that he likes?

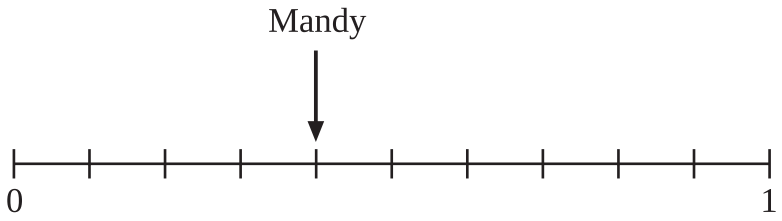
(c) Copy the following scale and draw an arrow to show the probability that Ken will get a sweet that he likes. Label the arrow 'Ken'.



(d) On your scale from (c), draw an arrow to show the probability that Colin will get a sweet that he likes. Label this arrow 'Colin'.



(e) Mandy buys one sweet. The arrow on the following scale shows the probability that Mandy gets a sweet that she likes.



Write a sentence that *could* describe which sweets Mandy likes.

(KS3/96/Ma/Tier 3-5/P2)

6.3 The Probability of Two Events

In this section we review the use of *listings*, *tables* and *tree diagrams* to calculate the probabilities of two events.

Example 1

An unbiased coin is tossed twice.

(a) List *all* the possible outcomes.

(b) What is the probability of obtaining *two heads*?

(c) What is the probability of obtaining a *head* and a *tail* in any order?

Solution

(a) The possible outcomes are:

H H
H T
T H
T T

So there are 4 possible outcomes that are all equally likely to occur as the coin is not biased.

(b) There is only one way of obtaining 2 heads, so:

$$p(\text{2 heads}) = \frac{1}{4}$$

(c) There are two ways of obtaining a head and a tail, H T and T H, so:

$$\begin{aligned} p(\text{a head and a tail}) &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Example 2

A red dice and a blue dice, both unbiased, are rolled at the same time. The scores on the two dice are then added together.

(a) Use a table to show all the possible outcomes.

(b) What is the probability of obtaining:

(i) a score of 5,

(ii) a score which is *greater than* 3,

(iii) a score which is an *even number*?

Solution

(a) The following table shows all of the 36 possible outcomes:

		Red Dice					
		1	2	3	4	5	6
Blue Dice	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(b) (i) There are 4 ways of scoring 5, so:

$$p(5) = \frac{4}{36}$$

$$= \frac{1}{9}$$

(ii) There are 33 ways of obtaining a score greater than 3, so:

$$p(\text{greater than 3}) = \frac{33}{36}$$

$$= \frac{11}{12}$$

(iii) There are 18 ways of obtaining a score which is an even number, so:

$$p(\text{even score}) = \frac{18}{36}$$

$$= \frac{1}{2}$$

Example 3

A card is taken at random from a pack of 52 playing cards, and then replaced. A second card is then drawn at random from the pack.

Use a tree diagram to determine the probability that:

(a) *both* cards are Diamonds,

(b) *at least one* card is a Diamond,

(c) *exactly one* card is a Diamond,

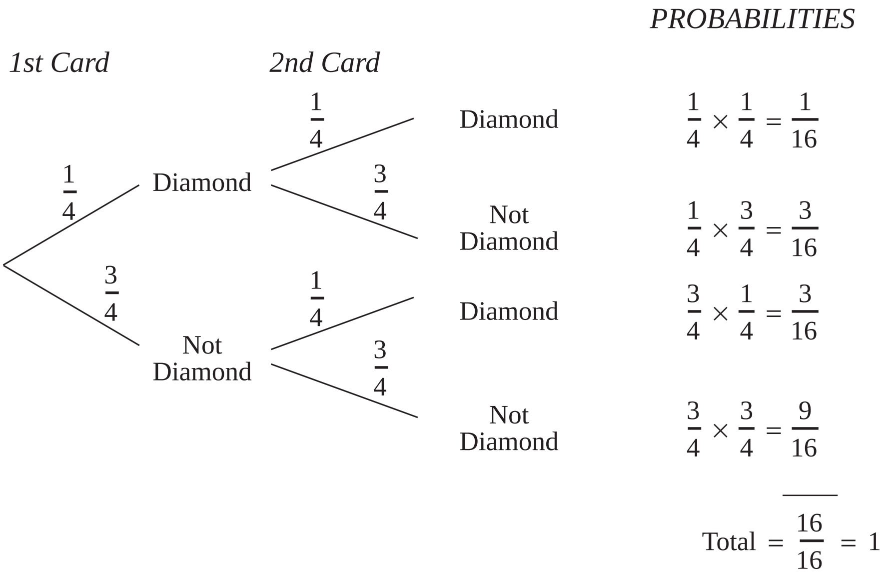
(d) *neither* card is a Diamond.

Solution

We first note that, for a single card drawn from the pack,

$$p(\text{Diamond}) = \frac{13}{52} = \frac{1}{4} \text{ and } p(\text{not Diamond}) = \frac{39}{52} = \frac{3}{4}.$$

We put these probabilities on the branches of the tree diagram below:



Note also that the probability for each combination, for example, two Diamonds, is determined by *multiplying* the probabilities along the branches.

(a) $p(\text{both Diamonds}) = \frac{1}{16}$

(b) $p(\text{at least one Diamond}) = \frac{1}{16} + \frac{3}{16} + \frac{3}{16}$

$$= \frac{7}{16}$$

(c) $p(\text{exactly one Diamond}) = \frac{3}{16} + \frac{3}{16}$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$

(d) $p(\text{neither card a Diamond}) = \frac{9}{16}$

Exercises

1. The faces of an unbiased dice are painted so that 2 are *red*, 2 are *blue* and 2 are *yellow*. The dice is rolled twice. Three of the possible outcomes are listed below:

   R R   R B   R Y

   (a) List all 9 possible outcomes.

   (b) What is the probability that:

      (i) both faces are *red*,

      (ii) both faces are the *same colour*,

      (iii) the faces are of *different colours*?

2. A spinner is marked with the letters A, B, C and D, so that each letter is equally likely to be obtained. The spinner is spun twice.

   (a) List the 16 possible outcomes.

   (b) What is the probability that:

      (i) A is obtained *twice*,

      (ii) A is obtained *at least onc*e,

      (iii) *both* letters are the *same*,

      (iv) the letter B is *not* obtained at all?

3. Two fair dice are renumbered so that they have the following numbers on their faces:

   1, 1, 2, 3, 4, 6

   The dice are rolled at the same time, and their scores added together.

   (a) Draw a table to show the 36 possible outcomes.

   (b) What is the probability that the total score is:

      (i) 6,   (ii) 3,

      (iii) greater than 10,   (iv) less than 5 ?

4. A red spinner is marked with the numbers 1 to 4 and a blue spinner is marked with the numbers 1 to 5. On each spinner all the numbers are equally likely to be obtained. The two spinners are spun at the same time and the two scores are added together.

   (a) Draw a table to show the 20 possible outcomes.

   (b) What is the probability that the total score on the two spinners is:

      (i) an *even* number,   (ii) the number 7,

      (iii) a number greater than 4,   (iv) a number less than 7 ?

5. An unbiased dice is rolled and a fair coin is tossed at the same time.

   (a) *Either* list all the possible outcomes *or* show them in a table.

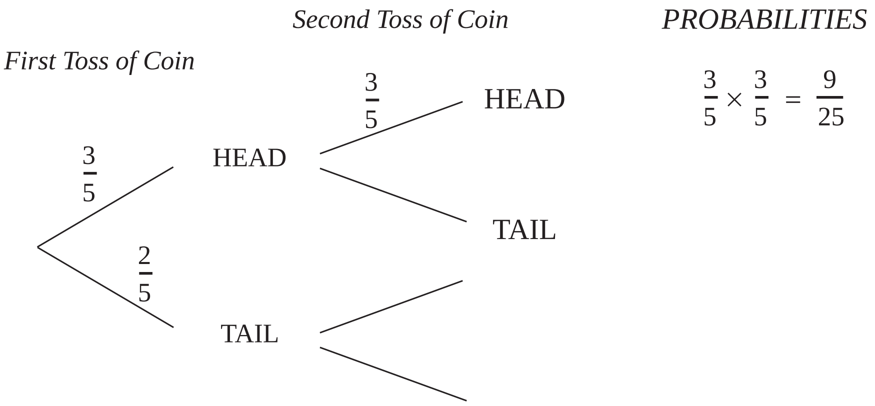
   (b) What is the probability of obtaining:

      (i) a head and a 6, (ii) a tail and an odd number,

      (iii) a tail and a number less than 5 ?

6. A coin is biased so that the probability of obtaining a head is $\frac{3}{5}$ and the probability of obtaining a tail is $\frac{2}{5}$.

   (a) Copy and complete the following tree diagram to show the possible outcomes and probabilities if the coin is tossed twice.



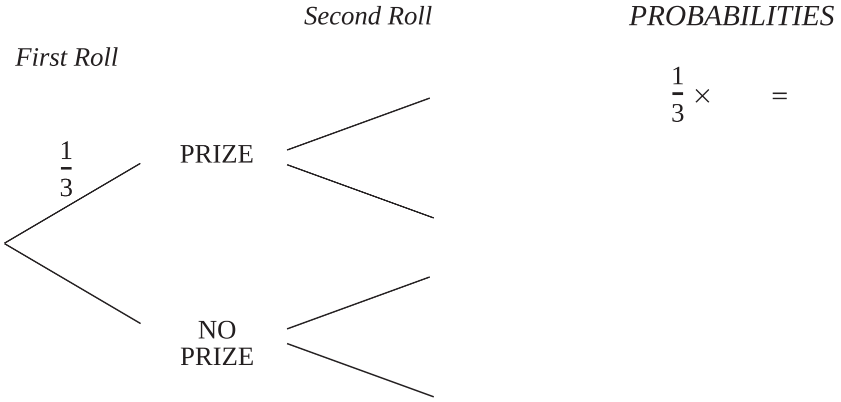
   (b) What is the probability of obtaining:

      (i) 2 heads, (ii) at least one head,

      (iii) 2 tails, (iv) exactly 1 tail ?

7. An unbiased dice is rolled twice in a game. If a 1 or a 6 is obtained, you win a prize.

   (a) Copy and complete the following tree diagram:



   (b) What is the probability that a player wins:

      (i) 2 prizes, (ii) 1 prize, (iii) at least 1 prize ?

8. A card is taken at random from a pack of 52 playing cards. It is replaced and a second card is then taken at random from the pack.
A card is said to be a 'Royal' card if it is a *King*, *Queen* or *Jack*.

Use a tree diagram to calculate the probability that:

(a) *both* cards are Royals,
(b) *one* card is a Royal,
(c) *at least one* card is a Royal,
(d) *neither* card is a Royal.

9. The probability that a school bus is late on any day is $\frac{1}{10}$. Use a tree diagram to calculate the probability that on two consecutive days, the bus is:

(a) late *twice*,
(b) late *once*,
(c) *never* late.

10. The probability that a piece of bread burns in a toaster is $\frac{1}{9}$. Two slices of bread are toasted, one after the other.

(a) Use a tree diagram to calculate the probability that at least one of these slices of bread burns in the toaster.

(b) Extend your tree diagram to include toasting 3 slices, one at a time. Calculate the probability of at least one slice burning in the toaster.

11. A coin has two sides, heads and tails.

(a) Chris is going to toss a coin. What is the probability that Chris will get heads? Write your answer as a fraction.

(b) Sion is going to toss 2 coins. Copy and complete the following table to show the different results he could get.

First coin	*Second coin*
heads	heads

(c) Sion is going to toss 2 coins. What is the probability that he will get tails with both his coins? Write your answer as a fraction.

(d) Dianne tossed one coin. She got tails.
Dianne is going to toss another coin.
What is the probability that she will get tails again with her next coin?
Write your answer as a fraction.

(KS3/99/Ma/Tier 3-5/P1)

12. I have two fair dice. Each of the dice is numbered 1 to 6.

    (a) The probability that I will throw *double 6 (*both dice showing number 6) is

    $$\frac{1}{36}$$

    What is the probability that I will *not* throw double 6 ?

    (b) I throw both dice and get double 6. Then I throw both dice again.

    Which one answer from the list below describes the probability that I will throw *double 6* this time?

    less than $\frac{1}{36}$

    $\frac{1}{36}$

    more than $\frac{1}{36}$

    Explain your answer.

    I start again and throw both dice.

    (c) What is the probability that I will throw *double 3* (both dice showing number 3) ?

    (d) What is the probability that I will throw a double? (It could be double 1 or double 2 or any other double.)

(KS3/98/Ma/Tier 4-6/P2)

13. On a road there are two sets of traffic lights. The traffic lights work independently.

    For each set of traffic lights, the probability that a driver will have to *stop* is 0.7.

    (a) A woman is going to drive along the road.

    (i) What is the probability that she will have to *stop* at *both* sets of traffic lights?

    (ii) What is the probability that she will have to *stop* at *only one of the two sets* of traffic lights?

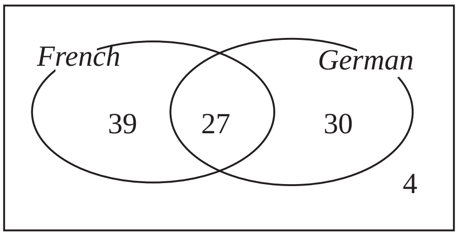
    Show your working.

    (b) In one year, a man drives 200 times along the road. Calculate an estimate of the number of times he drives through *both sets* of traffic lights *without stopping.* Show your working.

(KS3/99/Ma/Tier 6-8/P2)

14. 100 students were asked whether they studied French or German.

Results:



27 students studied both French *and* German.

(a) What is the probability that a student chosen at random will study only *one* of the languages?

(b) What is the probability that a student who is studying German is also studying French?

(c) Two of the 100 students are chosen at random.

From the following calculations, write down one which shows the probability that *both* students study French and German.

$$\frac{27}{100} \times \frac{26}{100} \qquad \frac{27}{100} + \frac{26}{99} \qquad \frac{27}{100} + \frac{27}{100}$$

$$\frac{27}{100} \times \frac{26}{99} \qquad \frac{27}{100} \times \frac{27}{100}$$

(KS3/98/Ma/Tier 6-8/P1)

15. A company makes computer disks. It tested a random sample of the disks from a large batch. The company calculated the probability of any disk being defective as 0.025.

Glenda buys 2 disks.

(a) Calculate the probability that *both* disks are defective.

(b) Calculate the probability that *only one* of the disks is defective.

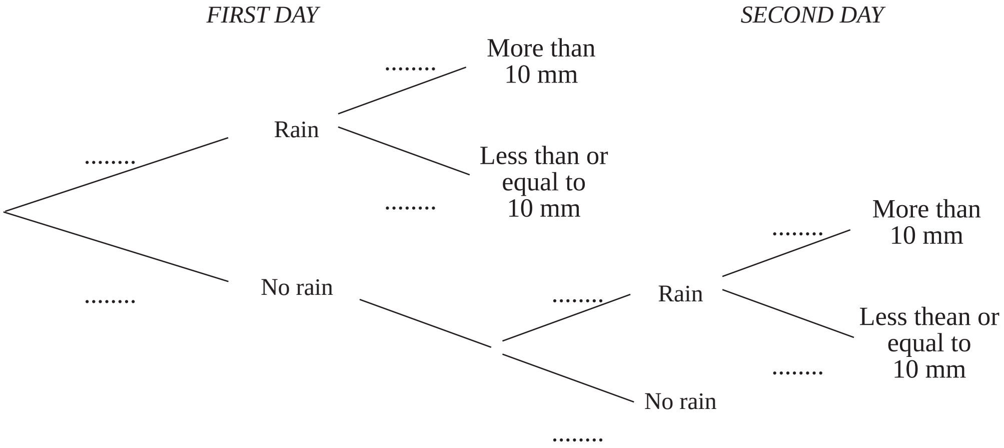
(c) The company found 3 defective disks in the sample they tested.
How many disks were likely to have been tested?

(KS3/96/Ma/Tier 6-8/P2)

16. On a tropical island the probability of it raining on the first day of the rainy season is $\frac{2}{3}$. If it does not rain on the first day, the probability of it raining on the second day is $\frac{7}{10}$. If it rains on the first day, the probability of it raining more than 10 mm on the first day is $\frac{1}{5}$. If it rains on the second day

but not on the first day, the probability of it raining more than 10 mm is $\frac{1}{4}$.

You may find it helpful to copy and complete the tree diagram before answering the questions.



(a) What is the probability that it rains more than 10 mm on the second day, and does not rain on the first? Show your working.

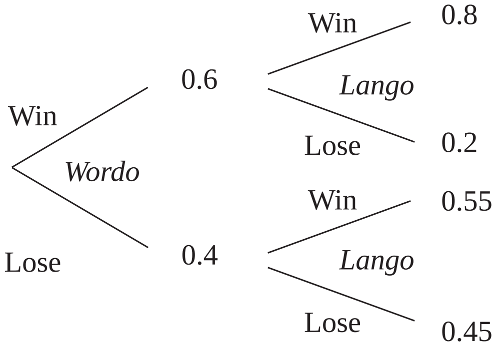
(b) What is the probability that it has rained by the end of the second day of the rainy season? Show your working.

(c) Why is it not possible to work out the probability of rain on both days from the information given?

(KS3/96/Ma/Ext)

17. Pupils at a school invented a word game called Wordo. They tried it out with a large sample of people and found that the probability of winning Wordo was 0.6.

The pupils invented another word game, Lango. The same sample who had played Wordo then played Lango. The pupils drew this tree diagram to show the probabilities of winning.



(a) What was the probability of someone from the sample winning Lango?

(b) What was the probability of someone from the sample winning *only* one of the two word games?

(c) The pupils also invented a dice game. They tried it out with the same sample of people who had already played Wordo and Lango.

The probability of winning the dice game was 0.9. This was found to be independent of the probabilities for Wordo and Lango.

Calculate the probability of someone from the sample winning two out of these three games.

(d) Calculate the probability of someone from the sample winning *only* one of these three games.

(KS3/95/Ma/Levels 9-10)

6.4 Theoretical and Experimental Probabilities

In this section we compare *theoretical* and *experimental* probabilities.

The term 'theoretical probabilities' describes those which have been calculated, for example by the methods described in sections 6.2. and 6.3.

'Experimental probabilities' are estimates for probabilities that cannot be determined logically. They can be derived from the results of experiments, but often they are obtained from the analysis of statistical data or historical records.

Here we obtain experimental probabilities from simple experiments and compare them with the theoretical probabilities.

Example 1

An unbiased dice is to be rolled 240 times.

(a) Calculate the number of times you would expect to obtain each of the possible scores.

(b) Now roll the dice 240 times and collect some experimental results, presenting them in a bar chart.

(c) Compare the theoretical and experimental results.

Solution

(a) $p(6) = \frac{1}{6}$

$$\text{Expected number of 6s} = \frac{1}{6} \times 240$$

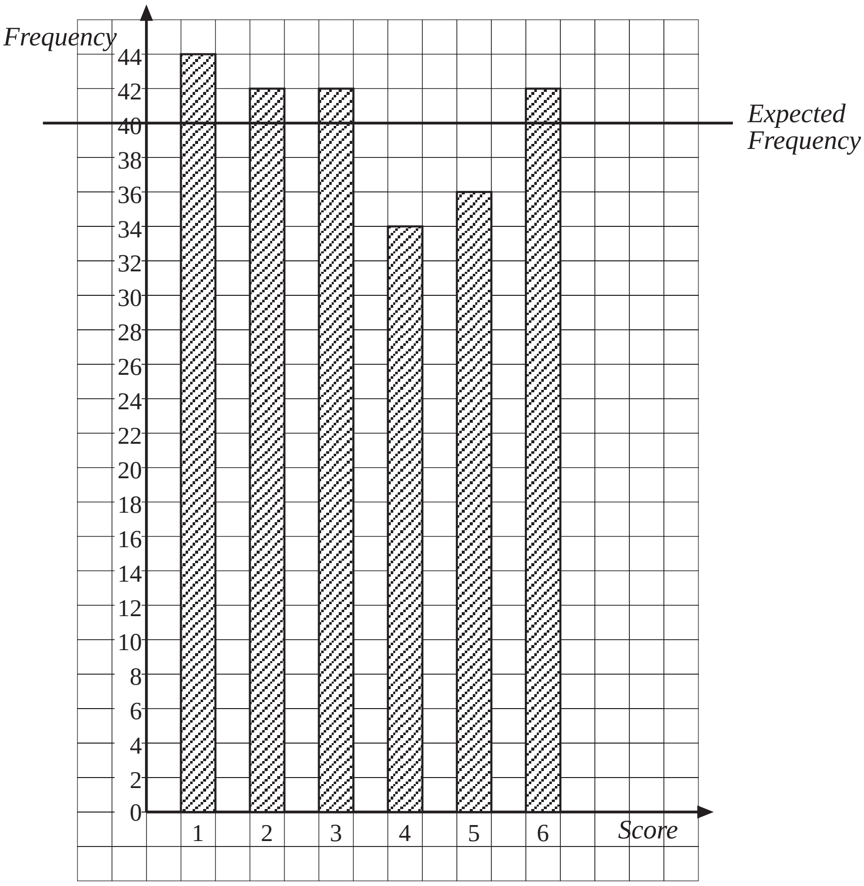
$$= 40$$

Similarly, you would expect to obtain each of the possible scores 40 times.

(b) The results of the experiment are recorded in the following table:

Score	*Tally*	*Frequency*
1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	44
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷	42
3	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷	42
4	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	34
5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷	36
6	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷	42

These results are illustrated in the following bar chart. A horizontal line has been drawn to show the expected frequencies for the scores.



Note that none of the bars is of the expected height; some are above and some are below. However, all the bars are *close* to the predicted number. We would not expect to obtain *exactly* the predicted number. The more times the experiment is carried out, the closer the experimental results will be to the theoretical predictions.

Exercises

1. (a) A fair coin is tossed 100 times. How many heads and how many tails would your expect to obtain?

   (b) Toss a fair coin 100 times and display your results using a bar chart.

   (c) Compare your theoretical predictions with your experimental results.

2. Two fair coins are to be tossed at the same time.

   (a) Calculate the probability of obtaining:

   (i) 2 heads, (ii) a head and a tail, (iii) 2 tails.

   (b) Calculate the number of times you would expect to obtain each outcome if the coins are tossed 100 times.

   (c) Toss two coins 100 times and illustrate your results using a bar chart.

   (d) Compare your theoretical predictions with your experimental results.

3. (a) List the 8 possible outcomes when 3 fair coins are tossed at the same time.

   (b) If three fair coins were tossed 32 times, how many times would you expect to obtain:

   (i) 3 heads, (ii) 2 heads,

   (iii) 1 head, (iv) 0 heads ?

   (c) Carry out an experiment and compare your theoretical predictions with your experimental results.

4. (a) What are the expected frequencies of the totals 2, 3, 4, ..., 11, 12 when two fair dice are thrown at the same time and the experiment is repeated 36 times?

   (b) Carry out the experiment in (a) and compare the predicted and experimental frequencies.

   (c) Repeat (a) and (b) for 144 throws.

   (d) Comment on how carrying out the experiment more times influences the differences between the predicted and experimental frequencies.

5. A fair coin and an unbiased dice are thrown at the same time. A score is then calculated using the following rules:

   - *if the coin shows a head, you double the score shown on the dice;*
   - *if the coin shows a tail, you subtract 1 from the score on the dice.*

   (a) Use a two-way table to show all the possible scores.

   (b) Draw up a table showing the theoretical probabilities for the various scores.

(c) If the coin and the dice are thrown 120 times, how many times would you expect to obtain each score?

(d) Conduct an experiment and compare your experimental results with your answers to part (c).

6. A dice with 4 faces has one blue, one green, one red and one yellow face.

Five pupils did an experiment to investigate whether the dice was biased or not.

The following table shows the data they collected.

Pupil's Name	*Number of Throws*	*Face Landed On*			
		Red	*Blue*	*Green*	*Yellow*
Peter	20	9	7	2	2
Caryl	60	23	20	8	9
Shana	250	85	90	36	39
Keith	40	15	15	6	4
Paul	150	47	54	23	26

(a) Which pupil's data is most likely to give the best estimate of the probability of getting each colour on the dice? Explain your answer.

The pupils collected all the data together.

Number of Throws	*Face Landed On*			
	Red	*Blue*	*Green*	*Yellow*
520	179	186	75	80

(b) Consider the data. Write down whether you think the dice is biased or unbiased, and explain your answer.

(c) From the data, work out the probability of the dice landing on the blue face.

(d) From the data work out the probability of the dice landing on the green face.

(KS3/95/Ma/Levels 5-7/P1)

7. Some pupils threw 3 fair dice. They recorded how many times the numbers on the dice were the same.

Name	*Number of throws*	*Results*		
		all different	*2 the same*	*all the same*
Morgan	40	26	12	2
Sue	140	81	56	3
Zenta	20	10	10	0
Ali	100	54	42	4

(a) Write the name of the pupil whose data are *most likely* to give the best estimate of the probability of getting each result. Explain your answer.

(b) This table shows the pupils' results collected together:

Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	171	120	9

Use these data to estimate the *probability* of throwing numbers that are *all different.*

(c) The theoretical probability of each result is shown below:

	all different	*2 the same*	*all the same*
Probability	$\frac{5}{9}$	$\frac{5}{12}$	$\frac{1}{36}$

Use these probabilities to calculate, for 300 throws, *how many times* you would theoretically expect to get each result. Copy and complete the table below.

Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	....	....	....

(d) Explain why the pupils' results are not the same as the theoretical results.

(KS3/98/Ma/Tier 5-7/P2)

UNIT 6 *Probability* Activities

Activities

ACTIVITY 6.1 *Evens and Odds*

This is a simple game, where you throw a dice which controls the position of your counter on a 3×3 board.

FINISH		
		START

Place your counter at the START square. Throw a dice.

If you get an EVEN number, you move your counter one square *upwards*.

If you get an ODD number, you move your counter one square *left*.

If your counter moves off any side of the board, you lose!

If your counter reaches the FINISH square, you have won.

Play the game a few times and see if you win.

1. How many 'odds' and how many 'evens' do you need to get to win?

2. What is the probability of winning?

Extensions

1. Play the game 20 or 30 times to find the experimental probability.
   Check your answers with that of question 2, above.

2. Analyse the same game on a 4×4, 5×5, . . ., board.

ACTIVITY 6.2 *A Russian Fable*

This is the method traditionally used in some Russian villages to see which of the girls in the village are to be married next year! You take three blades of grass, folded in two, and hold them in your hand so that the six ends are hanging down. A young girl ties the ends together in pairs. If, on release, a large loop is formed, the girl will be married next year.

1. What are the possible outcomes for this experiment in terms of small, medium and large loops?

2. By labelling the six ends (say *a* and A for the two ends of one blade of grass), consider all the possible outcomes and hence find the probability of getting the large loop.

3. Test your predicted probabilities by using short lengths of string and getting the class to work in pairs, recording their answers. Collect all the data together and use it to work out the experimental probabilities. Compare these to the theoretical values found in question 2.

4. If a Russian village has 30 young girls and they all go through this ritual, how many do you estimate will be predicted to marry next year?

Extension

What happens if either 4 or 5 blades of grass (string) are used? What is the probability of now obtaining one large loop?

ACTIVITY 6.3 *Birthdays*

First try this experiment. Find out the birthdays of as many of your family as possible. Do any of them have birthdays on the same date?

Now try the same experiment with all the members of your class. We will see how likely it is that two members of a group have the same birthday.

Consider each member of a group, one by one. The first person will have his/her birthday on a particular day.

1. What is the probability of the second person having a different birthday from the first?

2. What is the probability of the third person having a birthday different from both the first and second person?

3. What is the probability that at least two of the first three people have the same birthday?

This solves the problem of a group of three people. As expected, it is not likely that any 2 out of 3 people will have the same birthday.

4. Repeat the problem above for 4 people. What is the probability that at least 2 of them have the same birthday?

5. Using either a computer programme or a calculator, solve the problem for a group of n people, where $n = 10, 20, 30$, etc.

6. What is the probability that 2 members of your class have the same birthday?

Extension

How many people are needed in the group to be 95% sure that there will be at least two with the same birthday?

ACTIVITIES 6.1 - 6.3 *Notes and Solutions*

Notes and solutions given only where appropriate.

6.1 1. 3 'odds' and 3 'evens' in the first six throws of the dice.

2. This has a probability of

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3 \times 20 = 0.3125$$

since there are 20 distinct ways of arranging three 'evens' and three 'odds'.

6.2 1. 3 small loops ; 1 small loop and 1 medium loop ; 1 large loop.-

2. Probabilities $\frac{1}{15}, \frac{6}{15}, \frac{8}{15}$

4. About 16

6.3 1. $\frac{364}{365}$

2. $\frac{363}{365}$

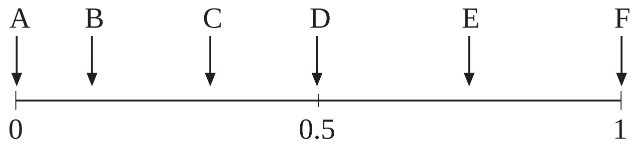
3. $1 - \frac{364}{365} \cdot \frac{363}{365} \approx 0.008$

4. 0.0164

5. $n = 10 \Rightarrow p = 0.117$; $n = 20 \Rightarrow p = 0.411$; $n = 30 \Rightarrow p = 0.706$

UNIT 6 *Probability* Extra Exercises 6.1

1. The following probability line shows the probabilities of 6 events, A, B, C, D, E and F.



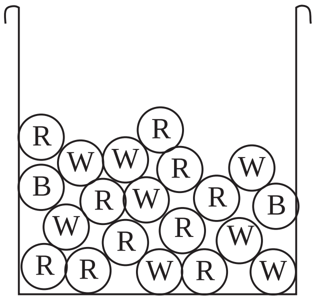
(a) Which event is *certain* to occur?

(b) Which event is the *most unlikely* to occur, but is *not impossible*?

(c) Which event is *impossible*?

(d) Which events are *more likely* to occur than C?

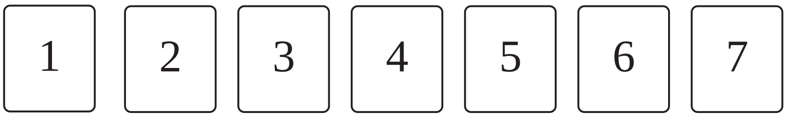
2. The diagram shows a jar containing *red* (R), *blue* (B) and *white* (W) balls. One of the balls is taken at random.



(a) What colour is this ball *most likely* to be?

(b) What colour is this ball *least likely* to be?

3. In a game you are given one of the following cards at random:



(a) Are you more likely to be given an *odd* number or an *even* number?

(b) Are you more likely to be given a 7 than a 5 ?

(c) Are you more likely to be given a number *greater* or *less* than 5?

UNIT 6 *Probability* Extra Exercises 6.2

1. A sweet jar contains 10 toffees, 8 mints and 12 chocolates. A sweet is taken at random from the jar. What is the probability that the sweet is:

   (a) a mint,

   (b) a toffee,

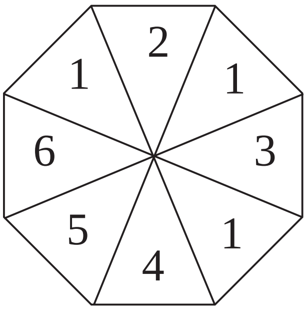
   (c) a chocolate,

   (d) a mint or a toffee,

   (e) not a mint,

   (f) a chocolate or a toffee?

2. The diagram shows a spinner. What is the probability that, with one spin, your score is:



   (a) 1,

   (b) 2,

   (c) greater than 2,

   (d) less than 4,

   (e) an even number?

3. Ahmed rolls a fair dice 300 times. How many times would he expect to obtain:

   (a) 6,

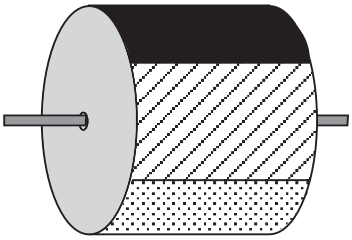
   (b) an even number,

   (c) a number greater than 1,

   (d) a number less than 3,

   (e) a 2 or a 5 ?

UNIT 6 *Probability* Extra Exercises 6.3



1. The curved outer surface of a drum is painted red (R), yellow (Y), purple (P), silver (S) and black (B), in 5 equal sections.

   The drum is spun twice and the colour uppermost noted.

   (a) Copy and complete this list of possible outcomes:

   R R
   R Y
   R P
   R S
   R B

   Y R
   Y Y
   ....
   ....
   ....

   (b) What is the probability that you obtain:

   (i) 2 reds,
   (ii) the same colour on both spins,
   (iii) a yellow and a red in any order,
   (iii) no yellows.

2. In a game a card is taken at random from a full pack of 52 playing cards. It is then replaced, and a second card is taken.

   Use a tree diagram to calculate the probabilities that:

   (a) both cards are diamonds,
   (b) *neither* card is a diamond,
   (c) one of the cards is a diamond,
   (d) at least one card is a diamond.

3. Two fair dice have faces that are numbered,

   1, 1, 2, 2, 3, 6

   (a) Draw a table to list the outcomes when the two dice are rolled together and the two scores added to give a total score.

   (b) Calculate the probabilities that the total score on the two dice is:

   (i) 7,
   (ii) greater than 5,
   (iii) an even number,
   (iv) less than 4
   (v) a multiple of 3.

UNIT 6 *Probability* Extra Exercises 6.4

1. Sally rolls a dice 300 times.
   (a) How many sixes would you expect her to obtain?
   (b) Should she be surprised if she obtained 55 sixes?

2. (a) If you toss a fair coin 10 times, how many heads would you expect to obtain?
   (b) Toss a coin 10 times and record the number of heads that you obtain.
   (c) Comment on how your answers to parts (a) and (b) compare.

3. Two unbiased dice are rolled at the same time. The scores are then multiplied together.
   (a) Use a table to list all the possible outcomes.
   (b) If the dice were rolled 72 times, how often would you expect to get each score?
   (c) Conduct an experiment in which you roll the two dice 70 times and compare your results with your expected results calculated in part (b).

Extra Exercises 6.1 Answers

1. (a) F (b) B (c) A (d) D, E, F

2. (a) Red (b) Blue

3. (a) odd number (b) no; equally likely (c) less than 5

Extra Exercises 6.2 Answers

1. (a) $\frac{8}{30} = \frac{4}{15}$ (b) $\frac{10}{30} = \frac{1}{3}$ (c) $\frac{12}{30} = \frac{2}{5}$

   (d) $\frac{18}{30} = \frac{3}{5}$ (e) $\frac{22}{30} = \frac{11}{15}$ (f) $\frac{22}{30} = \frac{11}{15}$

2. (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{4}{8} = \frac{1}{2}$

   (d) $\frac{5}{8}$ (e) $\frac{3}{8}$

3. (a) 50 (b) 150 (c) 250

   (d) 100 (e) 100

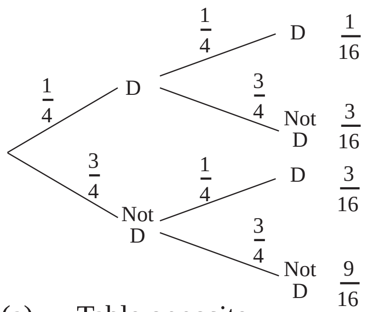
Extra Exercises 6.3 Answers

1. (a)

R R	Y R	P R	S R	B R
R Y	Y Y	P Y	S Y	B Y
R P	Y P	P P	S P	B P
R S	Y S	P S	S S	B S
R B	Y B	P B	S B	B B

   (b) $\frac{1}{25}$ (ii) $\frac{5}{25} = \frac{1}{5}$ (iii) $\frac{2}{25}$ (iv) $\frac{16}{25}$

2.



(a) $\frac{1}{16}$ (b) $\frac{9}{16}$

(c) $\frac{6}{16} = \frac{3}{8}$ (d) $\frac{7}{16}$

3. (a) Table opposite

   (b) (i) $\frac{4}{36} = \frac{1}{9}$ (ii) $\frac{12}{36} = \frac{1}{3}$

   (iii) $\frac{18}{36} = \frac{1}{2}$ (iv) $\frac{12}{36} = \frac{1}{3}$

   (v) $\frac{12}{36} = \frac{1}{3}$

Dice B

Dice A

	1	1	2	2	3	6
1	2	2	3	3	4	7
1	2	2	3	3	4	7
2	3	3	4	4	5	8
2	3	3	4	4	5	8
3	4	4	5	5	6	9
6	7	7	8	8	9	12

Extra Exercises 6.4 Answers

1. (a) 50 (b) no

2. 5

3. (a)

Dice B

Dice A	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

(b)

Score	*Expected Frequency*
1	2
2	4
3	4
4	6
5	4
6	8
8	4
9	2
10	4
12	8
15	4
16	2
18	4
20	4
24	4
25	2
30	4
36	2

UNIT 6 *Probability* Lesson Plans **St**

These are based on 45/50 minute lessons.

Lesson No.	*Suggested Plan*	*References*
1.	**Probability Scale**	
	Recap concept	OS 6.1
	Exercises	PB 6.1, Q2
	Review answers	
	Recap probability scale	OS 6.2
	Exercises	PB 6.1, Q3
	Review answers	
	Exercises	PB 6.1, Q11
	Review answers	
	Set homework	PB 6.1, Q13
2.	**Probability of Single Event 1**	
	Discuss homework	
	Introduction	OS 6.3
	Exercises	PB 6.2, Q1
	Review answers	
	Exercises	PB 6.2, Q4
	Review answers	
	Set homework	PB 6.2, Q6 and Q13
3.	**Probability of Single Event 2**	
	Discuss homework	
	Activity	Activity 6.1 or Activity 6.2
	Exam-type questions	PB 6.2, Q14
	Review answers	
	Mental Test	M 6.1
	Review answers	
	Set homework	PB 6.2, Q16
4.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 6.1
5.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 6 *Probability* Lesson Plans A

Lesson No.	*Suggested Plan*	*References*
1.	**Probability Scale**	
	Recap concept	OS 6.1
	Exercises	PB 6.1, Q2
	Review answers	
	Recap probability scale	OS 6.2
	Exercises	PB 6.1, Q3
	Review answers	
	Exercises	PB 6.1, Q11
	Review answers	
	Set homework	PB 6.1, Q12 and Q13
2.	**Probability of Single Event**	
	Discuss homework	
	Introduction	OS 6.3
	Exercises	PB 6.2, Q4
	Review answers	
	Exercises	PB 6.2, Q13
	Review answers	
	Activity	Activity 6.1
	Set homework	Complete Activity 6.1 or PB 6.2, Q14 and Q16
3.	**Probability of Two Events 1**	
	Discuss homework	
	Introduction to concept	OS 6.4
	Exercises	PB 6.3, Q1
	Review answers	
	Numbered spinners	OS 6.5
	Exercises	PB 6.3, Q4
	Review answers	
	Set homework	PB 6.3, Q11 and Q12
4.	**Probability of Two Events 2**	
	Discuss homework	
	Tree diagrams	OS 6.6
	Exercises	PB 6.3, Q6
	Review answers	
	Mental Test	M 6.2
	Review answers	
	Set homework	PB 6.3, Q13 and Q15

UNIT 6 *Probability* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
5.	**Theoretical and Experimental Probabilities**	
	Discuss homework	
	Introduction (using Example 1, PB 6.4 or equivalent)	OS 6.7
	Exercises	Choose from PB 6.4, Q1-Q4 or Activity 6.2
	Review results	
	Exercises	PB 6.4, Q6
	Review answers	
	Set homework	PB 6.4, Q7
6.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 6.2
7.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 6 *Probability* Lesson Plans E

Lesson No.	*Suggested Plan*	*References*
1.	**Probability of Single Event**	
	Recap concept	OS 6.3
	Exercises	PB 6.2, Q4
	Review answers	
	Exercises	PB 6.2, Q13
	Review answers	
	Activity	Activity 6.1 or Activity 6.2
	Set homework	PB 6.2, Q11, Q14 and Q15
2.	**Probability of Two Events**	
	Discuss homework	
	Recap methods	OS 6.4, OS 6.5 and OS 6.6
	Exercises	PB 6.2, Q6
	Review answers	
	Exercises	PB 6.2, Q12
	Review answers	
	Activity	Activity 6.3
	Set homework	Complete Activity 6.3 or PB 6.2, Q13 and Q14
3.	**Theoretical and Experimental Probabilities**	
	Discuss homework	
	Introduction (using Example 1, PB 6.4 or equivalent)	OS 6.7
	Exercises	Choose from PB 6.4, Q1-Q4 or Activity 6.2
	Review results	
	Exercises	PB 6.4, Q6
	Review answers	
	Mental Test	M 6.3
	Review answers	
	Set homework	PB 6.4, Q6 and Q7
4.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 6.3
5.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 6 Probability Mental Tests

M 6.1 Standard Route *(no calculator)*

What is the probability of obtaining:

1. a HEAD when tossing a fair coin, $(\frac{1}{2})$

2. a 6 when rolling a fair dice, $(\frac{1}{6})$

3. a 5 or a 6 when rolling a fair dice, $(\frac{1}{3})$

4. an *even number* when rolling a fair dice, $(\frac{1}{2})$

5. a number *less than* 4 when rolling a fair dice ? $(\frac{1}{2})$

6. A biased coin is such that $p(\text{head}) = \frac{1}{3}$.

   What is the probability of obtaining TAILS ? $(\frac{2}{3})$

Refer to Diagram A on the Information Sheet for questions 7 - 9.

What is the probability of obtaining

7. *RED*, $(\frac{1}{5})$

8. *RED* or *YELLOW*, $(\frac{2}{5})$

9. *NOT RED* ? $(\frac{4}{5})$

10. If a fair dice is rolled 12 times, how many 6s would you expect to obtain? (2)

UNIT 6 Probability Mental Tests

M 6.2 Academic Route *(no calculator)*

What is the probability of obtaining:

1. a 4 when rolling a fair dice, $\left(\frac{1}{6}\right)$

2. a number *less than* 3 when rolling a fair dice, $\left(\frac{1}{3}\right)$

3. a number *greater than* 3 when rolling a fair dice, $\left(\frac{1}{2}\right)$

4. two HEADS when tossing a fair coin, $\left(\frac{1}{4}\right)$

5. two 6s when rolling two fair dice twice ? $\left(\frac{1}{36}\right)$

6. A biased coin is such that $p(\text{head}) = \frac{1}{3}$. If it is thrown twice,

   what is the probability of obtaining 2 TAILS ? $\left(\frac{4}{9}\right)$

Refer to Diagram B on the Information Sheet for questions 7 - 9.

What is the probability of obtaining

7. number 5, $\left(\frac{3}{8}\right)$

8. an even number, $\left(\frac{3}{8}\right)$

9. a number other than number 4 ? $\left(\frac{3}{4}\right)$

10. If a fair dice is rolled 120 times, how many 6s would you expect to obtain? (20)

UNIT 6 Probability Mental Tests

M 6.3 Express Route *(no calculator)*

What is the probability of obtaining:

1. a number *less than* 3 when rolling a fair dice, ($\frac{1}{3}$)

2. a number *greater than* 3 when rolling a fair dice, ($\frac{1}{2}$)

3. a HEAD and a TAIL when tossing two fair coins, ($\frac{1}{2}$)

4. three HEADS when tossing a fair coin three times, ($\frac{1}{8}$)

5. a total sum of 11 or 12 when throwing two fair dice ? ($\frac{1}{12}$)

6. A biased coin is such that $p(\text{head}) = \frac{1}{3}$. If it is thrown twice,

   what is the probability of obtaining 2 TAILS ? ($\frac{4}{9}$)

Refer to Diagram B on the Information Sheet for questions 7 and 8.

When the spinner is spun three times, what is the probability of obtaining:

7. two number 5s, ($\frac{9}{64}$)

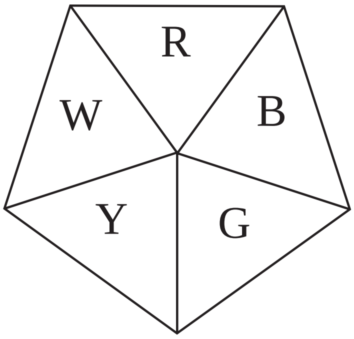
8. number 1 and number 2, in any order, ($\frac{1}{32}$)

9. If a fair dice is rolled twice, what is the probability of obtaining an even number in both throws? ($\frac{1}{2}$)

10. If a fair dice is rolled 600 times, how many 6s would you expect to obtain? (100)

UNIT 6 Probability Mental Tests

Information Sheet



KEY

R : Red

B : Blue

G : Green

Y : Yellow

W: White

Diagram A 5-sided spinner

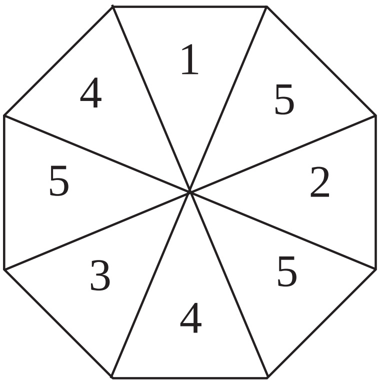


Diagram B 8-sided spinner

UNIT 6 *Probability* Overhead Slides

Overhead Slides

OS 6.1 *Assessing Probabilities*

For each question below, select the more reasonable statement from each pair:

1. The probability that it snows on Christmas Day in London is 0.9.

   The probability that it does *not* snow on Christmas Day in London is 0.9.

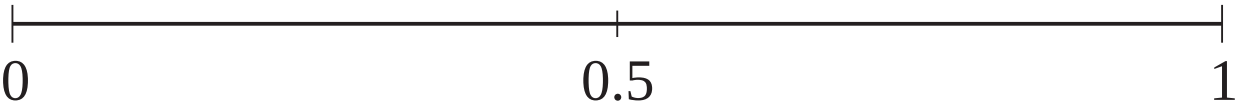
2. The probability that the school bus is *late* is less than 0.5.

   The probability that the school bus is *on time* is greater than 0.5.

3. I can be certain that I will *not win* a prize in the school raffle if I buy one ticket.

   I am very *unlikely to win* a prize in the school raffle if I buy one ticket.

OS 6.2 *Probability Scale*



Mark an estimate of the probability of each event listed below on the probability scale shown.

A *Your school football team wins a trophy.*

B *You roll an unbiased dice and obtain a number greater than 1.*

C *You toss a fair coin and obtain a head.*

D *Your school is struck by lightning.*

E *You draw a 'Diamond' from a pack of 52 playing cards.*

OS 6.3 *Probability of a Single Event*

A jar contains

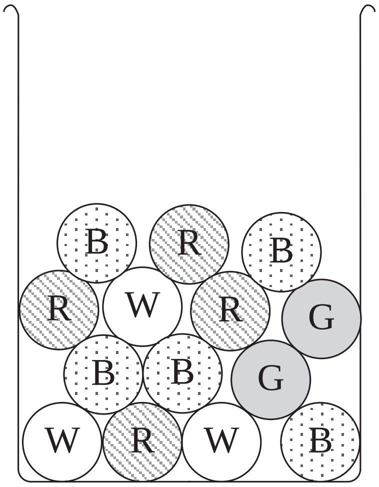
3 *white* balls,

2 *green* balls,

4 *red* balls

and

5 *blue* balls.



What is the probability that a ball taken at random is:

(a) *red*,

(b) *not blue*,

(c) *green* or *white*,

(d) *yellow* ?

(a) $p\,(red) = $ ______

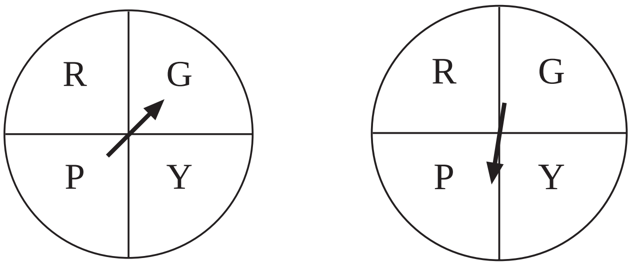
(b) $p\,(not\ blue) = $ ______

(c) $p\,(green \text{ or } white) = $ ______

(d) $p\,(yellow) = $ ______

OS 6.4 *Probability of Two Events 1*

The four sections of two identical spinners are coloured Red (R), Green (G), Yellow (Y) and Pink (P).



The spinners are spun at the same time. List the 16 possible outcomes.

1. .... 5. .... 9. .... 13. ....
2. .... 6. .... 10. .... 14. ....
3. .... 7. .... 11. .... 15. ....
4. .... 8. .... 12. .... 16. ....

Complete the following statements:

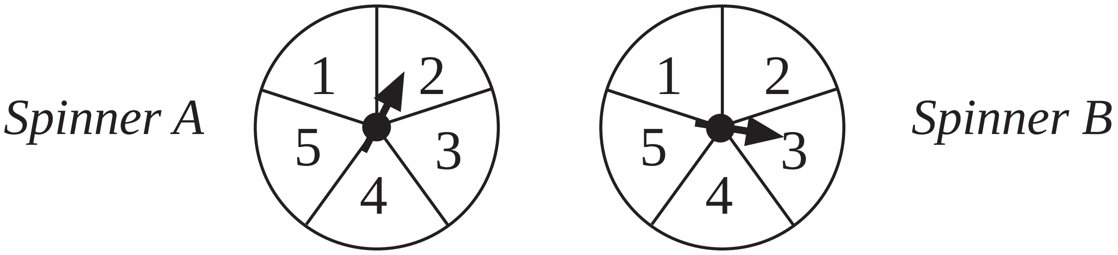
p (2 *red* sections) =

p (2 sections of *different* colours) =

p (no *yellow* sections) =

p (both sections the *same colour*) =

OS 6.5 *Two- Numbered Spinners*



These two spinners are spun at the same time. Complete the table to show the possible outcomes for the total of the scores when added together.

Spinner B

		1	2	3	4	5
	1					
	2					
Spinner A	3					
	4					
	5					

Complete the following statements:

p (score *greater than* 3) = ______

p (*even* score) = ______

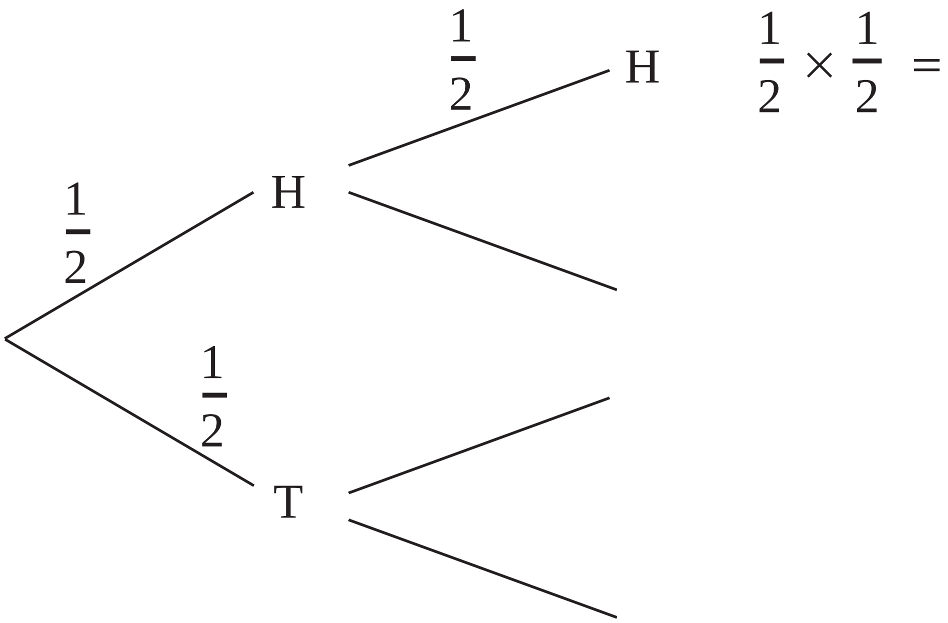
p (score *less than* 5) = ______

p (score *of* 8) = ______

OS 6.6 *Tossing Two Coins*

Two fair coins are tossed at the same time.

Complete the tree diagram to show the possible outcomes.



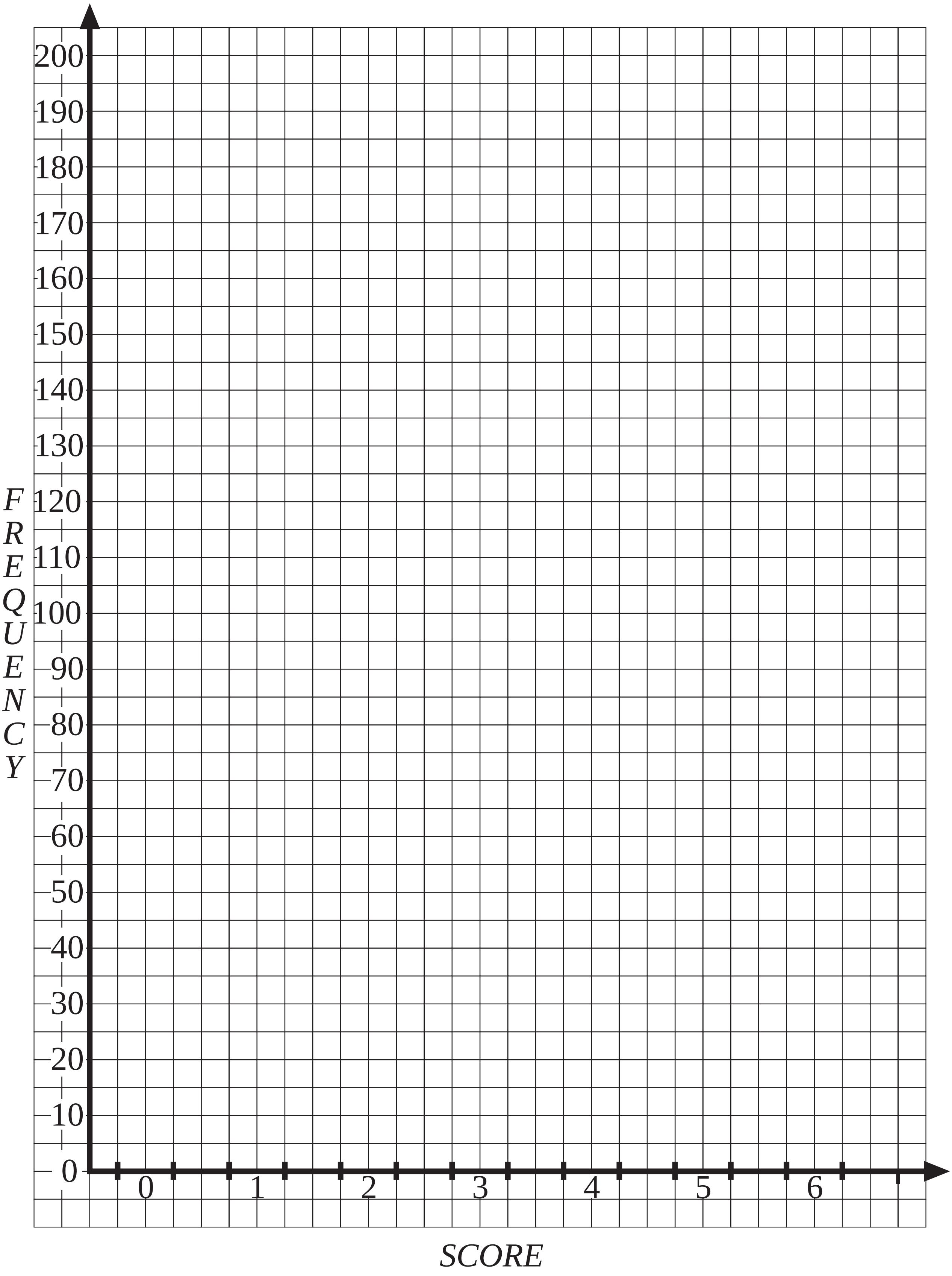
Complete the following statements:

$p\,(2 \text{ tails}) \quad = \quad \_\_\_\_$

$p\,(\textit{no} \text{ tails}) \quad = \quad \_\_\_\_$

$p\,(\text{a head and a tail}) \quad = \quad \_\_\_\_$

OS 6.7 *Experimental Probabilities*



Practice Book *UNIT 6 Probability* Answers

6 .1 The Probability Scale

1. Pupils' individual descriptions. It is best to discuss these with the whole class.

2. (a) This depends on the year and the political climate.
   (b) Not reasonable – this estimate is far too high.
   (c) Reasonable, if in a dry spell; not reasonable if in a wet spell.
   (d) Not reasonable – the probability of the school being hit by lightning is *very* remote.

3. Answers here will depend on the pupil concerned. It may be best to invite individual students to present their order and scale to the rest of the class.

4. Because it is impossible to be the first person to walk on the moon as Neil Armstrong did that in 1969.

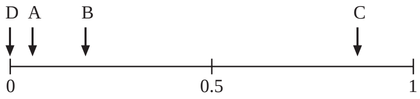
5. Pupils' individual descriptions. It is best to discuss these with the whole class.

6. (a) No
   (b) Assuming that there are aliens, there is a very small chance of being abducted, so the probability will be very close to 1, but not equal to 1.

7. Pupils' individual descriptions. It is best to discuss these with the whole class.

8. Pupils' individual descriptions. It is best to discuss these with the whole class. Suggestions might include getting an even number when you throw a fair dice or a rat turning right in a maze when presented with a left/right choice at a T-junction.

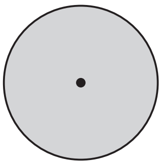
9. C, B, A, D



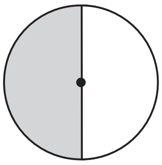
10. A, because the probability 0.1 implies that the train is late only once in every 10 occasions.

11. (a) Joe is wrong because there is an equal number of squares and triangles, so both have a probability of $\frac{1}{2}$.
   "It is certain that the number on Sara's card will be smaller than 10."
   "It is likely that the number on Sara's card will be an odd number."
   (b) "On the next card, △ is more likely than ■."
   or "On the next card an even number is less likely than an odd number."
   Higher than 5 is more likely, because the remaining cards have the number 2, 3, 4, 7, 8, 9, and 9, i.e. there are 4 numbers above 5 and only 3 numbers below 5."

12. (a) S (b) R (zero probability) (c)



(d)



13. (a) (i) D (ii) B (iii) E
   (iv) C (v) A
   (b) 6 or more
   (c) 10 black and 10 white

6.2 The Probability of a Single Event

1. (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{5}{6}$

2. (a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) 0 (do *not* allow $\frac{0}{30}$ or $\frac{0}{10}$)

3. (a) $\frac{1}{26}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{1}{4}$ (e) $\frac{3}{4}$

4. (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

5. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

6. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{8}$
   (e) 0 (do *not* allow $\frac{0}{8}$) (f) 1 (do *not* allow $\frac{8}{8}$)

7. (a) 180
   (b) No - the number of trials is not really large enough to have the probability settle down.

8. (a) 2 (b) 50 (c) 80

9. (a) 20 (b) 10 (c) 130 (d) 65 (e) 100

10. (a) 90 (b) 150 (c) 60 (d) 30

11. (a) $\frac{56}{200} = 0.28$ (b) $0.28 \times 1200 = 336$

12. (a) ATLN, ATNL, LTNA, LTAN, NTAL, NTLA (b) $\frac{7}{8}$

13. (a) $\frac{1}{2}$ (b) Any 8 sectors shaded. (c) Any 8 sectors shaded.

14. (a) Maria is correct. Owen is wrong because the selection is 1 black from a total of 6 in the bag.
    (b) $\frac{6}{13}$
    (c) $7n$ black beads and $6n$ white beads (for any positive integer n), e.g. 7B, 6W or 14B, 12W.
    (d) $7m$ black beads and $6m$ white beads (for any positive integer m greater than the n given in part (c), e.g. 30W and 35 B.

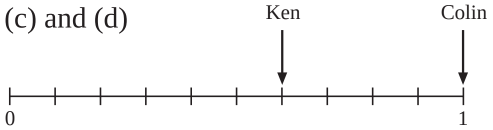
6.2 Answers

15. (a) 0.88 (b) 0.08 (c)

Time	*Failed*	*Still working*
At 1000 hours	0.07	0.93
At 2000 hours	0.57	0.43

16. (a) $\frac{3}{5}$ (b) 1

(c) and (d)



(e) Mandy likes only 2 out of the 5 sweets (or any equivalent comment, e.g. there are 3 sweets that she does not like).

6.3 The Probability of Two Events

1. (a) RR, RB, RY, BR, BB, BY, YR, YB, YY

(b) (i) $\frac{1}{9}$ (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$

2. (a) AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD

(b) (i) $\frac{1}{16}$ (ii) $\frac{7}{16}$ (iii) $\frac{1}{4}$ (iv) $\frac{9}{16}$

3. (a)

First Dice

Second Dice		1	1	2	3	4	6
	1	2	2	3	4	5	7
	1	2	2	3	4	5	7
	2	3	3	4	5	6	8
	3	4	4	5	6	7	9
	4	5	5	6	7	8	10
	6	7	7	8	9	10	12

(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{9}$

(iii) $\frac{1}{36}$ (iv) $\frac{13}{36}$

4. (a)

Blue Spinner

Red Spinner		1	2	3	4	5
	1	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9

(b) (i) $\frac{1}{2}$ (ii) $\frac{3}{20}$

(iii) $\frac{7}{10}$ (iv) $\frac{7}{10}$

5. (a)

Dice

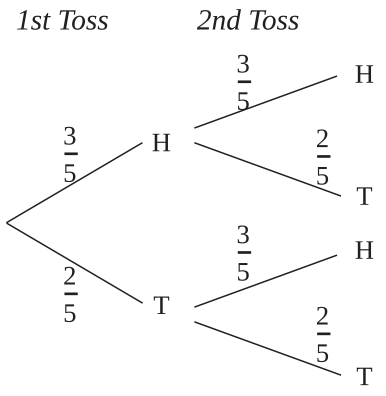
Coin		1	2	3	4	5	6
	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{4}$

(iii) $\frac{1}{3}$

6.3 Answers

6. (a)



PROBABILITIES

$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

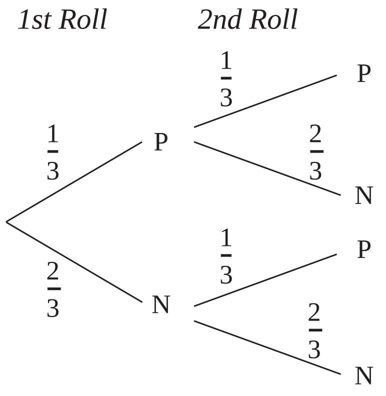
$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

(b) (i) $\frac{9}{25}$ (ii) $\frac{21}{25}$

(iii) $\frac{4}{25}$ (iv) $\frac{12}{25}$

7. (a) P = Prize, N = No Prize



PROBABILITIES

$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

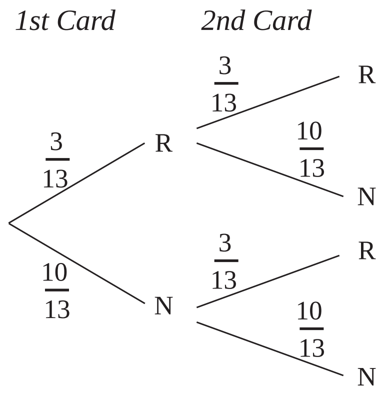
$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

(b) (i) $\frac{1}{9}$ (ii) $\frac{4}{9}$

(iii) $\frac{5}{9}$

8. R = Royal, N = Not Royal



PROBABILITIES

$\frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

$\frac{3}{13} \times \frac{10}{13} = \frac{30}{169}$

$\frac{10}{13} \times \frac{3}{13} = \frac{30}{169}$

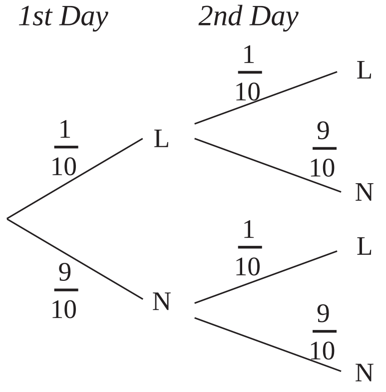
$\frac{10}{13} \times \frac{10}{13} = \frac{100}{169}$

(a) $\frac{9}{169}$ (b) $\frac{60}{169}$

(c) $\frac{69}{169}$ (d) $\frac{100}{169}$

6.3 Answers

9. L = Late, N = Not Late



PROBABILITIES

$$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$$

$$\frac{1}{10} \times \frac{9}{10} = \frac{9}{100}$$

$$\frac{9}{10} \times \frac{1}{10} = \frac{9}{100}$$

$$\frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$$

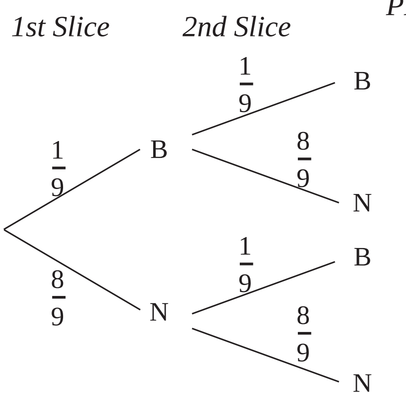
(a) $\frac{1}{100}$ (b) $\frac{9}{50}$

(c) $\frac{81}{100}$

10. (a) B = Burnt, N = Not Burnt

$p(\text{at least one slice burnt}) = \frac{17}{81}$

(see tree diagram opposite)



PROBABILITIES

$$\frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$$

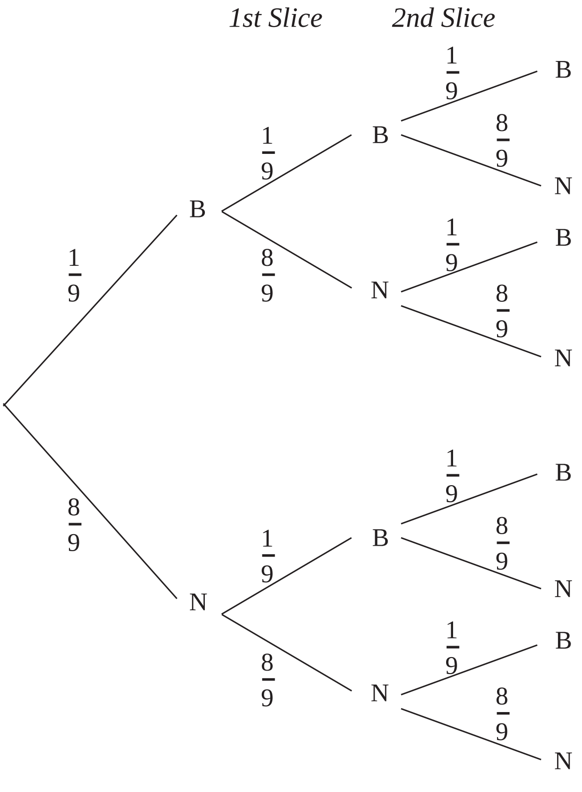
$$\frac{1}{9} \times \frac{8}{9} = \frac{8}{81}$$

$$\frac{8}{9} \times \frac{1}{9} = \frac{8}{81}$$

$$\frac{8}{9} \times \frac{8}{9} = \frac{64}{81}$$

(b) $p(\text{at least one slice burnt}) = \frac{217}{729}$

(see tree diagram below)



PROBABILITIES

$$\frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} = \frac{1}{729}$$

$$\frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{729}$$

$$\frac{1}{9} \times \frac{8}{9} \times \frac{1}{9} = \frac{8}{729}$$

$$\frac{1}{9} \times \frac{8}{9} \times \frac{8}{9} = \frac{64}{729}$$

$$\frac{8}{9} \times \frac{1}{9} \times \frac{1}{9} = \frac{8}{729}$$

$$\frac{8}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{64}{729}$$

$$\frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} = \frac{64}{729}$$

$$\frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} = \frac{512}{729}$$

6.3 Answers

11. (a) $\frac{1}{2}$

(b)

First coin	*Second coin*
heads	heads
heads	tails
tails	heads
tails	tails

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

12. (a) $\frac{35}{36}$

(b) $\frac{1}{36}$

(c) $\frac{1}{36}$ (d) $\frac{1}{6}$

13. (a) (i) $0.7 \times 0.7 = 0.49$ (ii) $(0.7 \times 0.3) + (0.3 \times 0.7) = 0.42$

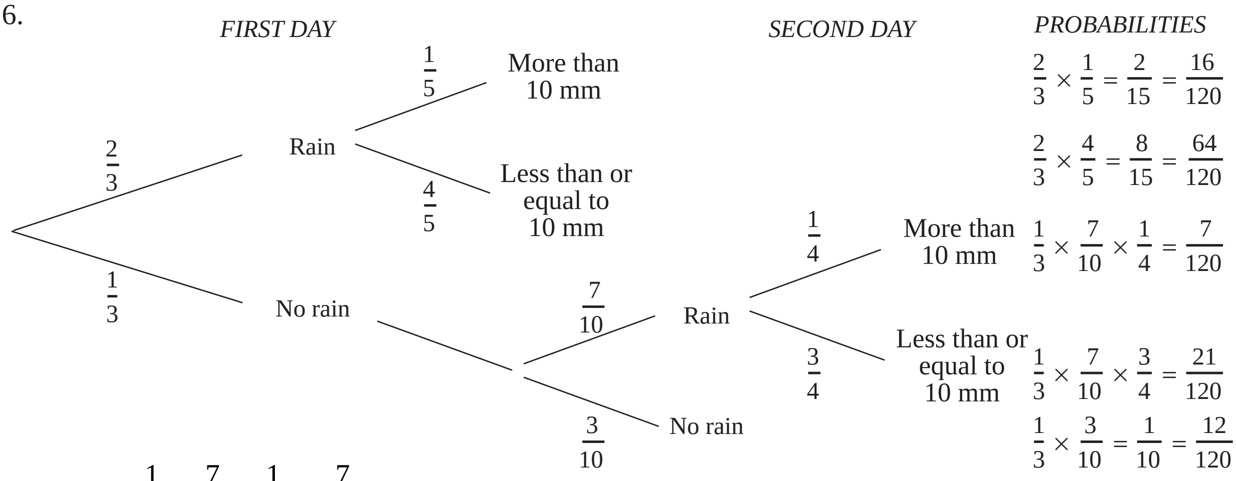
(b) p(drives through both sets of lights without stopping) $= 0.3 \times 0.3 = 0.09$,

so the estimated number of times he goes through unstopped $= 200 \times 0.09 = 18$.

14. (a) $\frac{69}{100}$ or 0.69 or 69% (b) $\frac{27}{57} = \frac{9}{19}$ (c) $\frac{27}{100} \times \frac{26}{100}$

15. (a) 0.000625 (b) 0.04875 (c) 120

16.



(a) $\frac{1}{3} \times \frac{7}{10} \times \frac{1}{4} = \frac{7}{120}$

(b) $\frac{2}{3} + \left(\frac{1}{3} \times \frac{7}{10}\right) = \frac{20}{30} + \frac{7}{30} = \frac{27}{30} = \frac{9}{10}$

(c) Because we are not given the probability that it rains in the second day if it rains on the first.

17. (a) $(0.6 \times 0.8) + (0.4 \times 0.55) = 0.7$ (b) $(0.6 \times 0.2) + (0.4 \times 0.55) = 0.34$

(c) $(0.6 \times 0.8 \times 0.1) + (0.6 \times 0.2 \times 0.9) + (0.4 \times 0.55 \times 0.9) = 0.354$

(d) $(0.6 \times 0.2 \times 0.1) + (0.4 \times 0.55 \times 0.1) + (0.4 \times 0.45 \times 0.9) = 0.196$

6.4 Answers

6.4 Theoretical and Experimental Probabilities

1. (a) 50 head and 50 tails (b) and (c) Pupils' own results, bar charts and comparisons.

2. (a) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{4}$

(b) 2 heads - 25 times; 1 head and 1 tail - 50 times; 2 tails - 25 times

(c) and (d) Pupils' own results and comparisons

3. (a) HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

(b) (i) 4 (ii) 12 (iii) 12 (iv) 4

4. (a)

Total	2	3	4	5	6	7	8	9	10	11	12
Expected Frequency	1	2	3	4	5	6	5	4	3	2	1

(b) Pupils' own results and comparisons

(c)

Total	2	3	4	5	6	7	8	9	10	11	12
Expected Frequency	4	8	12	16	20	24	20	16	12	8	4

(d) The results in (c) from 144 throws should be closer to the predictions than the results in (b) from 72 throws.

5. (a)

Dice

		1	2	3	4	5	6
Coin	*Heads*	2	4	6	8	10	12
	Tails	0	1	2	3	4	5

(b)

Total	0	1	2	3	4	5	6	8	10	12
Theoretical Probability	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

(c)

Total	0	1	2	3	4	5	6	8	10	12
Expected Frequency	10	10	20	10	20	10	10	10	10	10

(d) Pupils' own results and comparisons

6.4 Answers

6. (a) Shana: greater number of trials leads to better results.

   (b) The data does indicate that the dice is biased. If it was unbiased, we would expect to get frequencies of approximately 130 for each colour. However, the frequencies for red and blue are well above 130 and the frequencies for green and yellow are well below 130, which is fairly strong evidence that the dice is biased.

   (c) $p(\text{B}) = \frac{186}{520} = \frac{93}{260} = 0.358$ (to 3 d.p.) (d) $p(\text{G}) = \frac{75}{520} = \frac{15}{104} = 0.144$ (to 3 d.p.)

7. (a) Sue, because she conducted the experiment the greatest number of times.

   (b) $\frac{171}{300} = \frac{57}{100} = 0.57$

   (c)

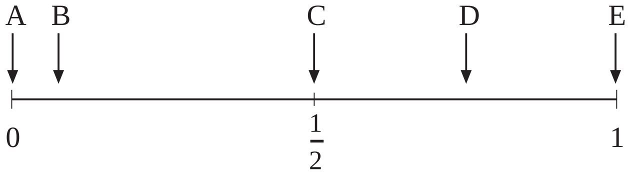
Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	$166\frac{2}{3}$ i.e. about 167 times	125	$8\frac{1}{3}$ i.e. about 8 times

   (d) There are insufficient trials for really accurate comparison.
   (The pupils' results are close to the theoretical results, i.e. they are within the margin of experimental error. In this case, it would be impossible to get the theoretical results exactly because the values are not all integers. Even if they were integers, we would still not expect to match the predicted results exactly because of experimental variation. The discrepancy between the predicted and experimental results should close as the experiment is conducted more often beyond the present 300 times.)

UNIT 6 *Probability* **Revision Test 6.1**

(Standard)

1. The following probability line shows the probabilities of 5 events:



(a) Which event is *certain*?

(b) Which event is *impossible*?

(c) Which event is *unlikely but not impossible*?

(d) Which event is *likely but not certain*?

(e) Which event is *equally likely to take place or not to take place*?

(5 marks)

2. (a) You toss an unbiased coin. What is the probability that you get a head?

(b) If you toss the coin 100 times, how many times would you expect to get a head?

(3 marks)

3. If you roll a fair dice, what is the probability that you get:

(a) a *six*,

(b) an *even* number,

(c) a number *less than* 3,

(d) a number *less than* 6 ?

(7 marks)

4. A packet contains different coloured sweets, as listed below:

8 *red* sweets
4 *green* sweets
6 *yellow* sweets
2 *blue* sweets

(a) How many sweets are there in the packet?

(b) What is the probability that a sweet taken from the bag at random is:

(i) *red* (ii) *yellow,* (iii) *blue* or *green,*

(iv) *not blue,* (v) *red* or *blue* ?

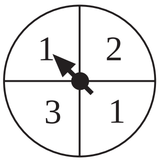
(11 marks)

Revision Test 6.1 (Standard)

5. The spinner shown is spun 200 times. How many times would you expect the pointer to land on:

(a) a 1,

(b) a 3 ?



(4 marks)

UNIT 6 *Probability*

Revision Test 6.2
(Academic)

1. A packet contains the sweets listed below:

18 *mints*
22 *chocolates*
10 *toffees*

A sweet is taken at random from the packet. What is the probability that it is:

(a) a *toffee*,
(b) a *toffee* or a *mint*,
(c) not a *chocolate*,
(d) not a *mint*.

(8 marks)

2. An unbiased dice is rolled 300 times. How many times would you expect to get:

(a) a *six*,
(b) an *odd* number,
(c) a number *less than 3* ?

(6 marks)

3. Two unbiased dice have faces marked

1, 2, 4, 6, 8, 10.

The two dice are rolled and their scores added together.

(a) Draw a table to list *all* the possible outcomes.
(b) What is the probability that the total score is:
  (i) 10,
  (ii) *less than* 8,
  (iii) *greater than* 5 ?

(9 marks)

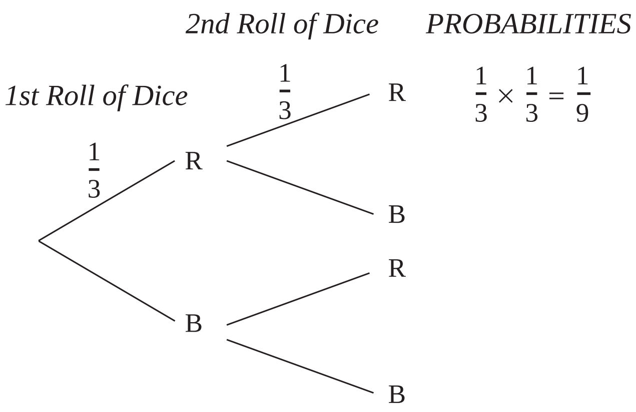
Revision Test 6.2 (Academic)

4. An unbiased dice is painted so that it has

2 *red* faces and 4 *blue* faces.

The dice is rolled twice.

(a) Copy and complete the following tree diagram:



(b) What is the probability of obtaining at least one *red* face?

(c) What is the probability of obtaining both faces the *same* colour?

(7 marks)

UNIT 6 *Probability*

Revision Test 6.3
(Express)

1. A bag contains 4 *red* balls, 12 *green* balls and 8 *yellow* balls. One ball is taken at random from the bag.

   What is the probability that it is:

   (a) *red,*

   (b) *yellow* or *green,*

   (c) *not green* ?

(6 marks)

2. An unbiased dice is painted so that it has 2 *red* faces and 4 *blue* faces. The dice is rolled twice. Use a tree diagram to determine the probability that the dice will show:

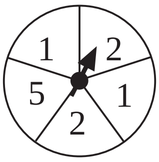
   (a) at least one *red* face,

   (b) two faces that are the *same* colour,

   (c) two faces that are *different* colours.

(10 marks)

3. The spinner shown is spun twice and the two scores added to give a total.



   (a) Draw a table to list all the possible outcomes.

   What is the probability that the total score is:

   (b) 6,

   (c) an *even* number,

   (d) *less than* 5 ?

(10 marks)

4. Two fair coins are tossed together 172 times. How many times would you expect to get:

   (a) two *heads,*

   (b) a *head* and a *tail,* in any order?

(4 marks)

Revision Test 6.1 (Standard) Answers

1.	(a)		E	B1	
	(b)		A	B1	
	(c)		B	B1	
	(d)		D	B1	
	(e)		C	B1	*(5 marks)*
2.	(a)		$\frac{1}{2}$	B1	
	(b)		$\frac{1}{2} \times 100 = 50$	M1 A1	*(3 marks)*
3.	(a)		$\frac{1}{6}$	B1	
	(b)		$\frac{3}{6} = \frac{1}{2}$	M1 A1	
	(c)		$\frac{2}{6} = \frac{1}{3}$	M1 A1	
	(d)		$\frac{5}{6}$	M1 A1	*(7 marks)*
4.	(a)		20	B1	
	(b)	(i)	$\frac{8}{20} = \frac{2}{5}$	M1 A1	
		(ii)	$\frac{6}{20} = \frac{3}{10}$	M1 A1	
		(iii)	$\frac{6}{20} = \frac{3}{10}$	M1 A1	
		(iv)	$\frac{18}{20} = \frac{9}{10}$	M1 A1	
		(v)	$\frac{10}{20} = \frac{1}{2}$	M1 M1	*(11 marks)*
5.	(a)		$\frac{1}{2} \times 200 = 100$	M1 A1	
	(b)		$\frac{1}{4} \times 200 = 50$	M1 A1	*(4 marks)*

(TOTAL MARKS 30)

Revision Test 6.2 (Academic) Answers

---

1. (a) $\frac{10}{50} = \frac{1}{5}$ M1 A1

(b) $\frac{28}{50} = \frac{14}{25}$ M1 A1

(c) $\frac{28}{50} = \frac{14}{25}$ M1 A1

(d) $\frac{32}{50} = \frac{16}{25}$ M1 A1 *(8 marks)*

2. (a) $\frac{1}{6} \times 300 = 50$ M1 A1

(b) $\frac{1}{2} \times 300 = 150$ M1 A1

(c) $\frac{1}{3} \times 300 = 100$ M1 A1 *(6 marks)*

3. (a)

Dice B

Dice A

	1	2	4	6	8	10
1	2	3	5	7	9	11
2	3	4	6	8	10	12
4	5	6	8	10	12	14
6	7	8	10	12	14	16
8	9	10	12	14	16	18
10	11	12	14	16	18	20

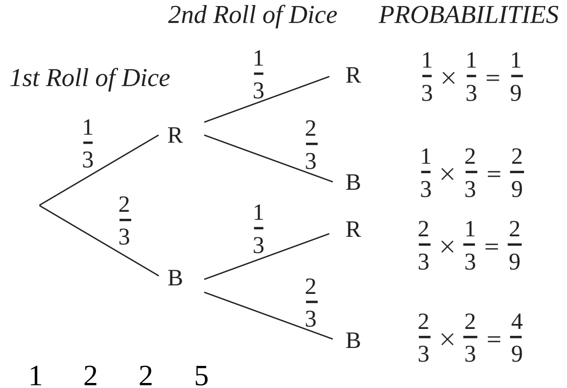
M1 A1 A1

(b) (i) $\frac{4}{36} = \frac{1}{9}$ M1 A1

(ii) $\frac{10}{36} = \frac{5}{18}$ M1 A1

(iii) $\frac{30}{36} = \frac{5}{6}$ M1 A1 *(9 marks)*

4. (a)



$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

M1 A1 A1

(b) $\frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}$ M1 A1

(c) $\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$ M1 A1 *(7 marks)*

(TOTAL MARKS 30)

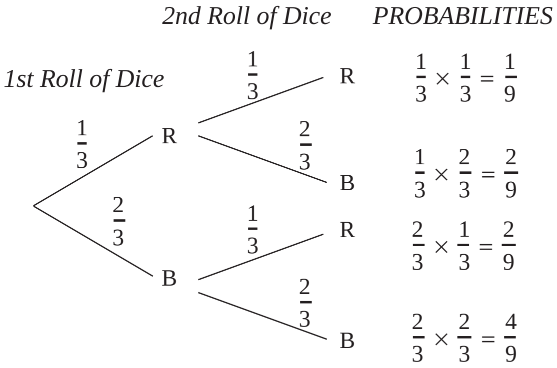
Revision Test 6.3 (Express) Answers

1. (a) $\frac{4}{24} = \frac{1}{6}$ M1 A1

(b) $\frac{20}{24} = \frac{5}{6}$ M1 A1

(c) $\frac{12}{24} = \frac{1}{2}$ M1 A1 *(6 marks)*

2.



M1 A1

M1 A1

(a) $\frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}$ M1 A1

(b) $\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$ M1 A1

(c) $\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$ M1 A1 *(10 marks)*

3. (a)

		First Spin				
		1	1	2	2	5
Second Spin	1	2	2	3	3	6
	1	2	2	3	3	6
	2	3	3	4	4	7
	2	3	3	4	4	7
	5	6	6	7	7	10

M1 A1 A1 A1

(b) $\frac{4}{25}$ M1 A1

(c) $\frac{13}{25}$ M1 A1

(d) $\frac{16}{25}$ M1 A1 *(10 marks)*

4. (a) $\frac{1}{4} \times 172 = 43$ M1 A1

(b) $\frac{1}{2} \times 172 = 86$ M1 A1 *(4 marks)*

(TOTAL MARKS 30)

UNIT 6 *Probability* Teaching Notes

This unit both revises and extends the earlier work on probability; additionally it emphasises the concept of theoretical and experimental frequencies and probabilities. Pupils should appreciate that experimental frequencies will not in general be exactly equal to the theoretical values, but that the theoretical model is still a valid one. An interesting source of material here is the national lottery results where the probability of any number being drawn is $\frac{1}{49}$, but the frequencies are not all equal! (See http:/lottery.merseyworld.com/).

Routes		**Standard**	**Academic**	**Express**
6.1	The Probability Scale	✓	(✓)	✗
6.2	The Probability of a Single Event	✓	✓	(✓)
6.3	The Probability of Two Events	✗	✓	✓
6.4	Theoretical and Experimental Probabilities	✗	(✓)	✓

Language	**Standard**	**Academic**	**Express**
Experimental and theoretical probabilities	✗	(✓)	✓

Misconceptions

- pupils must know that the probability of any event, p, must satisfy $0 \leq p \leq 1$, and that p can never be greater than 1 (or negative)
- pupils must know when probabilities must be *added* and when *multiplied*,

  e.g. $p\text{ (six)} = \frac{1}{6}$ so $p\text{ (2 sixes)} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$, etc. is one error seen over and over again!
- if you obtain 4 Heads in a row when tossing a fair coin, then the probability of Heads on the fifth throw is still $\frac{1}{2}$: this result often seems to be in conflict with the expectation that, over a period of many tosses of the coin, the number of Heads will approximately equate to the number of Tails. However, pupils must realise that each toss of the coin is an *independent event.*

(Other misconceptions are shown on Y7B, OS 21.10.)

Challenging Questions

The following questions are more challenging than others in the same section:

	Section	*Question No.*	*Page*
Practice Book Y9A	6.1	12	125
" "	6.2	11, 15	130/132
" "	6.3	13 - 17	140-143

UNITS 1-6 **Diagnostic Test 9B** (Standard)

You have ONE HOUR to complete this test.

1. $\boxed{1}$ $\boxed{7}$ $\boxed{3}$ $\boxed{5}$

Use some of the four number cards to make numbers that are *as close as possible* to the numbers written below.

Examples

80 → $\boxed{7}\boxed{5}$

30 → $\boxed{3}\boxed{1}$

You must *not* use the same card more than once in each answer.

50 → ☐☐

60 → ☐☐

4000 → ☐☐☐☐

1500 → ☐☐☐☐

1600 → ☐☐☐☐ *(5 marks)*

[KS3/99/Ma/Tier 3-5/P2]

UNITS 1-6

Diagnostic Test 9B (Standard)

2. Dan is doing a sponsored swim.

This is what Dan's friends promise to give for each length of the swimming pool he swims.

Ben says:	I will give Dan 20p a length.
Jan says:	I will give Dan 30p a length.
Cal says:	I will give Dan 25p a length.
Kim says:	I will give Dan 15p a length.
Wyn says:	I will give Dan 20p a length.

(a) Fill in the gaps in Dan's sponsor form.

Name	*Amount for each length*
Ben	20p
Cal	
	30p
	15p
	20p

(b) How much money will Dan collect altogether for each length he swims?

£

(c) Tom also did the sponsored swim. He swam 27 lengths.
He collected 75p for each length.

How much money did Tom collect for the swim?

£

UNITS 1-6

Diagnostic Test 9B (Standard)

(d) Nina swam 25 lengths in the sponsored swim.
She collected 72p for each length.
How much money did Nina collect for the swim?

£

(e) Nina's mother says:

> "Tell me how much you collected for your swim.
> I will give you a *quarter* of the amount."

How much should Nina's mother give her?

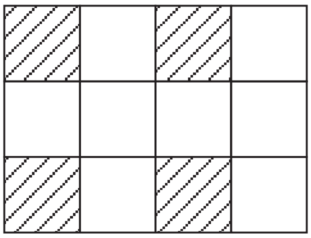
£

(5 marks)

[KS3/98/Ma/Tier 3-5/P2]

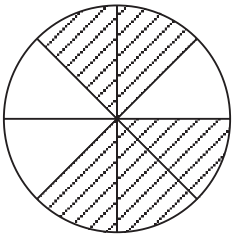
3. What fraction of each of the following shapes is shaded? Write each fraction in its simplest form.

(a)



Fraction shaded = $\frac{\square}{\square}$

(b)



Fraction shaded = $\frac{\square}{\square}$

(3 marks)

UNITS 1-6

Diagnostic Test 9B (Standard)

4. Calculate

$$\frac{5}{12} + \frac{2}{12} = \frac{\boxed{}}{\boxed{}}$$

(1 mark)

5. Calculate

$$\frac{2}{5} \text{ of } £20 = £\boxed{}$$

(2 marks)

6. Write 0.7 as a fraction.

$$0.7 \equiv \frac{\boxed{}}{\boxed{}}$$

(1 mark)

UNITS 1-6

Diagnostic Test 9B (Standard)

7. The cost of an old toy vehicle depends on its condition and on whether it is in its original box.

Condition	*Value*
excellent, and in its box	100%
good, and in its box	85%
poor, and in its box	50%
excellent, but not in its box	65%
good, but not in its box	32%
poor, but not in its box	15%

A Mail Van in excellent condition, and in its box, costs £125.

(a) How much is a Mail Van in *good* condition, and in its box?

£ ☐

(b) How much is a Mail Van in *good* condition, *but not in its box*?

£ ☐

(c) A Petrol Tanker in excellent condition, and in its box, costs £152.

Another Petrol Tanker should be sold for £98.80.
Using the chart above, what is its condition and does it have its box?

(3 marks)

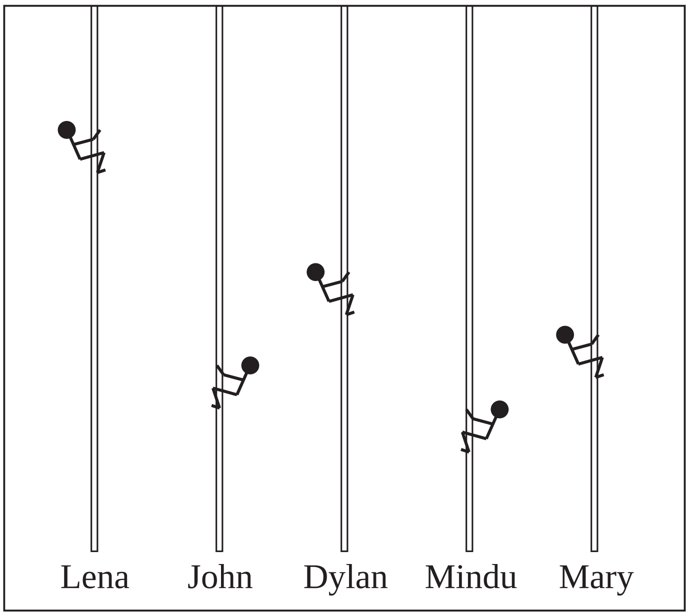
[KS3/97/Ma/Tier3-5/P2]

UNITS 1-6

Diagnostic Test 9B (Standard)

8. Some pupils are climbing up the ropes in the gym.

These are their positions after climbing for a few seconds.



(a) Dylan is about $\frac{1}{2}$ of the way *up* the rope.

Fill in each gap with a fraction.

Lena is about of the way up the rope.

John is about of the way up the rope.

UNITS 1-6

Diagnostic Test 9B (Standard)

(b) Dylan is about 50% of the way *up* the rope.

Fill in each gap with a percentage.

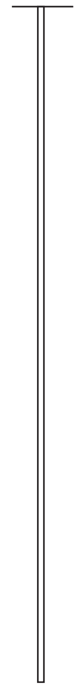
Mindu is about % of the way up the rope.

Mary is about % of the way up the rope.

(c) Anna is climbing a longer rope.

She has climbed $\frac{2}{5}$ of the way up the rope.

Put a × on the rope to show Anna's position.



Anna

(5 marks)

[KS3/95/Ma/Levels 4-6/P1]

UNITS 1-6

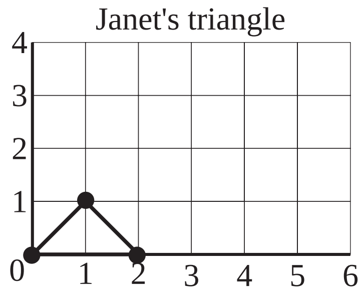
Diagnostic Test 9B (Standard)

9. Janet joins three points on a grid to make a triangle.

The co-ordinates of the points are:

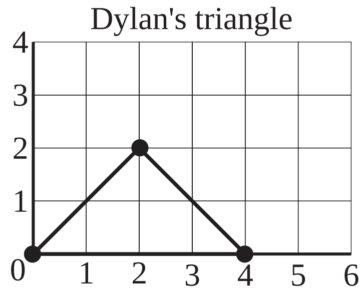
(0, 0)
(1, 1)
(2, 0)

The *area* of Janet's triangle is 1 cm^2.



Dylan *multiplies* each of Janet's co-ordinates by 2.

Janet's co-ordinates	× 2	*Dylan's co-ordinates*
(0, 0)	---►	(0, 0)
(1, 1)	---►	(2, 2)
(2, 0)	---►	(4, 0)



(a) What is the *area* of *Dylan's* triangle?

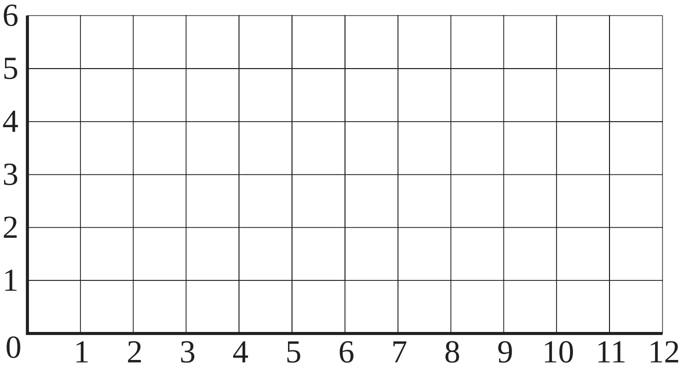
area = ... cm^2

(b) *Multiply* each of Janet's co-ordinates by 3.

Janet's co-ordinates	× 3	*Dylan's co-ordinates*
(0, 0)	---►	(....... , ...)
(1, 1)	---►	(....... , ...)
(2, 0)	---►	(....... , ...)

Plot the 3 points with the *new co-ordinates* on the grid below.

Join them up to make a triangle.



UNITS 1-6

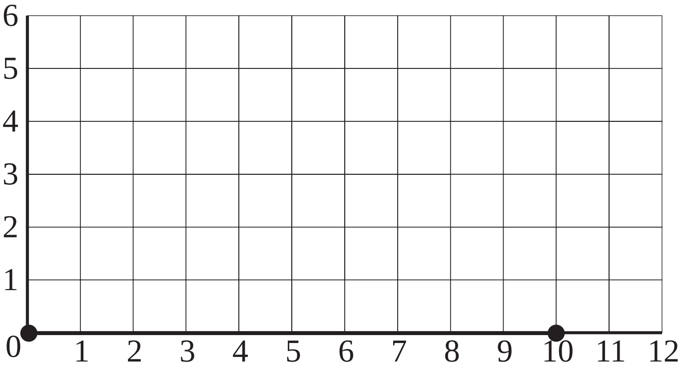
Diagnostic Test 9B (Standard)

(c) What is the *area* of your triangle?

area = ... cm^2

Nazir multiplies each of Janet's co-ordinates by *another* number.
He plots two of the points, (0, 0) and (10, 0), and joins them up.

(d) Plot Nazir's *third point.*



(e) *What number* did Nazir *multiply* Janet's co-ordinates by?

....

(5 marks)

[KS3/96/Ma/Tier 4-6/P2]

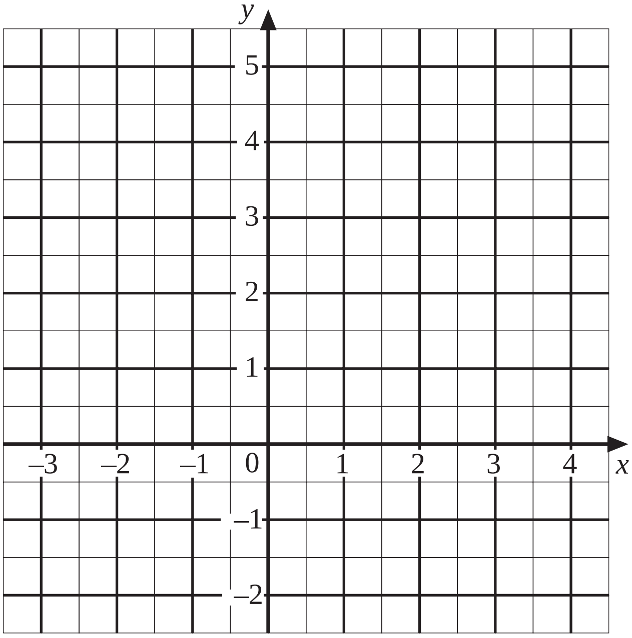
UNITS 1-6

Diagnostic Test 9B (Standard)

10. (a) Complete the following table for $y = x + 2$.

x	-3	-2	-1	0	1	2	3
y	-1	....	....	....	3	....	....

(b) Draw the graph of $y = x + 2$ on the following axes:



(4 marks)

11. Solve the following equations:

(a) $3x = 27$

$x = \boxed{}$

(b) $x + 5 = 14$

$x = \boxed{}$

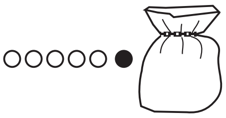
(2 marks)

UNITS 1-6

Diagnostic Test 9B (Standard)

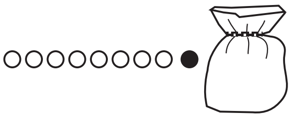
12. Sue puts 5 white beads and 1 black bead in a bag.

Sue takes a bead without looking.

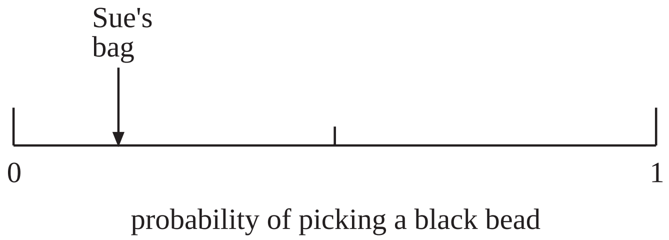


Ed puts 8 white beads and 1 black bead in a bag.

Ed takes a bead without looking.



(a) The arrow shows the probability that Sue gets a *black* bead from her bag.



Put an arrow on the line to show the probability that Ed gets a *black* bead from his bag. Put Ed's name above the arrow.

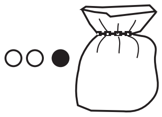
How did you decide which side of Sue's arrow to put Ed's arrow?

(b) Bob puts 2 white beads and 1 black bead in a bag.

Bob thinks he has a 50% chance of taking a *black* bead without looking.

He is wrong.

Explain why.



What is the chance that Bob gets a *black* bead?

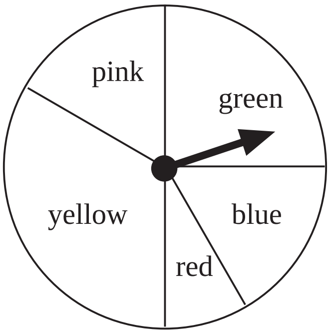
(c) Put an arrow on the line in part (a) to show the probability that Bob gets a *black* bead from his bag. Put Bob's name above the arrow.

(5 marks)

[KS3/94/Ma/4-6/P1]

UNITS 1-6

Diagnostic Test 9B (Standard)



13. Brenda wants to spin this spinner.

What colour is she *most* likely to get?

Explain why.

What colour is she *least* likely to get?

Explain why.

(b) Brenda says:

"I've got an equal chance of getting pink or blue."

Is Brenda right or wrong? ....

Give a reason for your answer.

(c) Brenda says:

"My chance of getting green is about $\frac{1}{2}$."

She is wrong.

Make a better estimate of the chance that Brenda will get green.

UNITS 1-6

Diagnostic Test 9B (Standard)

(d) Brenda says:

"The probability that I will get yellow is $\frac{1}{5}$ because there are 5 colours."

She is wrong.

Explain why.

Make a better estimate of the probability that Brenda will get yellow.

(e) Write down a rough estimate of the probability of getting each colour.

colour	yellow	green	pink	blue	red
probability	.... %	.... %	.... %	.... %	.... %

(9 marks)

[KS3/94/Ma3-5/P2]

UNITS 1-6 **Diagnostic Test 9B** (Standard) Answers

1.		51, 57, 3751, 1537, 1573	B1 B1 B1 B1 B1	*(5 marks)*
2.	(a)			

Name	*Amount for each length*
Ben	20p
Cal	25p
Jan	30p
Kim	15p
Wyn	20p

			B1	
	(b)	£1.10	B1	
	(c)	£20.25	B1	
	(d)	£18	B1	
	(e)	£4.50	B1	*(5 marks)*
3.	(a)	$\frac{4}{12} = \frac{1}{3}$	M1 A1	
	(b)	$\frac{5}{8}$	B1	*(3 marks)*
4.		$\frac{7}{12}$	B1	*(1 mark)*
5.		$\frac{2}{5} \times 20 = \frac{40}{5} = £8$	M1 A1	*(2 marks)*
6.		$\frac{7}{10}$	B1	*(1 mark)*
7.		£106.25	B1	
		£40	B1	
		Excellent, but not in its box	B1	*(3 marks)*
8.	(a)	$\frac{3}{4}, \frac{1}{3}$	B1 B1	
	(b)	25% , 40%	B1 B1	
	(c)	Anna's position	B1	*(5 marks)*
9.	(a)	4 cm^2	B1	
	(b)	(0, 0), (3, 3), (6, 0) and triangle	B1	
	(c)	9 cm^2	B1	
	(d)	Point (5, 5)	B1	
	(e)	5	B1	*(5 marks)*

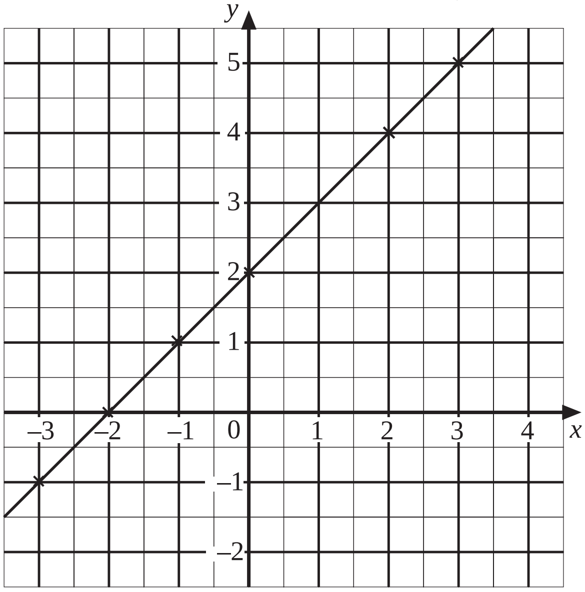
UNITS 1-6 Diagnostic Test 9B (Standard) Answers

10. (a)

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

(–1 for each mistake) B2

(b)



B2 *(4 marks)*

11. (a) $x = 9$ B1

(b) $x = 14 - 5 = 9$ B1 B1 *(2 marks)*

12. (a) Position to left of Sue, since probability B1
is less for Ed B1

(b) There are 2 white and 1 black beads (i.e. not equal) B1

p (black) $= \frac{1}{3}$ B1

(c) arrow B1 *(5 marks)*

13. (a) Yellow; as it has the largest sector. B1
Red; as it has the smallest sector. B1

(b) Yes; as they have equal-sized sectors B1

(c) $\frac{1}{4}$ B1

(d) Not all colours have equal chance B1

as sectors have different sizes; $\frac{1}{3}$ B1

(e) 30%-35%, 23%-27%, 15%-20%, 15%-20%, 6%-10% B3 *(9 marks)*

(TOTAL MARKS 50)

UNITS 1-6 **Diagnostic Test 9B** (Standard) Answers

Marks

Unit	**1-3**	**4**	**5**	**6**	
Question	1 - 2	3 - 8	9 - 11	12 - 13	
Total marks available	10	15	11	14	*Final total*
Total					

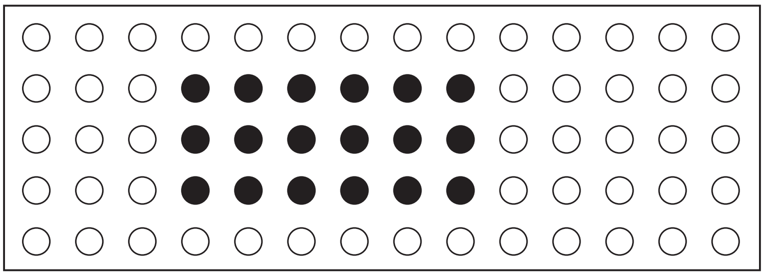
Assessment

45 +	Excellent – should be on *Academic Route*
40 - 44	Very good progress
30 - 39	Good progress: look carefully at mistakes
20 - 29	Steady progress, but you will need to work more carefully and/or make more effort
- 19	Struggling, so look carefully at your weak points to see where to target extra work

UNITS 1-6 **Diagnostic Test 9B** (Academic)

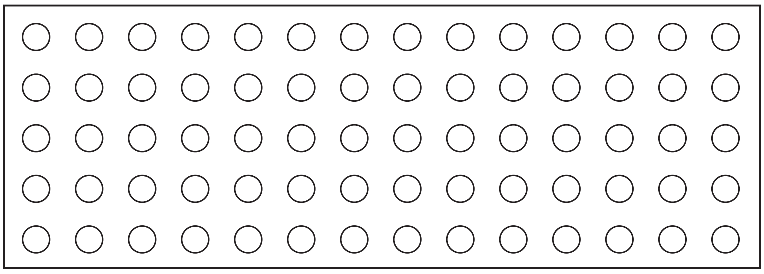
You have ONE HOUR to complete this test.

1. Gareth has some pegs and a pegboard.
He can make *a rectangle* with 18 pegs.



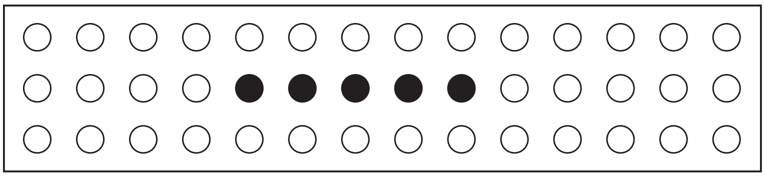
Gareth's rectangle is 6 pegs *long* and 3 pegs wide.

(a) Show how to use 18 pegs to make *another* rectangle with a *different* shape.
The rectangle must be more than 1 peg long and more than 1 peg wide.



How many pegs *long* is your rectangle?	How many pegs *wide* is your rectangle?
.... pegs *long*	.... pegs *wide*

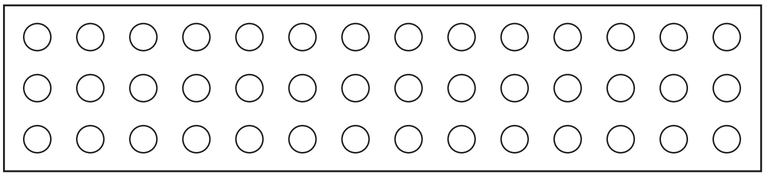
Gareth *cannot* make a rectangle with 5 pegs. He can only make a row.
This is because 5 is a *prime number*.



UNITS 1-6

Diagnostic Test 9B (Academic)

(b) Draw a row with a *prime number* of pegs which is *greater than 5*.



What is your prime number?

....

(c) Gareth says:

"9 is a *prime number.*"

Explain why Gareth is *wrong*.

You can write your answer, or draw a diagram.

(4 marks)

[KS3/97/Ma/Tier 3-5/P1]

2. Complete each of the following statements by filling in the missing numbers:

(a) $81 = 9^{\square}$

(b) $\left(3^{4}\right)^{3} = 3^{\square}$

(c) $5^{2} \times 5^{4} = 5^{\square}$

(3 marks)

UNITS 1-6

Diagnostic Test 9B (Academic)

3. Wendy is making a scale model of the Earth and the Moon for a museum.

She has found out the diameters of the Earth and the Moon, and the distance between them in centimetres.

Diameter of the Earth 1.28×10^{9} cm

Diameter of the Moon 3.48×10^{8} cm

Distance between Earth and Moon 3.89×10^{10} cm

(a) How many times bigger is the diameter of the Earth than the diameter of the Moon?

Show your working.

.... times

(b) In Wendy's scale model the diameter of the Earth is 50 cm.

What should be the distance between the Earth and the Moon in Wendy's model?

Show your working.

.... cm

(3 marks)

[KS3/96/Ma/Tier 6-8/P2]

UNITS 1-6

Diagnostic Test 9B (Academic)

4. Calculate the following, giving each answer in its simplest form:

$$\frac{5}{12} + \frac{3}{12} = \text{.................}$$

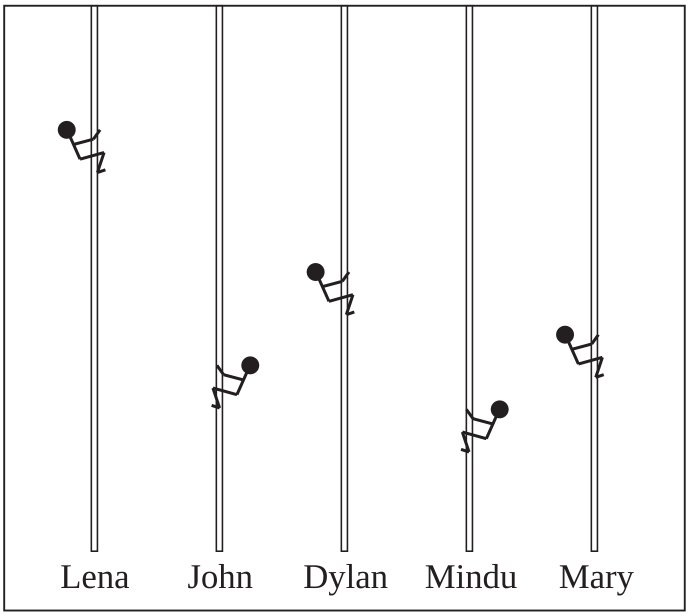
(2 marks)

5. Calculate $\frac{4}{9}$ of £63 = £ ☐

(2 marks)

6. Some pupils are climbing up the ropes in the gym.

These are their positions after climbing for a few seconds.



(a) Dylan is about $\frac{1}{2}$ of the way *up* the rope.

Fill in each gap with a fraction.

Lena is about of the way up the rope.

John is about of the way up the rope.

UNITS 1-6

Diagnostic Test 9B (Academic)

(b) Dylan is about 50% of the way *up* the rope.

Fill in each gap with a percentage.

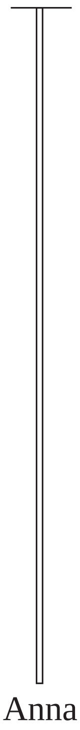
Mindu is about % of the way up the rope.

Mary is about % of the way up the rope.

(c) Anna is climbing a longer rope.

She has climbed $\frac{2}{5}$ of the way up the rope.

Put a × on the rope to show Anna's position.



(5 marks)

[KS3/95/Ma/Levels 4-6/P1]

UNITS 1-6

Diagnostic Test 9B (Academic)

7. Emlyn is doing a project on world population.

He has found some data about the population of the regions of the world in 1950 and 1990.

Regions of the World	*Population* (in millions) *in 1950*	*Population* (in millions) *in1990*
Africa	222	642
Asia	1558	3402
Europe	393	498
Latin America	166	448
North America	166	276
Oceania	13	26
World	2518	5292

(a) In 1950, what percentage of the world's population lived in *Asia*?

Show each step in your working.

.... %

(b) In 1990, for every person who lived in *North America* how many people lived in *Asia*?

Show your working.

.... people

UNITS 1-6

Diagnostic Test 9B (Academic)

(c) For every person who lived in *Africa* in 1950 how many people lived in *Africa* in 1990 ?

Show your working.

.... people

(d) Emlyn thinks that from 1950 to 1990 the population of *Oceania* went up by 100%.

Is Emlyn right?

Tick the correct box.

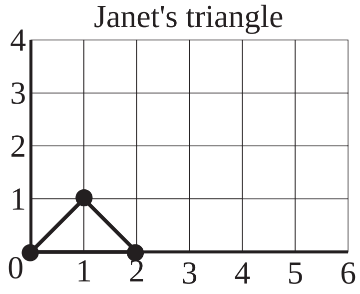
☐ Yes ☐ No ☐ Cannot tell

Explain your answer.

(5 marks)

[KS3/96/Ma/Tier 5-7/P2]

8. Janet joins three points on a grid to make a triangle.



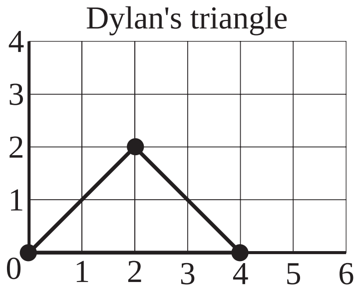
The co-ordinates of the points are:

(0, 0)
(1, 1)
(2, 0)

The *area* of Janet's triangle is 1 cm^2.

Dylan *multiplies* each of Janet's co-ordinates by 2.

Janet's co-ordinates	× 2	*Dylan's co-ordinates*
(0, 0)	---→	(0, 0)
(1, 1)	---→	(2, 2)
(2, 0)	---→	(4, 0)



(a) What is the *area* of *Dylan's* triangle?

area = ... cm^2

UNITS 1-6

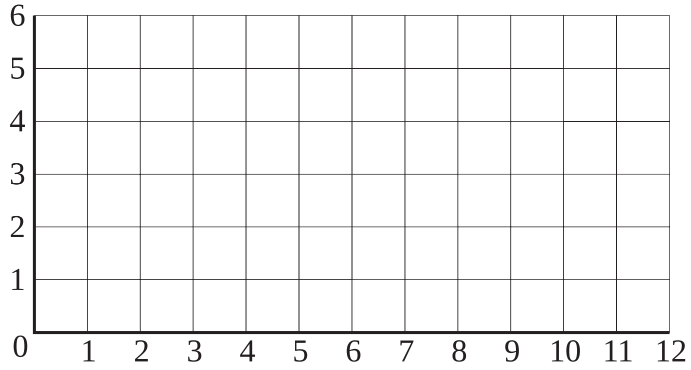
Diagnostic Test 9B (Academic)

(b) *Multiply* each of Janet's co-ordinates by 3.

Janet's co-ordinates	× 3	*Dylan's co-ordinates*
(0, 0)	--->	(....... , ...)
(1, 1)	--->	(....... , ...)
(2, 0)	--->	(....... , ...)

Plot the 3 points with the *new co-ordinates* on the grid below.

Join them up to make a triangle.

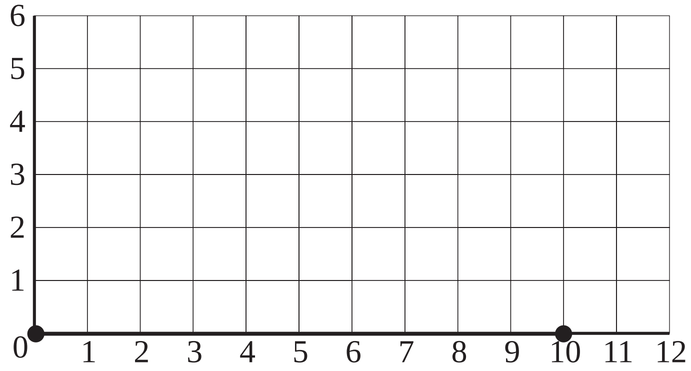


(c) What is the *area* of your triangle?

area = ... cm^2

Nazir multiplies each of Janet's co-ordinates by *another* number.
He plots two of the points, (0, 0) and (10, 0), and joins them up.

(d) Plot Nazir's *third point.*



(e) *What number* did Nazir *multiply* Janet's co-ordinates by?

....

(5 marks)

[KS3/96/Ma/Tier 4-6/P2]

UNITS 1-6

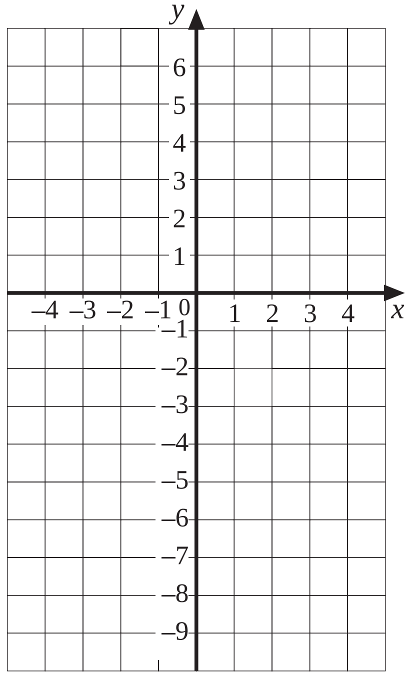
Diagnostic Test 9B (Academic)

9. (a) Complete the following table for $y = 2x - 3$.

x	-3	-2	-1	0	1	2	3
y	-9	....	....	-3	....	....	....

(2 marks)

(b) Draw the graph of $y = 2x - 3$ on the following axes:



(2 marks)

10. Solve the following equations:

(a) $2x + 7 = 19$

$x =$ ☐

(b) $3(x + 5) = 24$

$x =$ ☐

(3 marks)

UNITS 1-6

Diagnostic Test 9B (Academic)

11. Which of the following lines,

A : $y = x - 3$

B : $y = 3x + 1$

C : $y = 4 - 2x$

D : $y = 2x - 1$

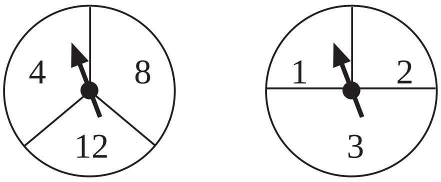
E : $y = 2 - 3x$

(a) is *parallel* to $y = 2x + 3$, line ☐

(b) is *perpendicular* to $y = 5 - \frac{1}{3}x$? line ☐

(2 marks)

12. Alun has these two spinners.



Alun spins both spinners and then adds up the numbers to get a total.

He starts to make a list of all the possible totals.

Complete Alun's list.

Totals

5

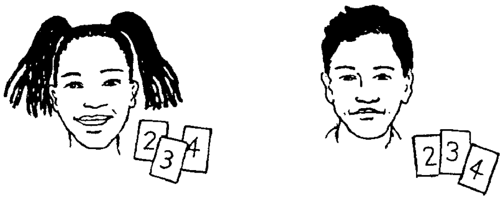
(2 marks)

[KS3/95/Ma/Levels 4-6/P1]

UNITS 1-6

Diagnostic Test 9B (Academic)

13. Karen and Huw each have three cards, numbered 2, 3 and 4.



They each take any *one* of their own cards.
Then they *add* together the numbers on the two cards.
The table shows all possible answers.

Karen

	+	**2**	**3**	**4**
Huw	**2**	4	5	6
	3	5	6	7
	4	6	7	8

(a) What is the *probability* that their answer is an *even* number?

(b) What is the *probability* that their answer is a number that is *greater than* 6 ?

UNITS 1-6

Diagnostic Test 9B (Academic)

(c) Both Karen and Huw still have their three cards, numbered 2, 3 and 4.

They each take any one of their own cards.
Then they *multiply* together the numbers on the two cards.

Draw a table to show all possible answers.

Use your table to fill in the gaps below:

The probability that their answer is a number that is

less than .... is $\frac{8}{9}$.

The probability that their answer is a number that is

less than .... is *zero*.

(5 marks)

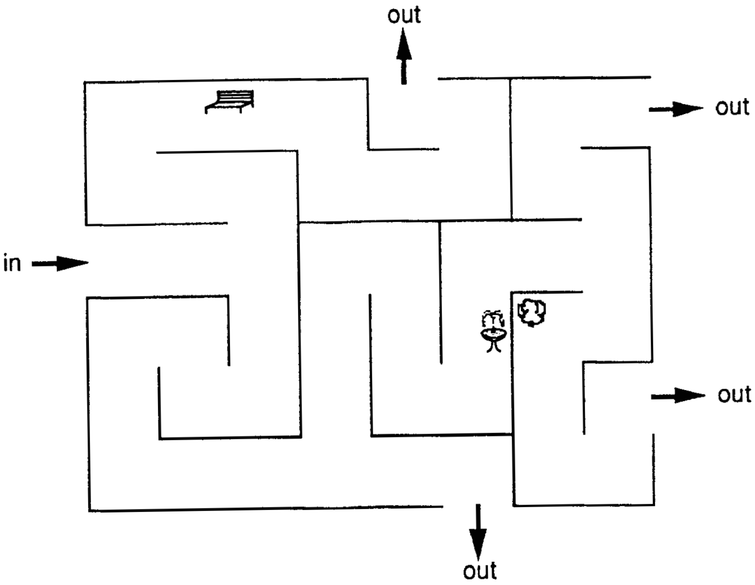
[KS3/97/Ma/Tier 4-6/P1]

UNITS 1-6

Diagnostic Test 9B (Academic)

14. Sara goes in a maze. In it there are:

a seat;
a fountain;
and a bush.



At each junction there is an equal chance of choosing each path so the probability of passing each object is:

seat	0.5
fountain	0.25
bush	0.125

(a) Work out the probability that Sara passes the seat *or* the fountain.
Show your working.

(b) Look at the maze.

Explain why the probability that Sara passes the fountain *or* the bush is *not* $0.25 + 0.125$.

UNITS 1-6

Diagnostic Test 9B (Academic)

(c) Sara and John each walk round the maze independently.
What is the probability that Sara and John will both pass the fountain?
Show your working.

(d) Explain one way in which the probability in part (c) would be different if they did not walk round the maze independently.

(5 marks)

UNITS 1-6 Diagnostic Test 9B (Academic) Answers

1.	(a)	9×2 on grid	B1	
		9, 2	B1	
	(b)	7, 11 or 13	B1	
	(c)	$9 = 3 \times 3$	B1	*(4 marks)*
2.	(a)	2 (b) 12 (c) 6	B1 B1 B1	*(3 marks)*
3.	(a)	$\frac{1280}{348} = 3.68$ (to 2 d.p.)	B1	
	(b)	$\frac{3890}{128} \times 50 = 1519.53$ cm (to 2 d.p.)	B1	*(3 marks)*
4.		$\frac{8}{12} = \frac{2}{3}$	M1 A1	*(2 marks)*
5.		$\frac{4}{9} \times £63 = 4 \times 7 = £28$	M1 A1	*(2 marks)*
6.	(a)	$\frac{3}{4}, \frac{1}{3}$	B1 B1	
	(b)	25% , 40%	B1 B1	
	(c)	Anna's position	B1	*(5 marks)*
7.	(a)	$\frac{1558}{2518} \times 100 = 61.87\%$	M1 A1	
	(b)	$\frac{3402}{276} = 12.32$	B1	
	(c)	$\frac{642}{222} = 2.89$	B1	
	(d)	Yes, the population was 13 and went up by 13.	B1	*(5 marks)*
8.	(a)	4 cm^2	B1	
	(b)	(0, 0), (3, 3), (6, 0) and triangle	B1	
	(c)	9 cm^2	B1	
	(d)	Point (5, 5)	B1	
	(e)	5	B1	*(5 marks)*

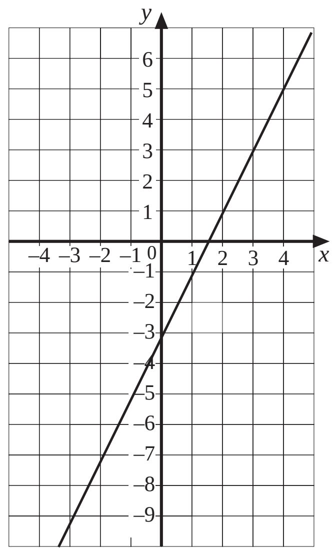
9. (a)

x	-3	-2	-1	0	1	2	3
y	-9	-7	-5	-3	-1	1	3

(–1 for each mistake) B2

UNITS 1-6 Diagnostic Test 9B (Academic) Answers

(b)



B2 *(4 marks)*

10. (a) $x = \frac{(19 - 7)}{2} = 6$ B1

(b) $x = \frac{24}{3} - 5 = 3$ M1 A1 *(3 marks)*

11. (a) D (b) B B1 B1 *(2 marks)*

12. 5, 9, 13 ; 6, 10, 14 ; 7, 11, 15 (–1 for each mistake) B2 *(2 marks)*

13. (a) $\frac{5}{9}$ B1

(b) $\frac{3}{9}\left(= \frac{1}{3}\right)$ B1

(c) 16 ; 4 B1 B1 B1 *(5 marks)*

	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

14. (a) $0.5 + 0.25 = 0.75$ B1

(b) They are not exclusive events. B1

(c) $(0.25)^2 = 0.0625$ M1 A1

(d) If they walked together, it would be 0.25. B1 *(5 marks)*

(TOTAL MARKS 50)

UNITS 1-6 Diagnostic Test 9B (Academic) Answers

Marks

Unit	**1-3**	**4**	**5**	**6**	*Final total*
Question	1 - 3	4 - 7	8 - 11	12 - 14	
Total marks available	10	14	14	12	
Total					

Assessment

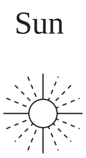
45 +	Excellent – should be on *Express Route*
40 - 44	Very good progress
30 - 39	Good progress: look carefully at mistakes
20 - 29	Steady progress, but you will need to work more carefully and/or make more effort
- 19	Struggling, so look carefully at the mistakes in your work; you might be better advised to transfer to the *Standard Route*

UNITS 1-6 **Diagnostic Test 9B** (Express)

You have ONE HOUR to complete this test.

1. Johann Bode was a mathematician and astronomer. In 1772 he made calculations which suggested that there was an unknown planet. His work led to the discovery of asteroids, which are pieces of rock moving around the sun.

Vanessa copied some of his methods. She recorded the average distance in kilometres of some planets from the sun.



Planet		*Distance from sun*
Venus	●	1.082×10^{8} km
Earth	●	1.496×10^{8} km
Mars	●	2.279×10^{8} km
Jupiter	●	7.783×10^{8} km
Saturn	●	1.427×10^{9} km

Vanessa said:

"Saturn is about 1200 million km further away from the sun than Mars."

(a) Is Vanessa right?

Show the calculation you did to help you decide.

Give a reason for your decision.

UNITS 1-6

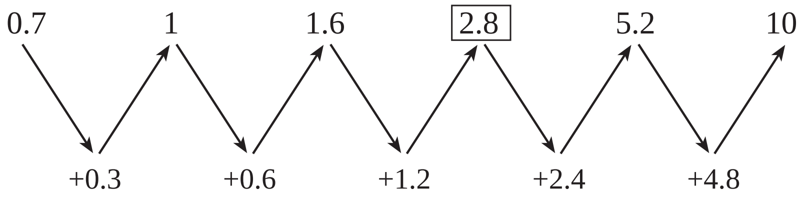
Diagnostic Test 9B (Express)

Vanessa calculated the ratio $\dfrac{\text{distance of planet from the sun}}{\text{distance of earth from the sun}}$.

(b) Work out the ratio for Jupiter and Saturn, correct to 2 decimal places.
Complete the table.

planet	Venus	Earth	Mars	Jupiter	Saturn
ratio	0.72	1	1.52	....	....

Johann Bode noticed that the ratios were similar to some of the numbers in this pattern:



(c) If there was an unknown planet, the ratio for it would be approximately 2.8.
Use the ratio 2.8 to calculate how far the unknown planet would be from the sun.

(d) You can use a formula to find the average distance of an object from the sun in kilometres.

The formula is $R = \dfrac{Gm}{V^2}$.

$R =$ the average distance of an object from the sun in kilometres

$V =$ the speed of the object in kilometres per second

$G =$ gravitational constant $= 6.67 \times 10^{-20}$

$m =$ mass of sun $= 1.993 \times 10^{30}$ kg

UNITS 1-6

Diagnostic Test 9B (Express)

Pieces of rock are moving round the sun with a speed of 17.5 km/s.
Use the formula to find out how far they are from the sun.

Show your working.

(e) Could these pieces of rock be parts of the missing planet between Mars and Jupiter?

Explain why or why not.

(10 marks)

[KS3/94/Ma/6-8/P1]

UNITS 1-6

Diagnostic Test 9B (Express)

2. Emlyn is doing a project on world population.

He has found some data about the population of the regions of the world in 1950 and 1990.

Regions of the World	*Population* (in millions) *in 1950*	*Population* (in millions) *in1990*
Africa	222	642
Asia	1558	3402
Europe	393	498
Latin America	166	448
North America	166	276
Oceania	13	26
World	2518	5292

(a) In 1950, what percentage of the world's population lived in *Asia*?

Show each step in your working.

.... %

(b) In 1990, for every person who lived in *North America* how many people lived in *Asia*?

Show your working.

.... people

UNITS 1-6

Diagnostic Test 9B (Express)

(c) For every person who lived in *Africa* in 1950 how many people lived in *Africa* in 1990 ?

Show your working.

.... people

(d) Emlyn thinks that from 1950 to 1990 the population of *Oceania* went up by 100%.

Is Emlyn right?

Tick the correct box.

☐ Yes ☐ No ☐ Cannot tell

Explain your answer.

(5 marks)

[KS3/96/Ma/Tier 5-7/P2]

3. A report on the number of police officers in 1995 said:

"There were 119 000 police officers.

Almost 15% of them were *women.*"

(a) The *percentage* was *rounded* to the nearest whole number, 15.

What is the *smallest* value the percentage could have been, to one decimal place?

Circle the correct answer below.

14.1% 14.2% 14.3% 14.4% 14.5%

14.6% 14.7% 14.8% 14.9%

Diagnostic Test 9B (Express)

(b) What is the *smallest number* of women police officers that there might have been in 1995?

(Use your answer to part (a) to help you calculate this answer.)

Show your working.

(c) A different report gives exact figures:

Number of women police officers	
1988	12 540
1995	17 468

Calculate the *percentage increase* in the number of women police officers from 1988 to 1995.

Show your working.

.... %

(d) The table opposite shows the *percentage* of police officers in 1995 and 1996 who were women.

1995	14.7%
1996	14.6%

Use the information in the table to decide which one of the statements below is true. Put a tick (√) by the true statement.

In 1996 there were *more* women police officers than in 1995. ....

In 1996 there were *fewer* women police officers than in 1995. ....

There is *not enough information* to tell whether there were more or fewer women police officers. ....

Explain your answer.

(6 marks)

KS3/99/Ma/Tier 6-8/P2]

UNITS 1-6

Diagnostic Test 9B (Express)

4. Calculate:

(a) $\frac{3}{5} \times \frac{10}{7} =$

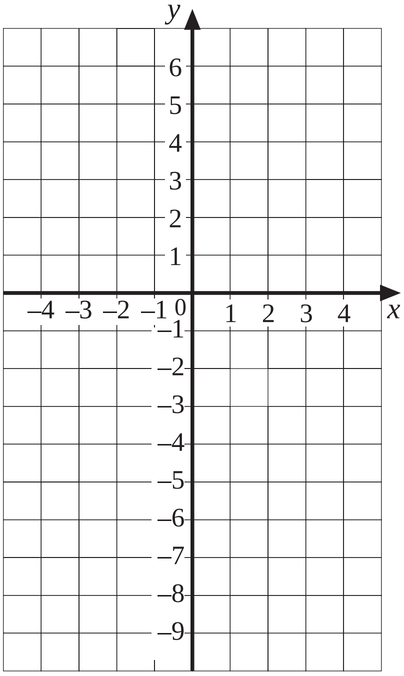
(b) $3\frac{3}{4} \div 1\frac{1}{4} =$

(4 marks)

5. (a) Complete the following table for $y = 2x - 3$.

x	– 3	– 2	– 1	0	1	2	3
y	– 9	....	....	– 3	....	....	....

(b) Draw the graph of $y = 2x - 3$ on the following axes:



(4 marks)

UNITS 1-6

Diagnostic Test 9B (Express)

6. Solve the following equation:

$$5(2x+9)=35$$

$x = \boxed{}$

(2 marks)

7. Class 9H were playing a number game.

Elin said:

"Multiplying my number by 4 and then subtracting 5
gives the same answer as
multiplying my number by 2 and then adding 1."

(a) Lena called Elin's number x and formed an equation:

$$4x-5=2x+1$$

Solve this equation and write down the *value* of x.

Show your working.

x = ...

Aled said:

"Multiplying my number by 2 and then adding 5
gives the same answer as
subtracting my number from 23."

(b) Call Aled's number y and form an equation.

.... = ...

Work out the *value* of Aled's number.

Aled's number is ....

UNITS 1-6

Diagnostic Test 9B (Express)

(c) Lena thought of two numbers which she called a and b.

She wrote down this information about them in the form of equations:

$$\begin{aligned} a + 3b &= 25 \\ 2a + b &= 15 \end{aligned}$$

Work out the values of a and b.

Show your working.

a = ...

b = ...

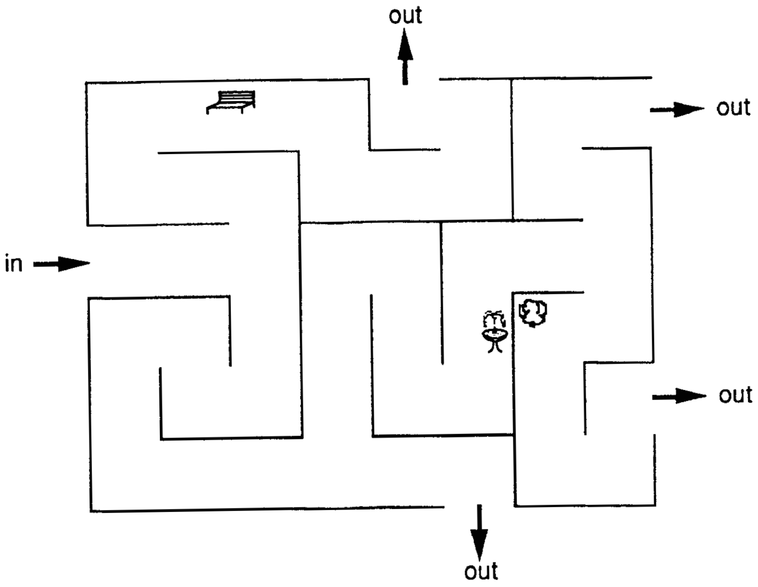
(7 marks)

UNITS 1-6

Diagnostic Test 9B (Express)

8. Sara goes in a maze. In it there are:

a seat; 
a fountain; 
and a bush. 



At each junction there is an equal chance of choosing each path so the probability of passing each object is:

seat	0.5
fountain	0.25
bush	0.125

(a) Work out the probability that Sara passes the seat *or* the fountain. Show your working.

(b) Look at the maze.

Explain why the probability that Sara passes the fountain *or* the bush is *not* $0.25 + 0.125$.

UNITS 1-6

Diagnostic Test 9B (Express)

(c) Sara and John each walk round the maze independently.

What is the probability that Sara and John will both pass the fountain?

Show your working.

(d) Explain one way in which the probability in part (c) would be different if they did not walk round the maze independently.

(5 marks)

9. Some pupils threw 3 fair dice.

They recorded how many times the numbers on the dice were the same.

Name	*Number of throws*	*Results*		
		all different	*2 the same*	*all the same*
Morgan	40	26	12	2
Sue	140	81	56	3
Zenta	20	10	10	0
Ali	100	54	42	4

(a) Write the name of the pupil whose data are *most likely* to give the best estimate of the probability of getting each result.

....

Explain your answer.

UNITS 1-6

Diagnostic Test 9B (Express)

(b) This table shows the pupils' results collected together:

Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	171	120	9

Use these data to estimate the *probability* of throwing numbers that are *all different.*

(c) The theoretical probability of each result is shown below:

	all different	*2 the same*	*all the same*
Probability	$\frac{5}{9}$	$\frac{5}{12}$	$\frac{1}{36}$

Use these probabilities to calculate, for 300 throws, *how many times* you would theoretically expect to get each result.

Number of throws	*Theoretical results*		
	all different	*2 the same*	*all the same*
300	....	....	....

(d) Explain why the pupils' results are not the same as the theoretical results.

UNITS 1-6

Diagnostic Test 9B (Express)

(e) Jenny throws the 3 dice twice.

Calculate the probability that she gets *all the same* on her first throw and gets *all the same* on her second throw.

Show your working.

(7 marks)

[KS3/98/Ma/Tier 6-8/P2]

UNITS 1-6 Diagnostic Test 9B (Express) Answers

1. (a) In million km,

$1.427 \times 10^{3} - 2.279 \times 10^{2} = 1427 - 227.9 = 1199.1$ M1 A1

Yes; distance ≈ 1200 million km A1

(b) Jupiter: $\frac{7.783}{1.496} \approx 5.20$ B1

Saturn: $\frac{1.427 \times 10}{1.496} \approx 9.54$ B1

(c) $2.8 \times 1.496 \times 10^{8}$ km $\approx 4.189 \times 10^{8}$ km M1 A1

(d) $R = \frac{6.67 \times 10^{-20} \times 1.993 \times 10^{30} \text{ km}}{(17.5)^{2}}$ M1

$= 4.34 \times 10^{8}$ km A1

(e) Yes; as the values are very close! B1 *(10 marks)*

2. (a) $\frac{1558}{2518} \times 100 = 61.87\%$ $(\approx 62\%)$ M1 A1

(b) $\frac{3402}{276} = 12.32$; i.e. 12 people B1

(c) $\frac{642}{222} = 2.89$; i.e. 3 people B1

(d) Yes, population was 13 and went up by 13. B1 *(5 marks)*

3. (a) 14.5% B1

(b) $119\,000 \times \frac{14.5}{100} \approx 17\,255$ M1 A1

(c) $\left(\frac{17\,468 - 12\,540}{12\,540}\right) \times 100 \approx 39.3\%$ M1 A1

(d) 3rd option. as actual total for 1996 is not known B1 *(6 marks)*

4. (a) $\frac{3}{5} \times \frac{10}{7} = \frac{6}{7}$ B2

(b) $3\frac{3}{4} \div 1\frac{1}{4} = 3$ B2 *(4 marks)*

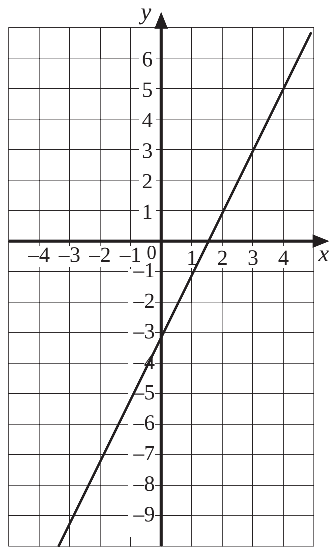
5. (a)

x	-3	-2	-1	0	1	2	3
y	-9	-7	-5	-3	-1	1	3

(–1 for each mistake) B2

UNITS 1-6 Diagnostic Test 9B (Express) Answers

(b)



B2 *(4 marks)*

6. $2x + 9 = 7 \Rightarrow x = -1$ M1 A1 *(2 marks)*

7. (a) $2x = 6 \Rightarrow x = 3$ M1 A1

(b) $2y + 5 = 23 - y$ B1

$y = 6$ B1

(c) $$\left.\begin{array}{l} 2a + 6b = 50 \\ \\ 2a + b = 15 \end{array}\right\} 5b = 35 \Rightarrow b = 7$$ M1 A1

$a = 4$ A1 *(7 marks)*

8. (a) $0.5 + 0.25 = 0.75$ B1

(b) They are not exclusive events. B1

(c) $(0.25)^2 = 0.0625$ M1 A1

(d) If they walked together, it would be 0.25. B1 *(5 marks)*

9. (a) Sue; as she has thrown the most B1

(b) $p = \frac{171}{300} = 0.57$ B1

(c) 167, 125, 8 (– 1 for each mistake) B2

(d) experimental variation; frequency will be close but not necessarily the same B1

(e) $\frac{1}{36} \times \frac{1}{36} = 0.0007716$ M1 A1

≈ 0.0008 *(7 marks)*

(TOTAL MARKS 50)

UNITS 1-6 Diagnostic Test 9B (Express) Answers

Marks

Unit	**1-3**	**4**	**5**	**6**	
Question	1	2 - 4	5 - 7	8 - 9	
Total marks available	10	15	13	12	*Final total*
Total					

Assessment

45 +	Excellent
40 - 44	Very good progress
35 - 39	Good progress: look carefully at mistakes
30 - 34	Steady progress, but you will need to work more carefully and/or make more effort
- 29	Struggling, so look carefully at the mistakes in your work; you might be better advised to transfer to the *Academic Route*