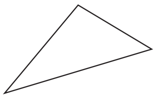
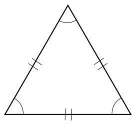
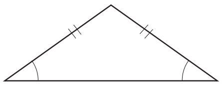
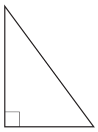
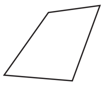
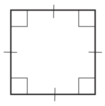
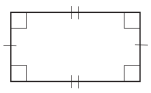
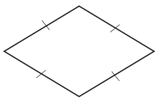
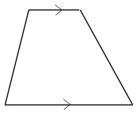
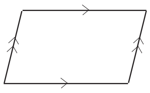
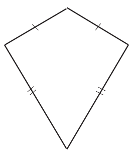
7 Transformations

7.1 Shapes

You should be familiar with the common 2-D shapes, but to recap, we give the names and definitions below.

NAME	*ILLUSTRATION*	*NOTES*
Triangle		3 straight sides
Equilateral Triangle		3 equal sides and 3 equal angles ($= 60\,°$)
Isosceles Triangle		2 equal sides and 2 equal angles
Right-angled Triangle		One angle $= 90\,°$
Quadrilateral		4 straight sides
Square		4 equal sides and 4 right angles
Rectangle		Opposite sides equal and 4 right angles
Rhombus		4 equal sides; opposite sides parallel
Trapezium		One pair of opposite sides parallel
Parallelogram		Both pairs of opposite sides equal and parallel
Kite		Two pairs of adjacent sides equal

Example 1

What could each one of the following shapes be if it has 4 sides and:

(a) opposite sides equal and parallel, (b) all sides equal,

(c) two adjacent angles are right angles?

Solution

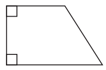
(a) It could be a

parallelogram, rhombus, rectangle or *square.*

(b) It could be a

rhombus or *square.*

(c) It could be a

trapezium, rectangle or *square.* (Trapezium )

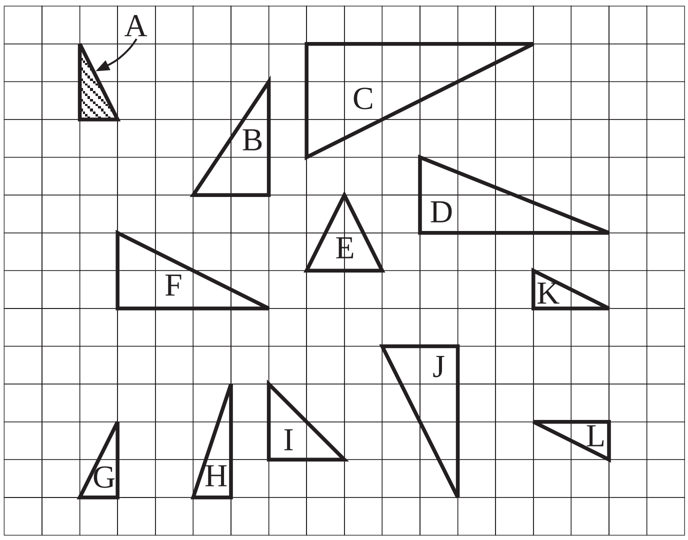
Example 2

For the grid opposite, name all shapes that are:

(a) *congruent,*

(b) *similar*

to shape A.



Solution

(a) *Congruent* to A means the *same size and shape* as A. The shapes congruent to A are G, L and K.

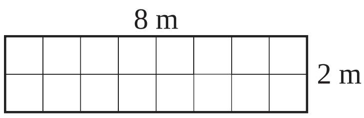
(b) *Similar* to A means the *same shape* as A but *not necessarily the same size* as A. The shapes similar to A are C, F, G, J, K and L.

Example 3

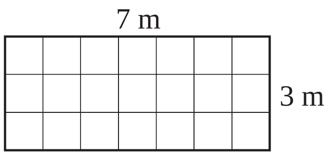
Using 20 m of fencing, design *four* different rectangular enclosures. For each one, find its area. Which shape gives the maximum area?

Solution

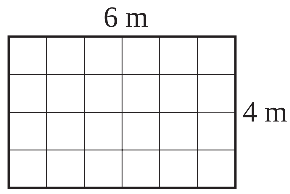
Possible shapes could be:



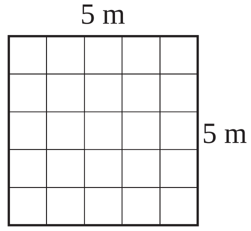
area $= 2 \times 8 = 16 \text{ m}^2$



area $= 3 \times 7 = 21 \text{ m}^2$



area $= 4 \times 6 = 24\ \text{m}^2$



area $= 5 \times 5 = 25\ \text{m}^2$

The square $(5\ \text{m} \times 5\ \text{m})$ gives the maximum area.

Exercises

1. What could each one of the following shapes be if it has 4 sides and:

   (a) all angles right angles,

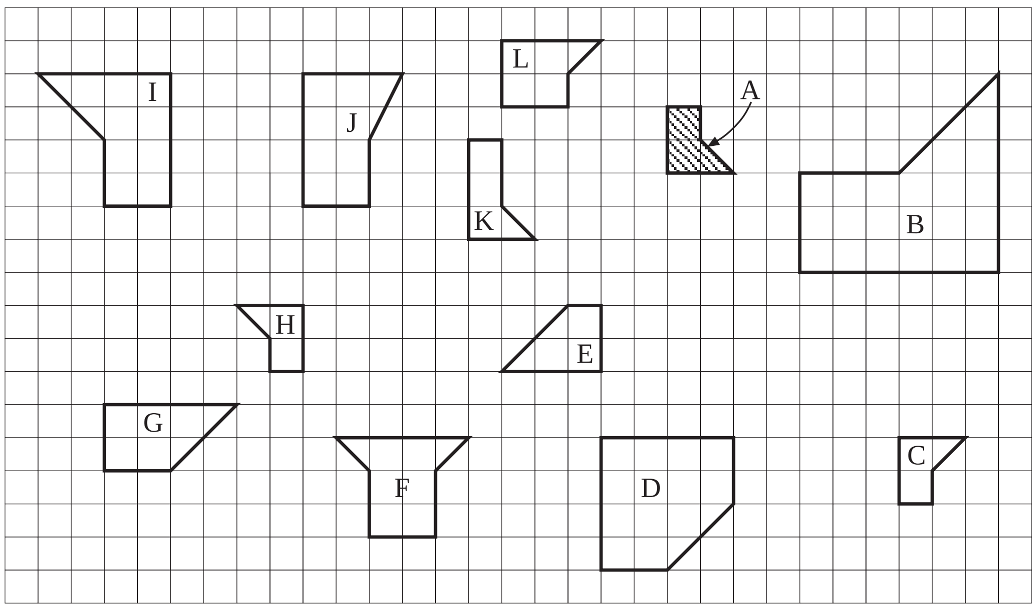
   (b) exactly one pair of opposite sides parallel, but not equal,

   (c) diagonals intersecting at right angles?

2. Which of the shapes in the diagram below are:

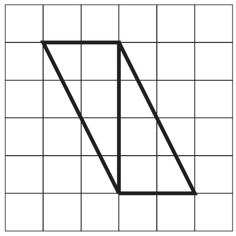
   (a) *congruent,* (b) *similar*

   to shape A ?

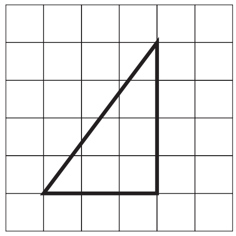


3. Using 40 cm of wire, design different rectangles. For each one, find its area. What shape gives the maximum area?

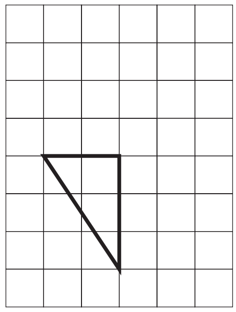
4. These two congruent triangles make a *parallelogram.*



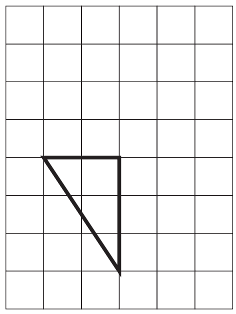
(a) On a copy of the grid opposite, draw another congruent triangle to make a *rectangle.*



(b) On a copy of the grid opposite, draw another congruent triangle to make a *bigger triangle.*

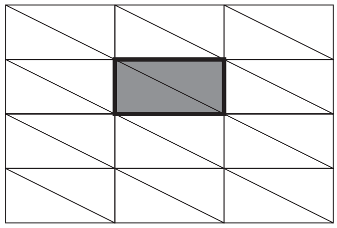


(c) On a copy of the grid opposite, draw another congruent triangle to make a *different bigger triangle.*

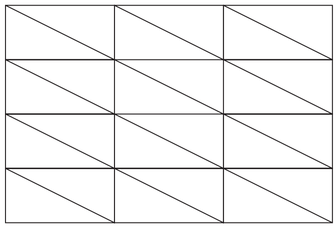


(KS3/98/Ma/Tier 3-5/P2)

5. Mike has a triangle grid. He shades in *2 triangles* to make a shape with 4 sides.



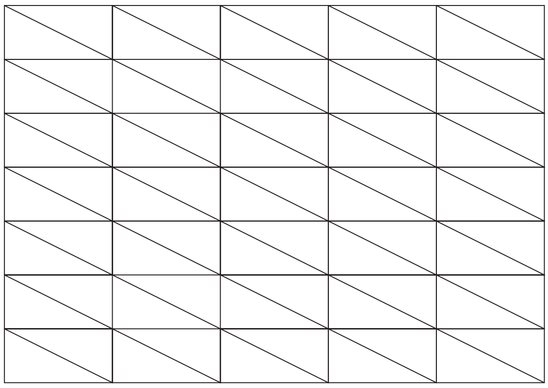
(a) Shade in *2 triangles* on a copy of the grid opposite to make a *different* shape with 4 sides.



(b) On another copy of the grid, shade in *2 triangles* to make another *different* shape with 4 sides.

(c) On another copy of the grid, shade in *4* small triangles to make a *bigger triangle.*

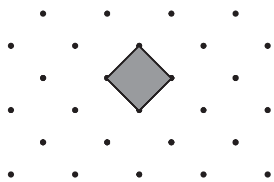
(d) On a copy of the grid below, shade in *more than 4* small triangles to make a *bigger triangle.*



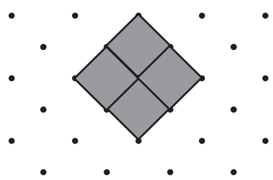
(KS3/97/Ma/Tier 3-5/P2)

6.

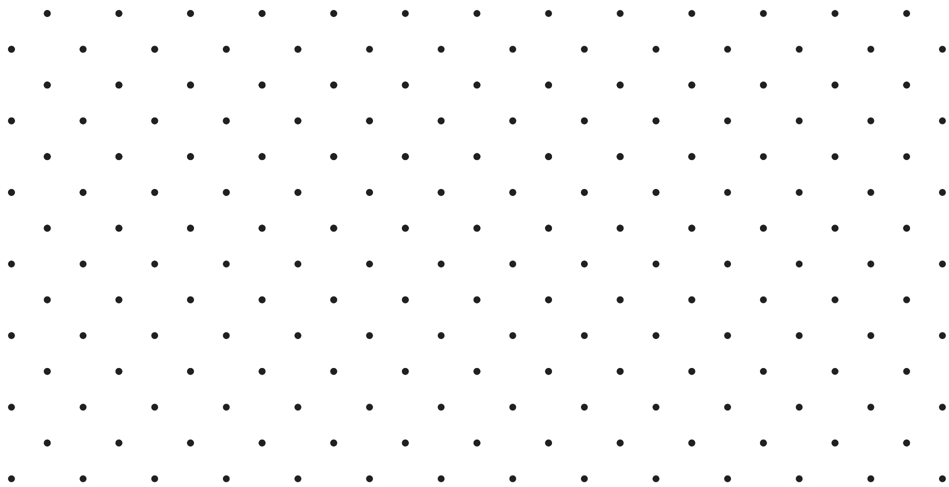
Kath puts 1 small square tile on a square dotty grid, like this:



Den makes a *bigger square* with 4 square tiles, like this:



(a) Scott has 9 small square tiles. On a copy of the following grid, show how Scott can make a *square* in the same way with 9 small square tiles.



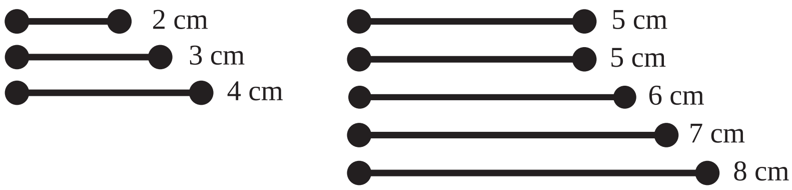
(b) On another copy of the grid, show how to make a *square* with *more than 9* of these small square tiles.

How many tiles are there in your square?

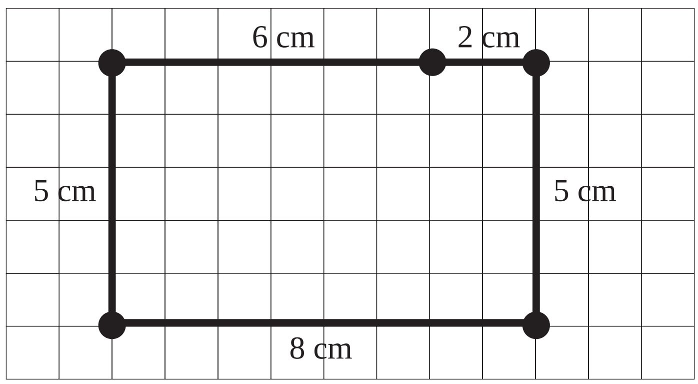
(c) Huw wants to make some more squares with the tiles. Write down 3 *other* numbers of tiles that he can use to make squares.

(KS3/96/Ma/Tier 3-5/P2)

7. Helen has *these eight rods.*



She can use 5 of her rods to make a rectangle.



(a) On a copy of the grid above, show how to make a *different rectangle* with a *different shape* with 5 of Helen's rods.

(b) On a larger grid, 13 squares by 10 squares, show how to make a rectangle with 6 of Helen's rods.

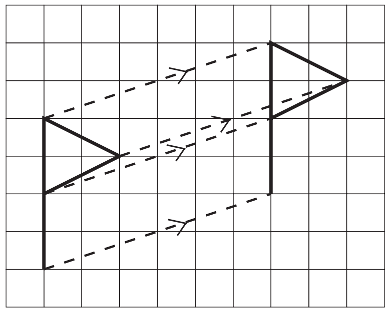
(c) On another large grid, show how to make a *square* with all 8 of Helen's rods.

(KS3/99/Ma/Tier 5-7/P1)

7.2 Translations

Under a *translation*, every point is moved by the *same amount* in the *same direction*. If each point moves distance a in the x-direction and distance b in the y-direction, we use the 'vector' notation $\begin{pmatrix} a \\ b \end{pmatrix}$ to describe this translation.

For example, the translation described by the column vector $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ is illustrated opposite; the translation moves the shape 6 units to the right and 2 units upwards.



Note that the actual shape *does not change its orientation*, only its position. It is *not reflected or rotated.*

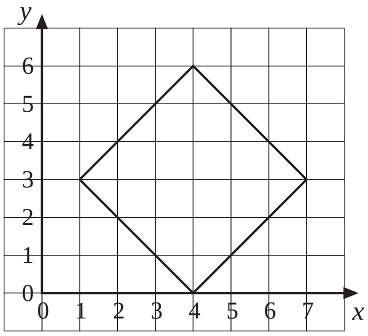
Example 1

(a) Draw the square with corners at the points with coordinates (4, 0), (1, 3), (4, 6) and (7, 3).

(b) The square is translated along the vector $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Draw the new square obtained by the translation.

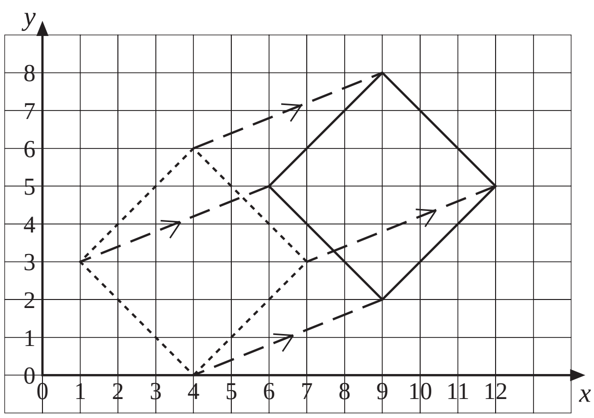
Solution

(a)



The diagram opposite shows the square.

(b)



For this translation each point should be moved 5 units to the right and 2 units up.

This diagram shows both squares and the vector that has been used to translate each corner.

Example 2

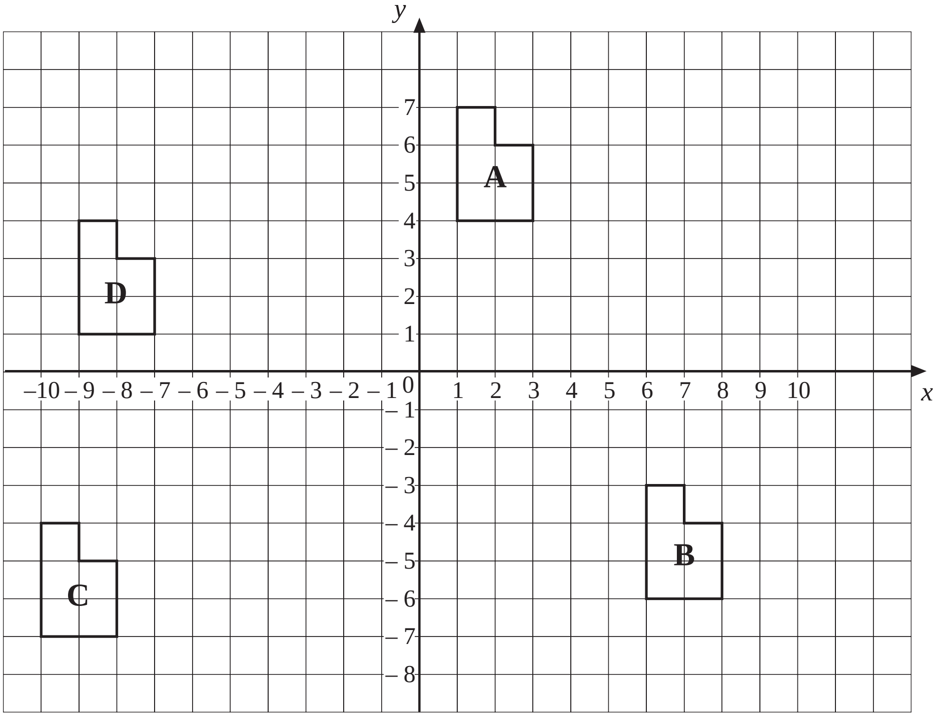
The diagram below shows the shapes A, B, C and D. Along what vector would you translate:

(a) D to A,

(b) C to D,

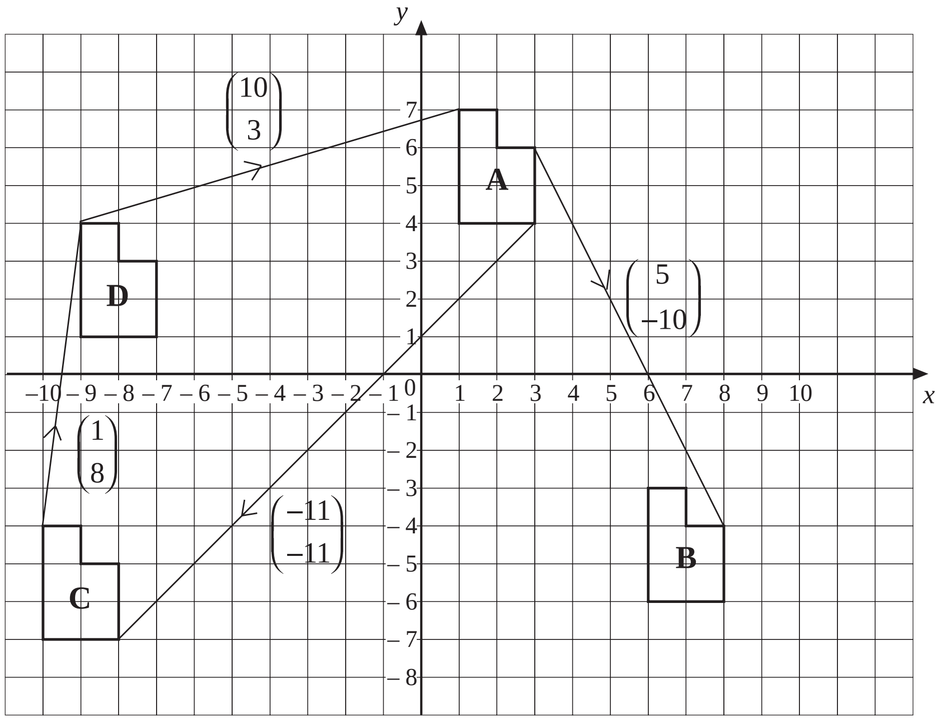
(c) A to B,

(d) A to C ?



Solution

The vector that describes each translation is shown on the following diagram:



(a) D to A $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$, 10 to the right and 3 up.

(b) C to D $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$, 1 to the right and 8 up.

(c) A to B $\begin{pmatrix} 5 \\ -10 \end{pmatrix}$, 5 to the right and 10 down.

(d) A to C $\begin{pmatrix} -11 \\ -11 \end{pmatrix}$, 11 to the left and 11 down.

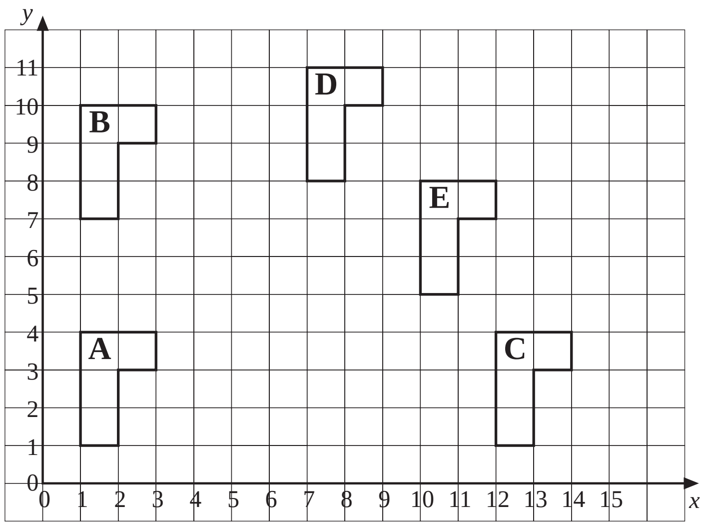
Exercises

1. (a) Draw the triangle which has corners at the points with coordinates (4, 1), (3, 5) and (1, 2).

   (b) Translate the triangle along the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

   (c) Write down the coordinates of the corners of the translated triangle.

2. The following diagram shows the shape A which is translated to give the shapes B, C, D and E:



   Write down the vector that describes the translation from:

   (a) A to B, (b) A to C, (c) A to D,

   (d) A to E, (e) B to D.

3. (a) Join the points with coordinates (1, 1), (2, 3) and (5, 4) to form a triangle. Label this triangle A.

(b) Translate the triangle A along the vector:

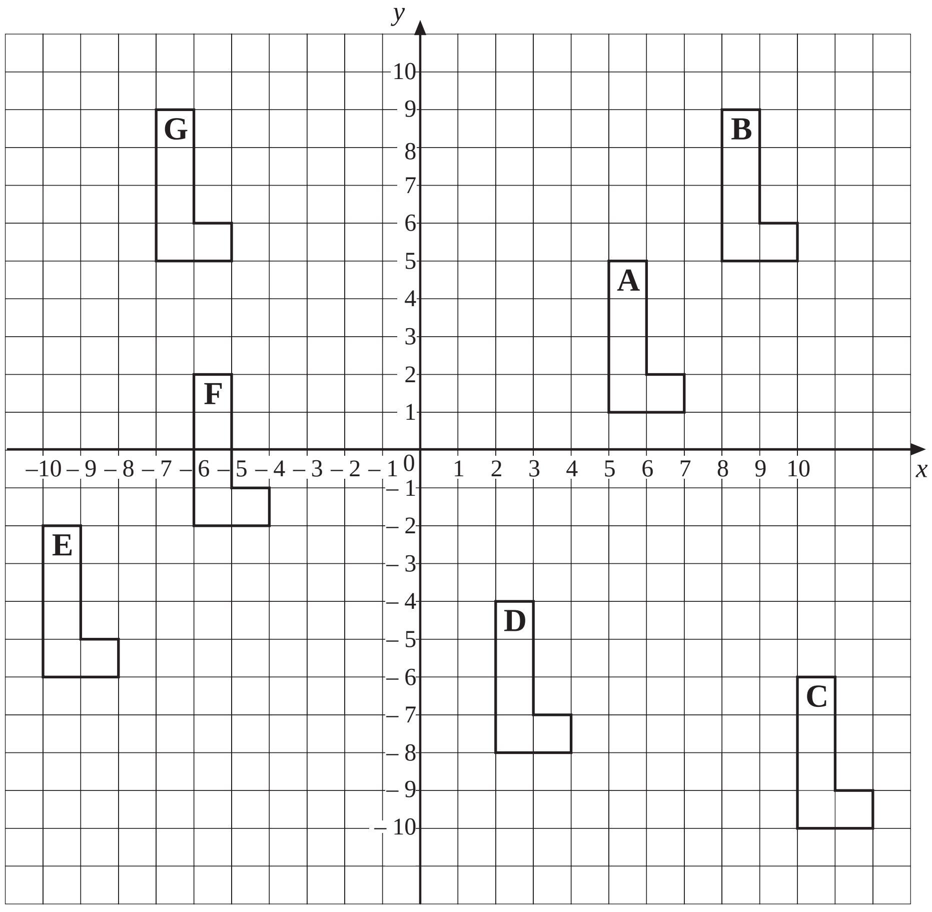
(i) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, to obtain B,

(ii) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, to obtain C,

(iii) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, to obtain D,

(iv) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$, to obtain E.

4. Write down the vector needed to translate the shape A to each of the other shapes shown on the following diagram:

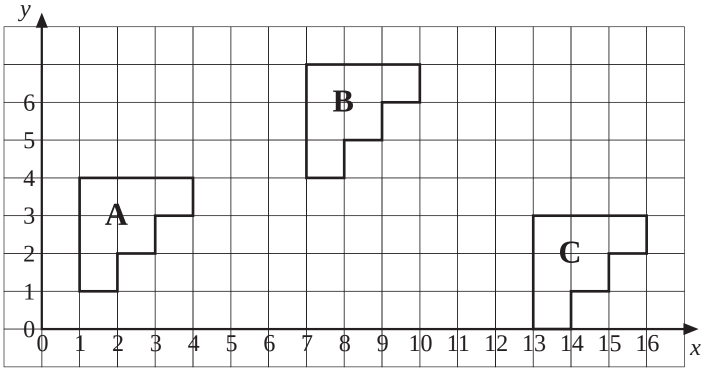


5. The point with coordinates (2, 3) is moved to the point with coordinates (7, 6) by a translation.

(a) Describe the translation using a column vector.

(b) Where would the point with coordinates (6, 1) move to under the same translation?

6. The diagram shows three shapes, A, B and C:



Write down the vector for the translation that moves:

(a) A to B,

(b) B to C,

(c) A to C.

Describe any relationship between these vectors.

7. The shape A has corners at the points with coordinates

(4, 2) (4, – 1) (6, – 3) and (6, 0)

(a) What is this shape?

(b) The shape is translated along the vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ to give shape B and then shape B is translated along the vector $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ to give C.

Draw A, B and C.

(c) What translation would take A straight to C ?

8. (a) Draw the triangle, A, that has corners at the points with coordinates (– 7, – 2), (– 5, – 5) and (– 4, – 2).

(b) Translate this shape along the vector $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ to obtain B.

(c) Describe the translation that would take B to A.

9. (a) Draw three lines by joining the points with coordinates

(4, 2) and (2, 4);

(6, 4) and (6, 6);

(2, 6) and (4, 8).

(b) Describe how to translate each line to form a hexagon made up of the original and translated lines.

10. A parallelogram has corners at the points A, B, C and D. The points A, B and C have coordinates (1, 2), (2, 5) and (5, 3) respectively.
    (a) Draw the parallelogram.
    (b) State the coordinates of the fourth corner, D.
    (c) Describe the translation that moves A B onto D C.
    (d) Describe the translation that moves A D onto B C.

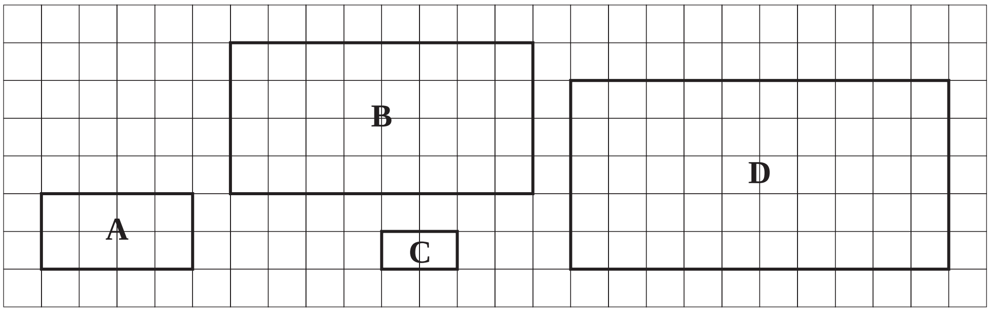
7.3 Enlargements

In this section we consider enlargements. We look at the use of the terms 'scale factor' and 'centre of enlargement'.

Example 1

The rectangle A, shown below, has been enlarged to give the shapes B, C and D.

Write down the scale factor for each enlargement.



Solution

A to B is scale factor 2 because the lengths are doubled.

A to C is scale factor $\frac{1}{2}$ because the lengths are halved.

A to D is scale factor 2.5 because the lengths are 2.5 times longer.

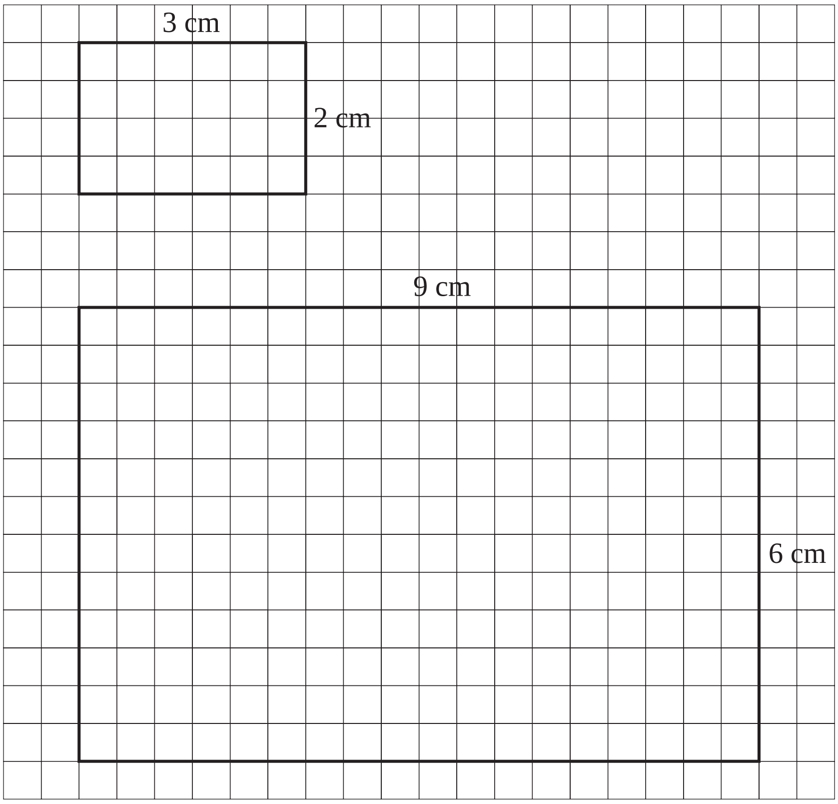
Example 2

A rectangle has sides of lengths 2 cm and 3 cm. It is enlarged with scale factor 3. Draw the original rectangle and the enlarged rectangle.

Solution

The lengths of the sides of the enlarged rectangle will be:

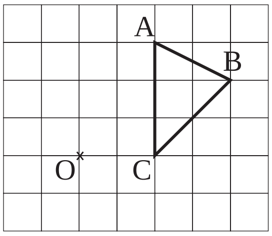
$$3 \times 2 \text{ cm} = 6 \text{ cm}$$
$$3 \times 3 \text{ cm} = 9 \text{ cm}$$



Examples 3 and 4 show how to use a centre of enlargement when enlarging a shape.

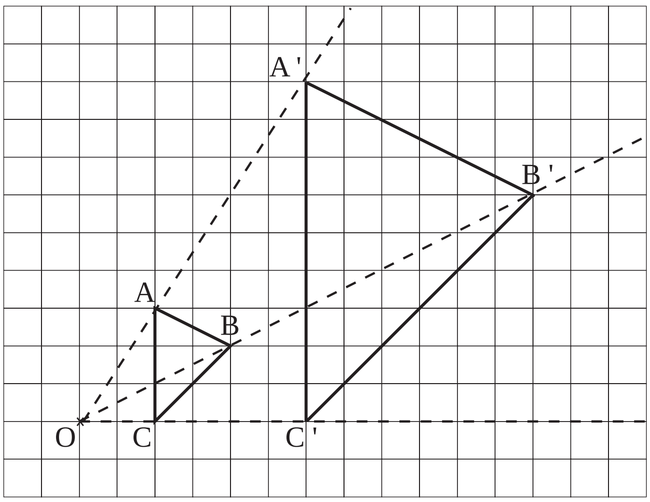
Example 3

The diagram shows the triangle A B C and the point O. Enlarge the triangle with scale factor 3, using O as the centre of enlargement.



Solution

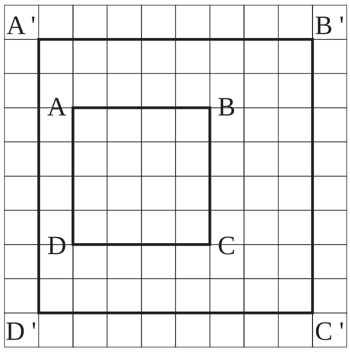
The diagram shows the 2 triangles; the explanation follows.

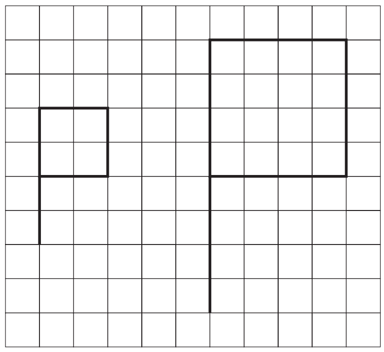


First draw lines from point O through A, B and C, as shown in the diagram. Measure the length O A and multiply it by 3 to get the distance from O of the image point A', i.e. $\text{O A}' = 3 \times \text{O A}$. Mark the point A' on the diagram. The images B' and C' can then be marked in a similar way and the enlarged triangle A' B' C' can then be drawn.

Example 4

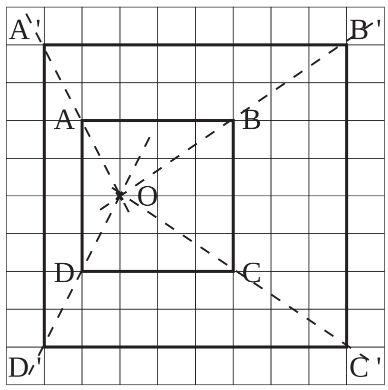
The following diagrams show two shapes that have been enlarged. Determine the centre of enlargement in each case.

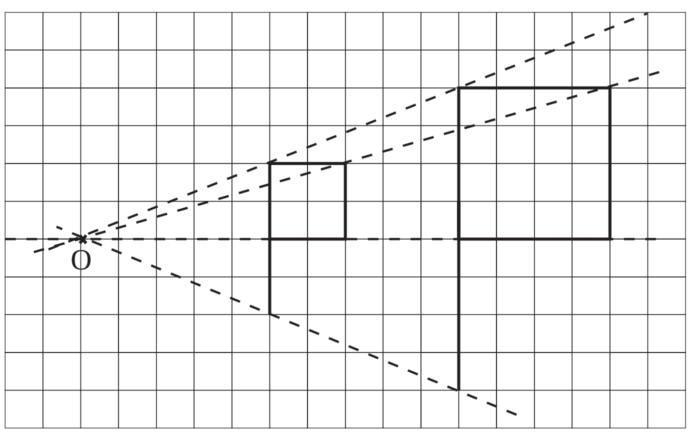




Solution

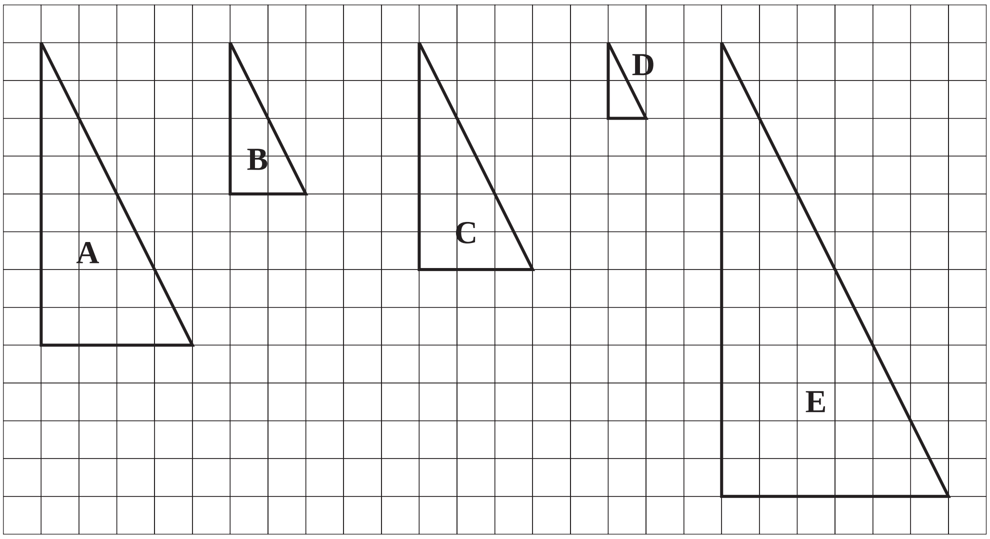
To find the centres of enlargement, draw lines through the corresponding corners of each shape. These lines will cross at the centre of enlargement, as shown below. The centres have been marked with the letter O in both diagrams.





Exercises

1. The following diagram shows 5 triangles, A, B, C, D and E:



What scale factor is used for each of the enlargements described below:

(a) B enlarged to A,
(b) C enlarged to E,
(c) D enlarged to E,
(d) D enlarged to A,
(e) B enlarged to C,
(f) B enlarged to D ?

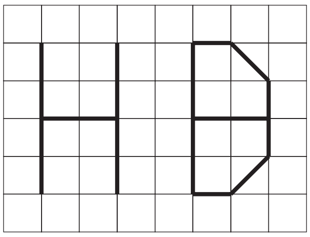
2. (a) Draw a rectangle that has sides of lengths 2 cm and 4 cm.

   (b) Draw enlargements of this rectangle using scale factors 2, 3, and $\frac{1}{2}$.

3. (a) Construct a triangle that has sides of lengths 3 cm, 4 cm and 5.5 cm.

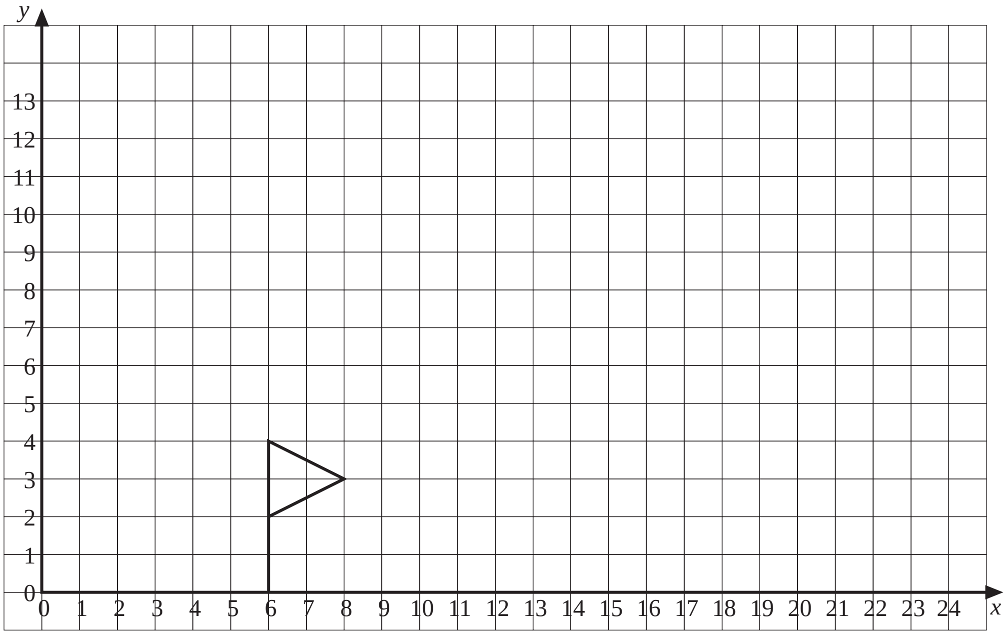
   (b) Draw enlargements of this triangle using scale factors 2 and 3.

4. Hannah writes her initials as shown:



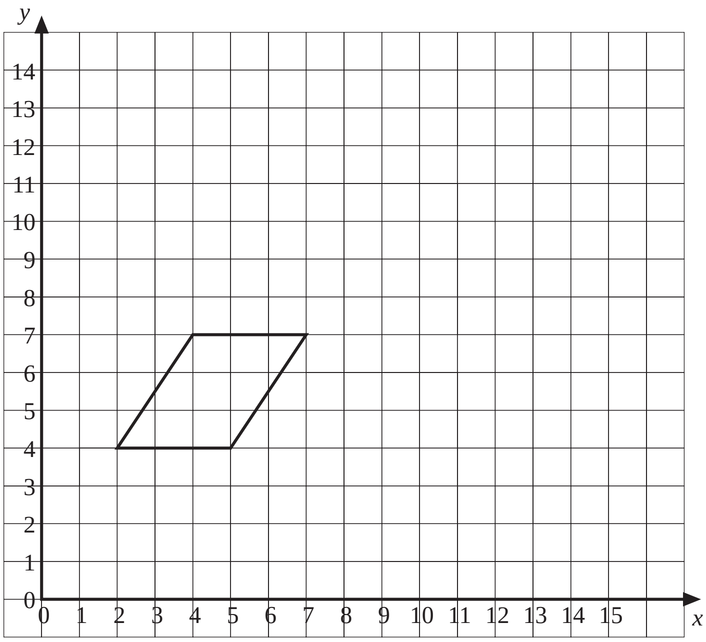
Enlarge her initials with scale factors 2 and $\frac{1}{2}$.

5. (a) Copy the following diagram:



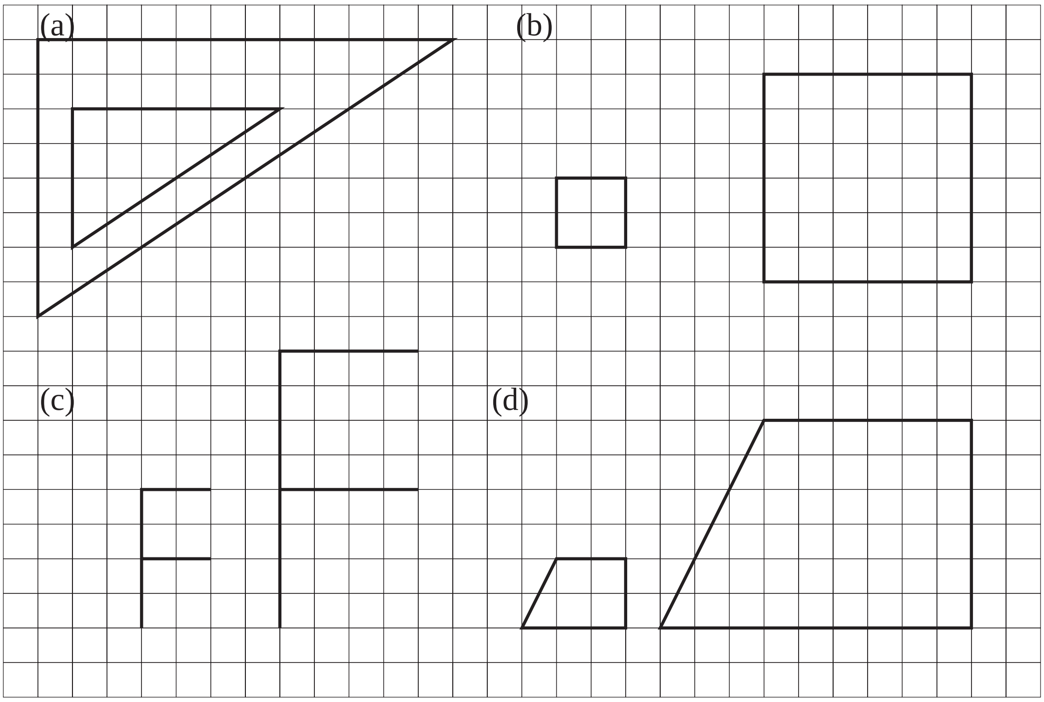
(b) Using (0, 0) as the centre of enlargement, enlarge the shape with scale factor 2 and scale factor 3.

6. (a) Copy the following diagram:



(b) Enlarge the shape with scale factor 2, using first (0, 0) as the centre of enlargement and then (1, 8) as the centre of enlargement.

7. For each of the following enlargements, copy the diagram and determine the centre of enlargement.



8. A triangle has corners at the points with coordinates (1, 2), (3, 3) and (0, 3). It is enlarged to give a triangle with corners at the points (5, 4), (11, 7) and (2, 7). Determine the scale factor of the enlargement and the coordinates of the centre of enlargement.

9. A trapezium has corners at the points with coordinates (1, 0), (3, 2), (3, 4) and (1, 5). It is enlarged with scale factor 3, using the point (0, 3) as the centre of enlargement.

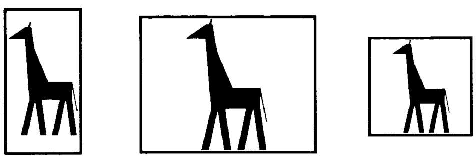
Determine the coordinates of the corners of the enlarged trapezium.

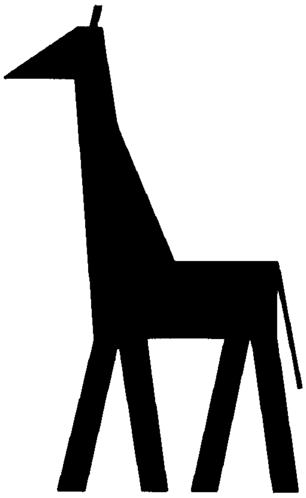
10. A parallelogram has corners at the points with coordinates (5, 1), (9, 3), (11, 9) and (7, 7). Enlarge this shape with scale factor $\frac{1}{2}$, using the point with coordinates (1, 3) as the centre of enlargement.

11. Jill has drawn an original picture of a giraffe for an animal charity.

It measures 6.5 cm high by 4 cm wide.

Different-sized copies of the original picture can be made to just fit into various shapes.

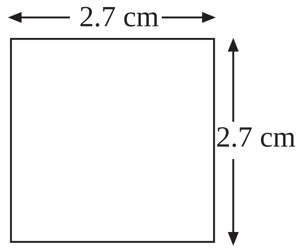




(a) Jill wants to enlarge the original picture so that it *just* fits inside a rectangle on a carrier bag. The rectangle measures 24 cm high by 12 cm wide.

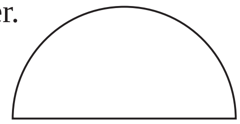
By what scale factor should she multiply the original picture? Show your working.

(b) Jill wants to multiply the *original picture* by a scale factor so that it *just* fits inside the square shown below for a badge.



By what scale factor should she multiply the original picture?

(c) The *original picture* is to be used on a poster. It must fit inside a shape like this.

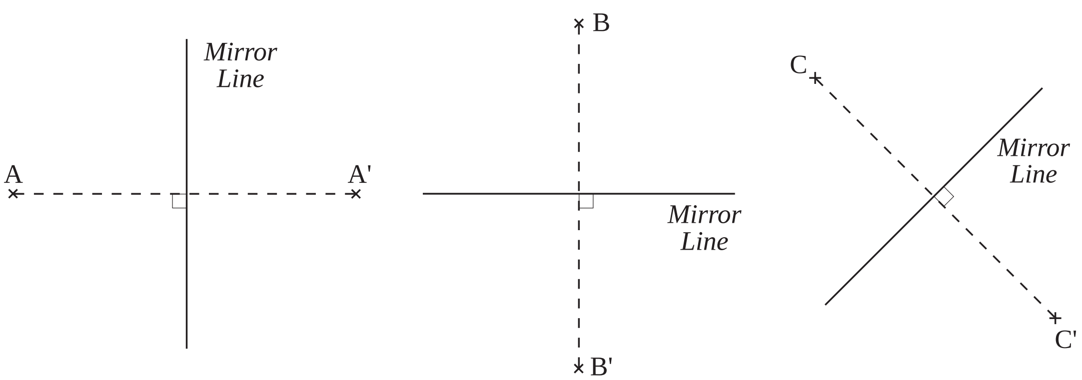


The shape is to be a semi-circle of radius 6.6 cm.
What would be the perimeter of the shape? Show your working.

(KS3/96/Ma/Tier 5-7/P2)

7.4 Reflections

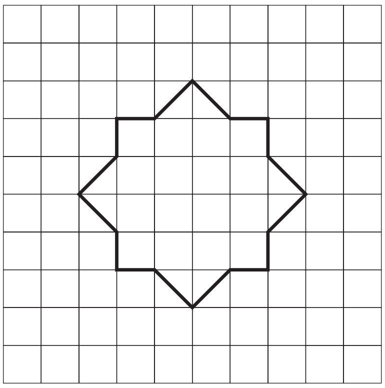
In this section we look at line symmetry and reflections of simple shapes, in horizontal, vertical and sloping lines. In a reflection, a point will move to a new position that will be the same distance from the mirror line, but on the other side. These distances will always be measured at right angles to the mirror line.



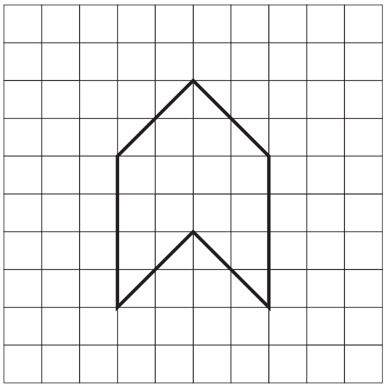
Example 1

Draw in the lines of symmetry for each of the following shapes:

(a)

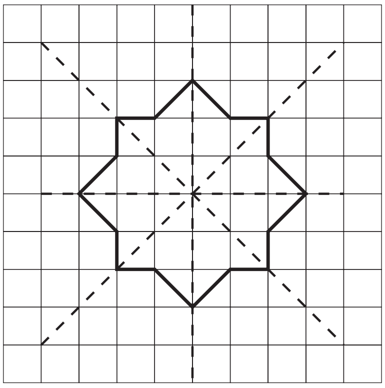


(b)



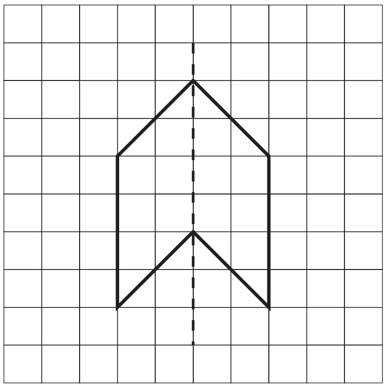
Solution

(a)



4 lines of symmetry

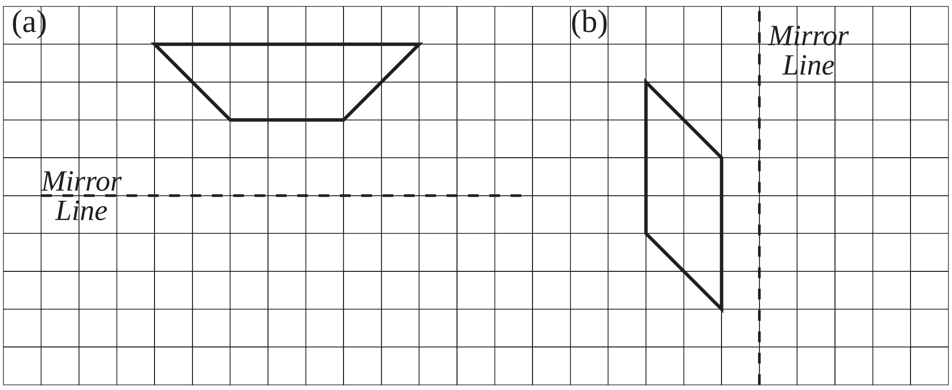
(b)



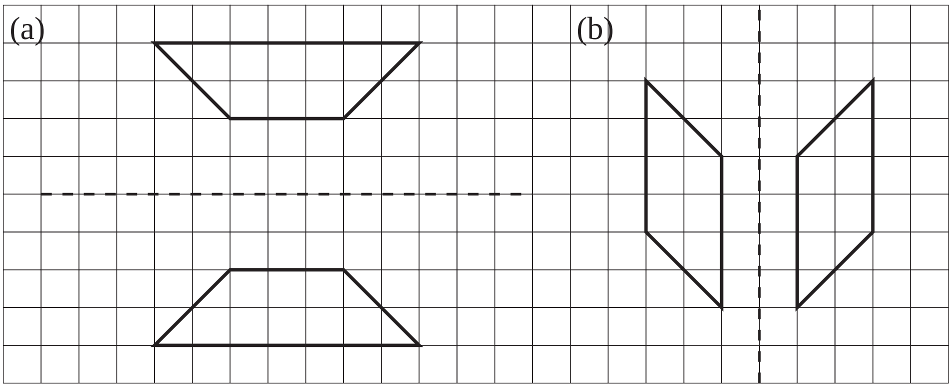
1 line of symmetry

Example 2

Draw the reflection of each of the following shapes in the given mirror line.



Solution



Example 3

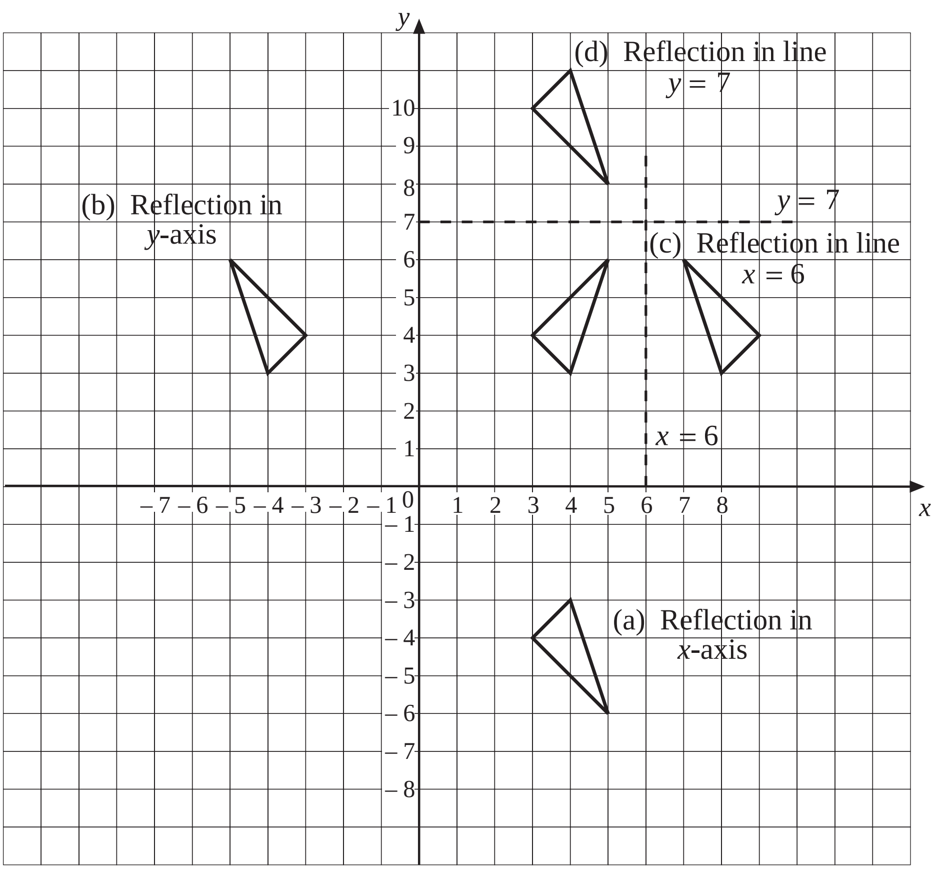
A triangle has corners at the points with coordinates (4, 3), (5, 6) and (3, 4).

Draw the reflection of the triangle in the:

(a) x-axis (b) y-axis,

(c) line $x = 6$ (d) line $y = 7$

Solution



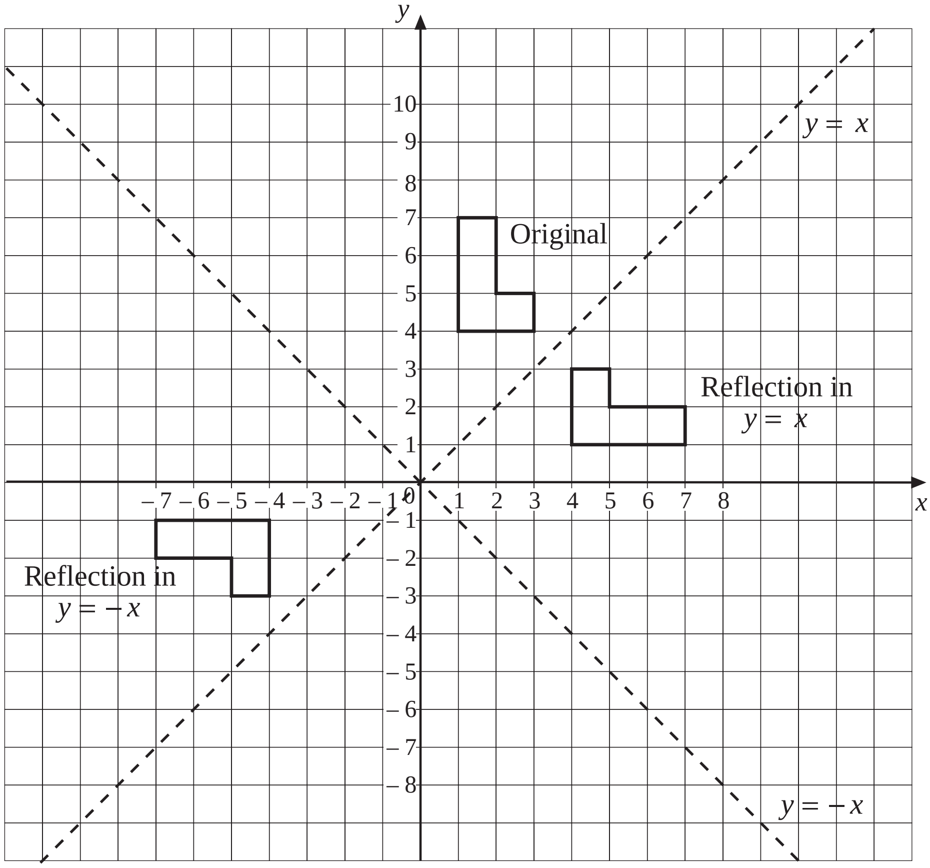
Example 4

An 'L' shape has corners at the points with coordinates (1, 4), (1, 7), (2, 7), (2, 5), (3, 5) and (3, 4).

Draw the reflection of the shape in the lines:

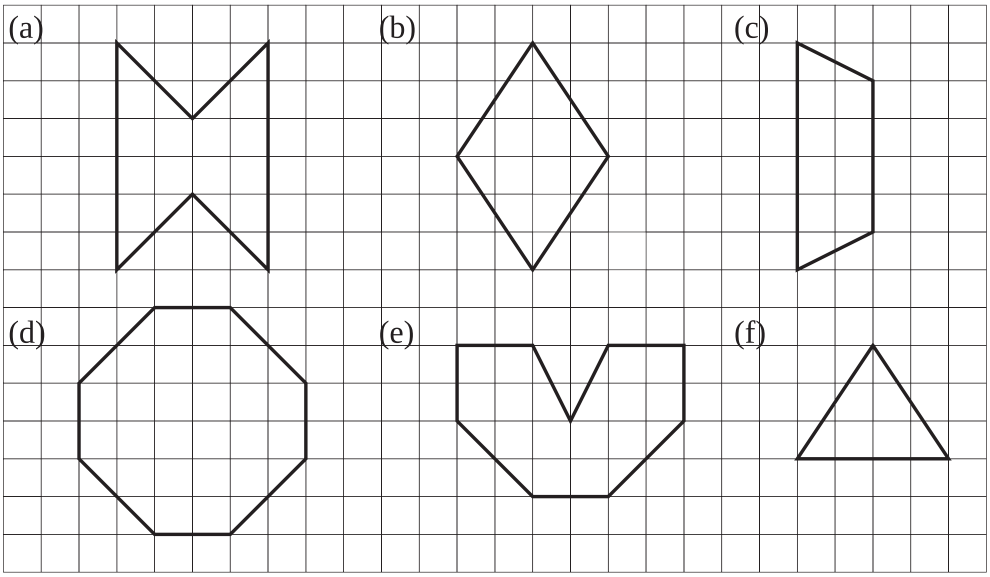
(a) $y = x$ (b) $y = -x$

Solution

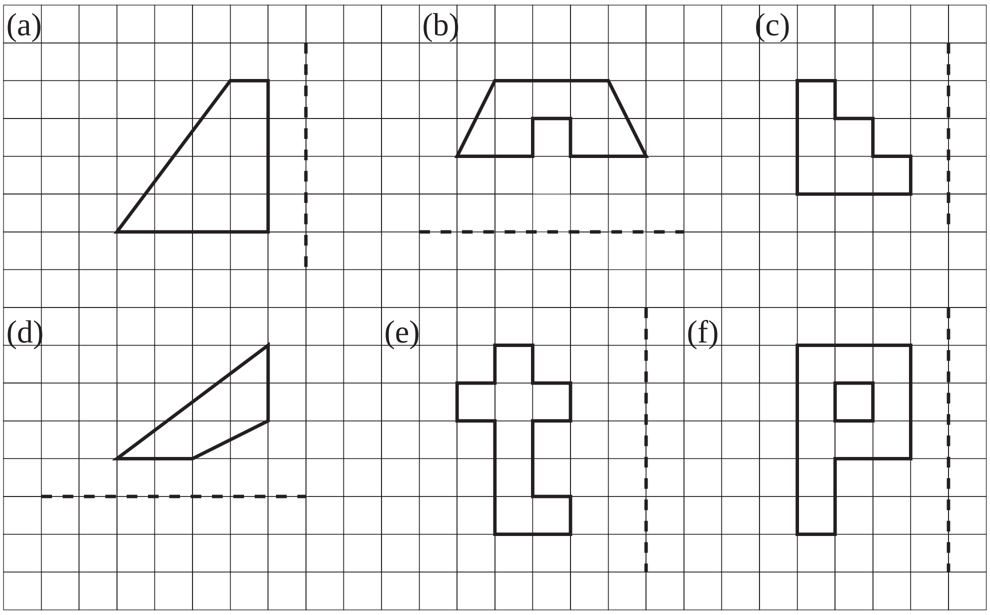


Exercises

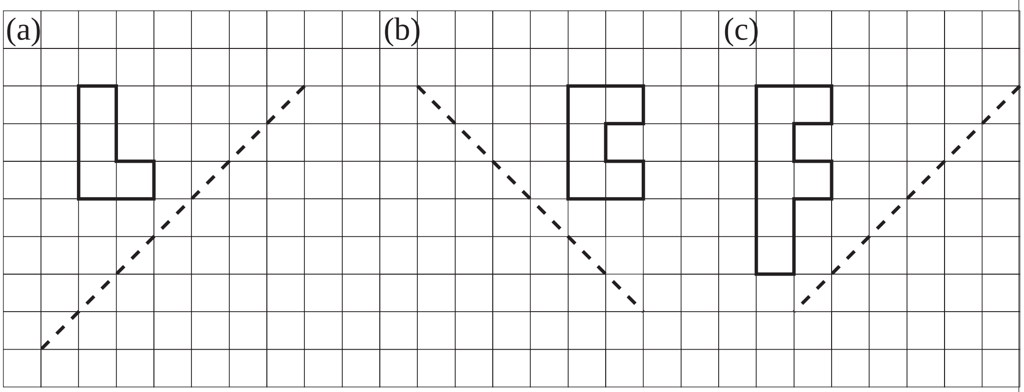
1. Copy the following shapes and draw in all their lines of symmetry.



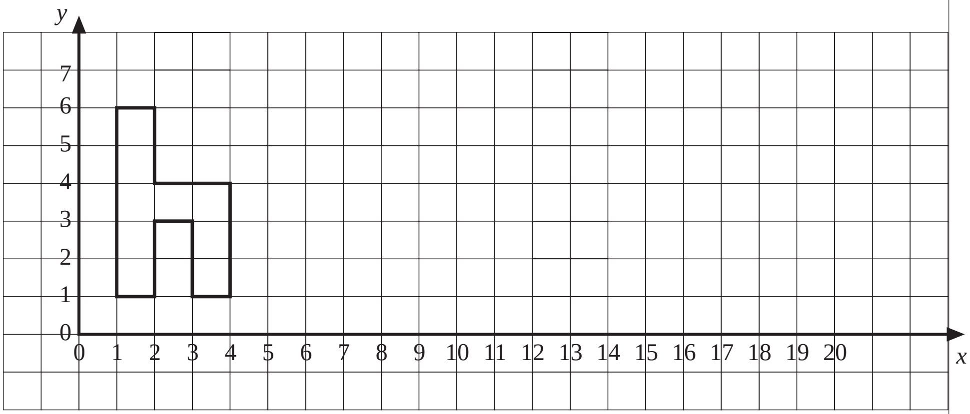
2. Draw the reflection of each of the following shapes in the line given:



3. Copy each of the following shapes and draw its reflection in the line shown:

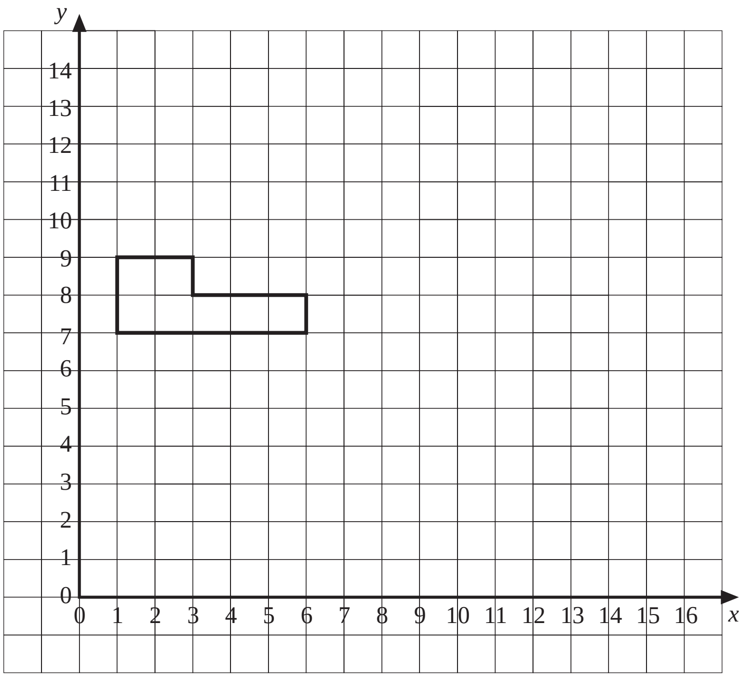


4. (a) Copy the following diagram:



(b) Reflect the shape in the lines $x = 8$ and $x = 11$.

5. (a) Copy the diagram shown.



(b) Reflect the shape in the lines $y = 10$, $y = 5$ and $x = 7$.

6. (a) Draw the triangle that has corners at the points with coordinates (1, 1), (4, 7) and (2, 5).

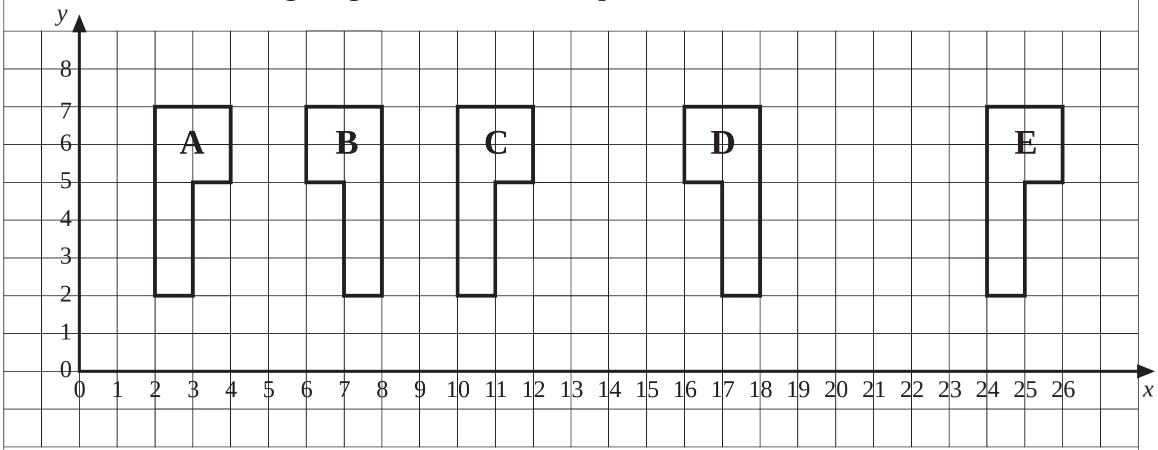
(b) Reflect the triangle in the lines:

(i) $x = 8$,

(ii) $x = -1$,

(iii) $y = -2$

7. The following diagram shows the shapes A, B, C, D and E.



Write down the equation of the mirror line for each of the following reflections:

(a) A to B (b) B to C (c) A to D

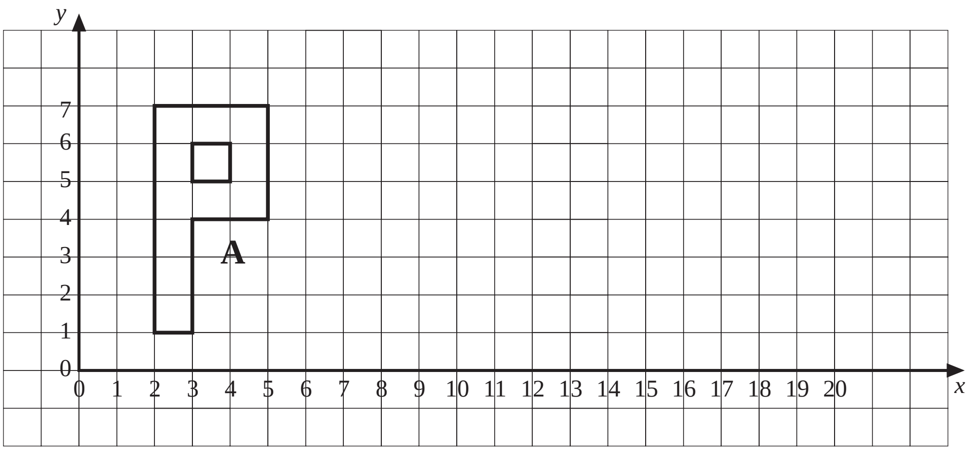
(d) B to E (e) D to E (f) C to D

8. (a) Draw the triangle which has corners at the points with coordinates (1, 4), (1, 7) and (3, 5).

(b) Reflect this shape in the line $y = x$ and state the coordinates of the corners of the reflected shape.

(c) Reflect the original triangle in the line $y = -x$ and state the coordinates of the corners of the reflected shape.

9. (a) Draw the shape A shown in the following diagram.



(b) Reflect the shape A in the line $x = 6$ to obtain shape B.

(c) Reflect the shape B in the line $x = 14$ to obtain shape C.

(d) Describe the translation that would take shape A straight to shape C.

10. Draw the triangle with corners at the points with coordinates (1, 3), (1, 8) and (6, 8). Reflect this triangle in the following lines:

(a) $x = 0$ (b) $y = 0$

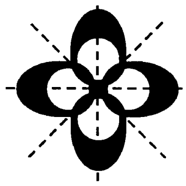
(c) $y = x$ (d) $y = -x$

11. These patterns have one or more lines of symmetry.

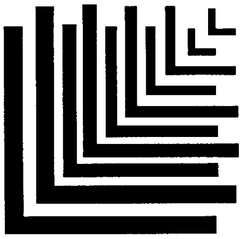
Draw *all* the lines of symmetry in each pattern.

You may use a mirror or tracing paper to help you.

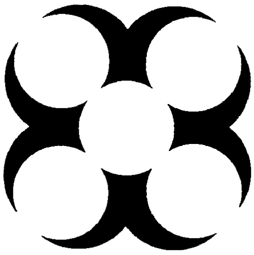
EXAMPLE



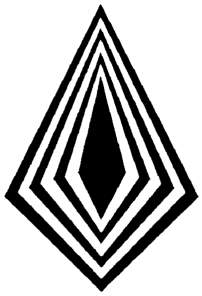
(a)



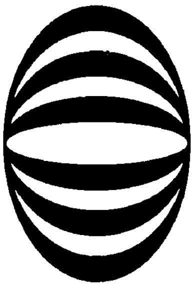
(b)



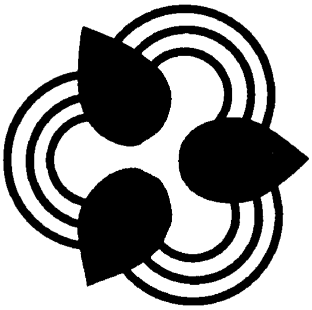
(c)



(d)

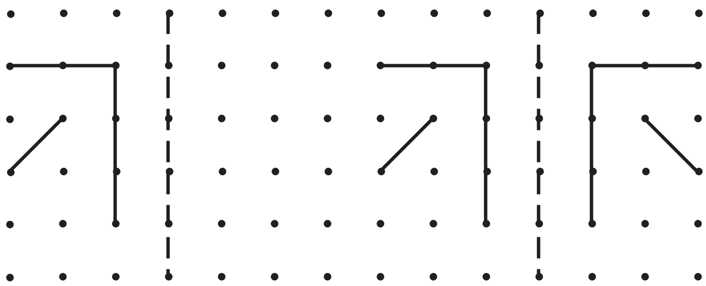


(e)



(KS3/94/Ma/3-5/P1)

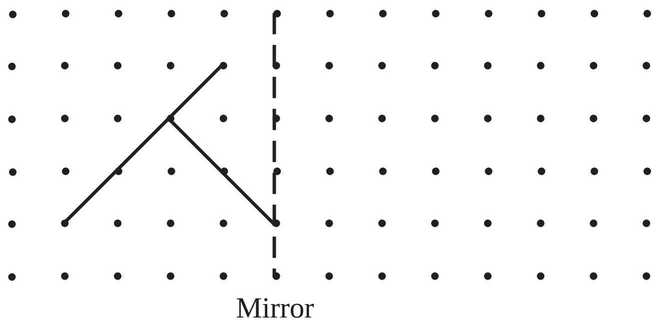
12. Nina is making Rangoli patterns. To make a pattern she draws some lines on a grid. Then she reflects them in a mirror line.



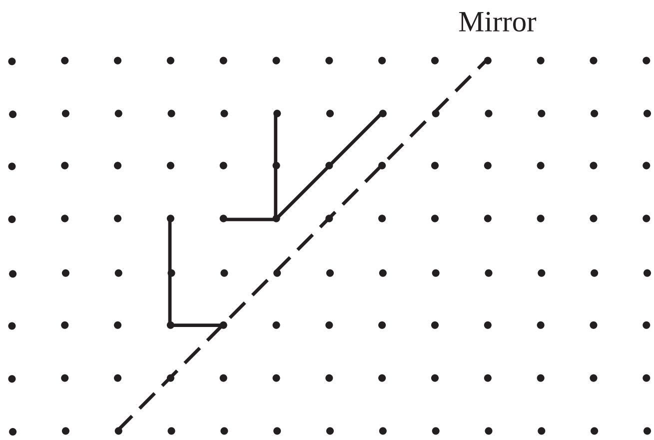
Make a copy each of the following grids and lines.

Reflect each group of lines in its mirror line to make a pattern. You may use a real mirror or tracing paper to help you.

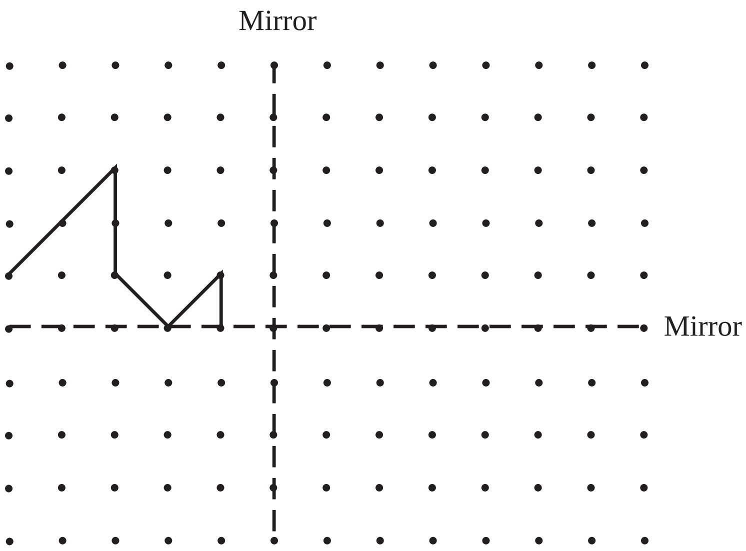
(a)



(b)



(c) Now use two mirror lines to make a pattern.
First reflect the group of lines in one mirror line to make a pattern.
Then reflect the whole pattern in the other mirror line.

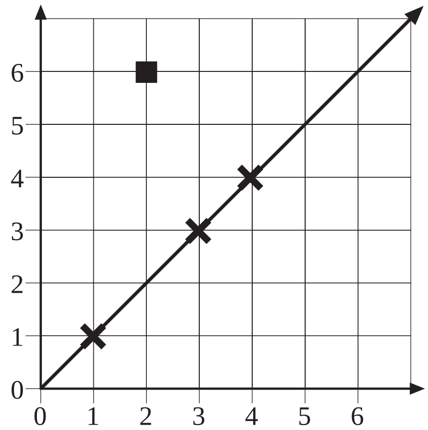


(KS3/95/Ma/Levels 3-5/P1)

13. (a) Three points on this line are marked with ✕.

Their coordinates are:
(1, 1), (3, 3) and (4, 4).

Look at the *numbers* in the coordinates of each point.
What do you notice?



(b) The point (? , $14\frac{1}{2}$) is *on* the line.
Write down its missing coordinate.

(c) The point ■ is *above* the line.

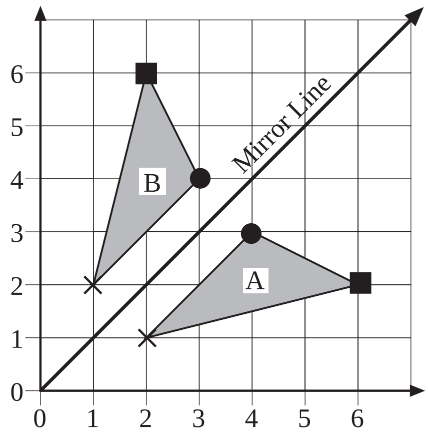
Four points are at (10, 10), (10, 12), (12, 10) and (12, 12).

Which one of these points is *above* the line? Explain why.

(d) The point (? , 15) is *above* the line. Write down a possible coordinate for the point.

(e) Look at triangles A and B.

	Triangle A	*Triangle B*
Coordinates of ●	(4, 3)	(3, 4)
Coordinates of ×	(2, 1)	(1, 2)
Coordinates of ■	(6, 2)	(2, 6)

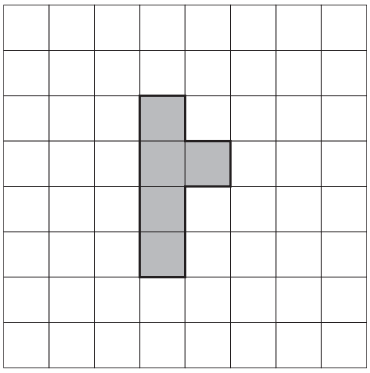


Triangle A was reflected onto triangle B.
What happened to the *numbers* in the coordinates of each corner?

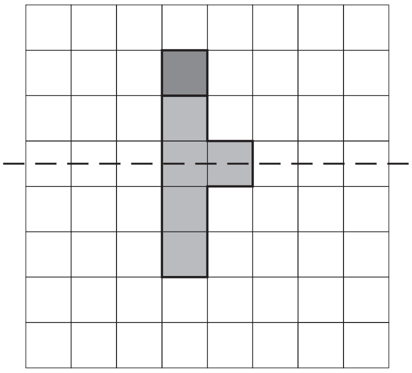
(f) Elen wants to reflect the point (20, 13) in the mirror line. What point will (20,13) go to?

(KS3/94/Ma/3-5/P2)

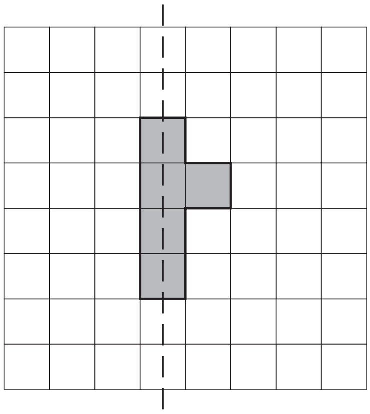
14. Catrin shades in a shape made of five squares on a grid:



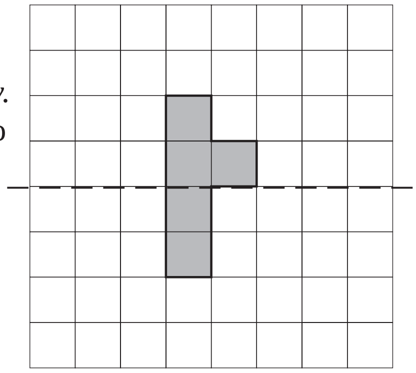
She shades in *1 more square* to make a shape which has the dashed line as a *line of symmetry*:



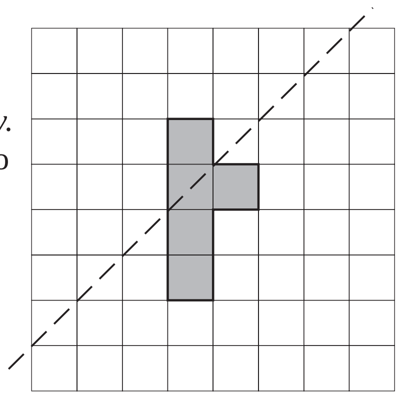
(a) On a copy of the grid opposite, shade in *1 more square* to make a shape which has the dashed line as a *line of symmetry.*



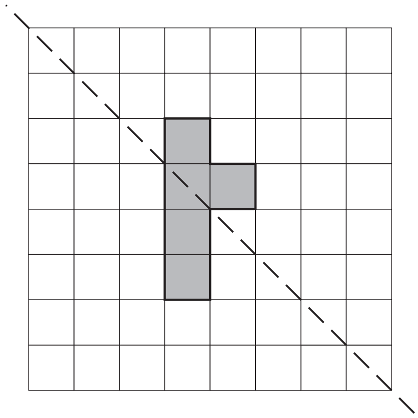
(b) On a copy of the grid opposite, shade in *1 more square* to make a shape which has the dashed line as a *line of symmetry.* You may use a mirror or tracing paper to help you.



(c) On a copy of the grid opposite, shade in *2 more squares* to make a shape which has the dashed line as a *line of symmetry.* You may use a mirror or tracing paper to help you.



(d) On a copy of the grid opposite, shade in 2 more *squares* to make a shape which has the dashed line as a *line of symmetry*. You may use a mirror or tracing paper to help you.



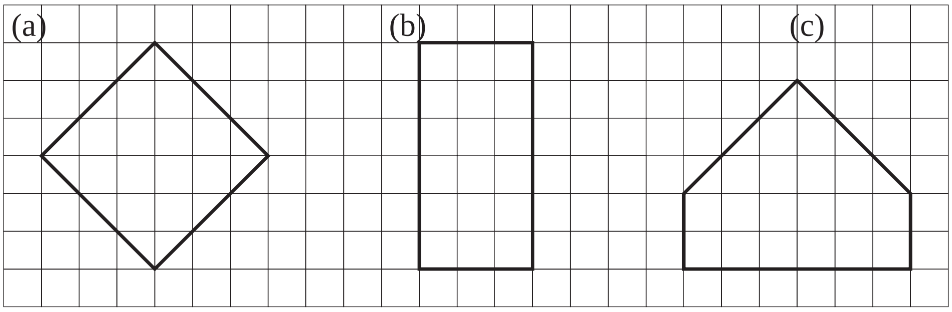
(KS3/96/Ma/Tier 3-5/P2)

7.5 Rotations

In this section we review rotational symmetry and draw rotations of shapes.

Example 1

State the order of rotational symmetry of each of the following shapes:



Solution

(a) Order 4. This means that the shape can be rotated 4 times about its centre before returning to its starting position. Each rotation will be through an angle of $90°$, and, after each one, the rotated shape will occupy the same position as the original square.

(b) Order 2

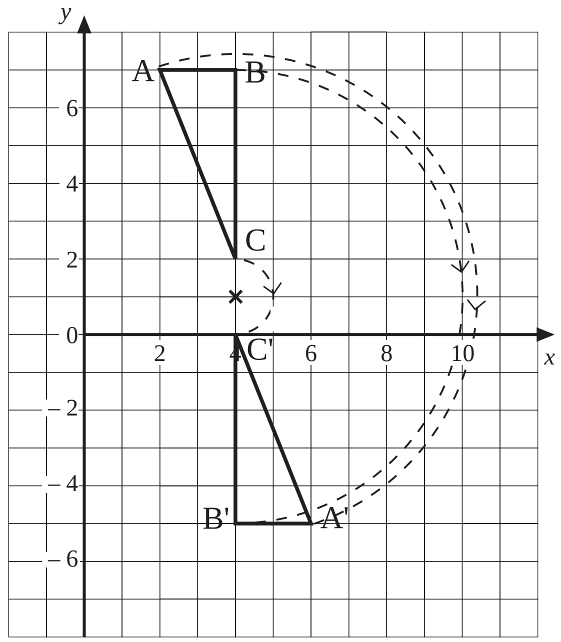
(c) Order 1. This means that the shape does *not* have rotational symmetry.

Example 2

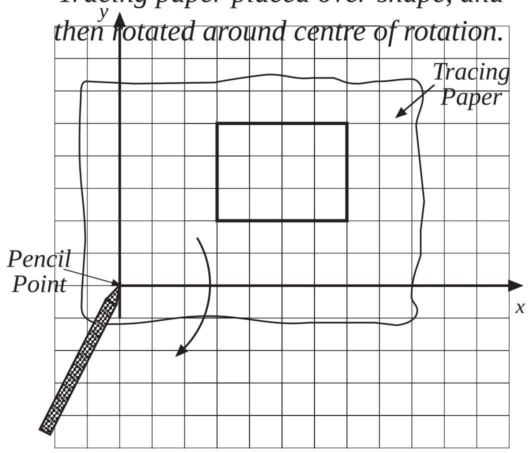
The corners of a rectangle have coordinates (3, 2), (7, 2), (7, 5) and (3, 5). The rectangle is to be rotated through $90°$ clockwise about the origin.
Draw the original rectangle and its position after being rotated.

Solution

The following diagram shows the original rectangle A B C D and the rotated rectangle A' B' C' D'. The curves show how each corner moves as it is rotated. The easiest way to rotate a shape is to place a piece of tracing paper over the shape, trace the shape, and then rotate the tracing paper about the centre of rotation, as shown.



Tracing paper placed over shape, and then rotated around centre of rotation.



Example 3

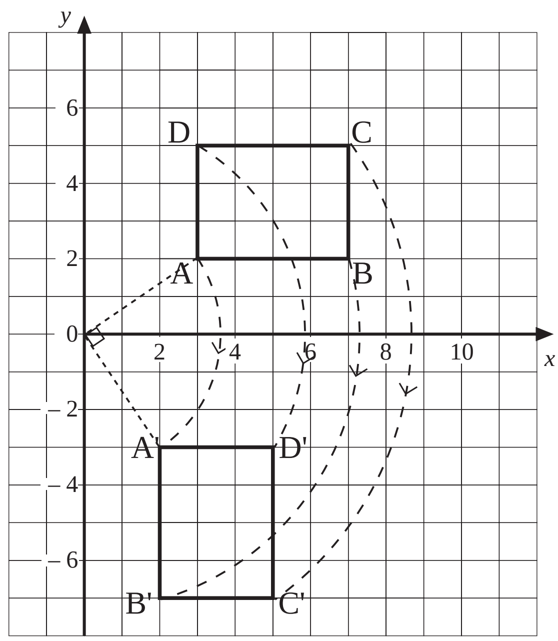
A triangle has corners at the points with coordinates (4, 7), (2, 7) and (4, 2).

(a) Draw the triangle.

(b) Rotate the triangle through 180 ° about the point (4, 1).

Solution

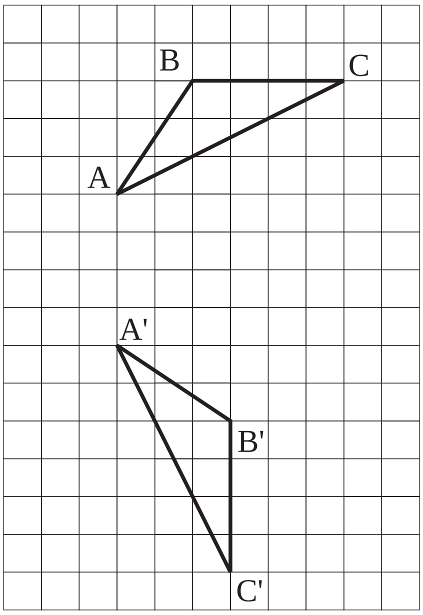
The diagram shows how the original triangle A B C is rotated about the point (4, 1) to give the triangle A' B' C'.



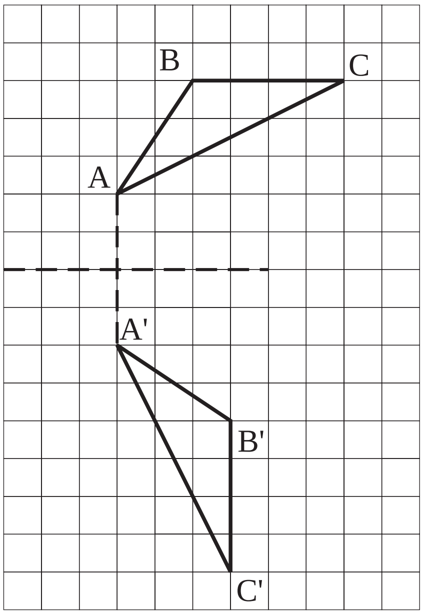
Example 4

The diagram shows the triangle A B C which is rotated through $90°$ to give A' B' C'.

Determine the position of the centre of rotation.



Solution

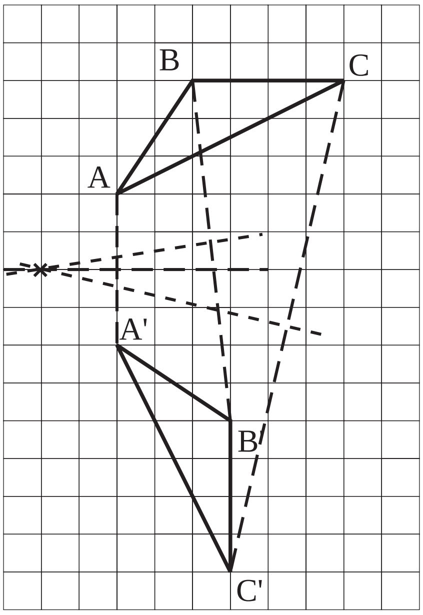


The first step is to join the points A and A' and draw the perpendicular bisector of this line.

The centre of rotation must be on this line.

Repeat the process, drawing the perpendicular bisectors of B B' and C C' as shown opposite.

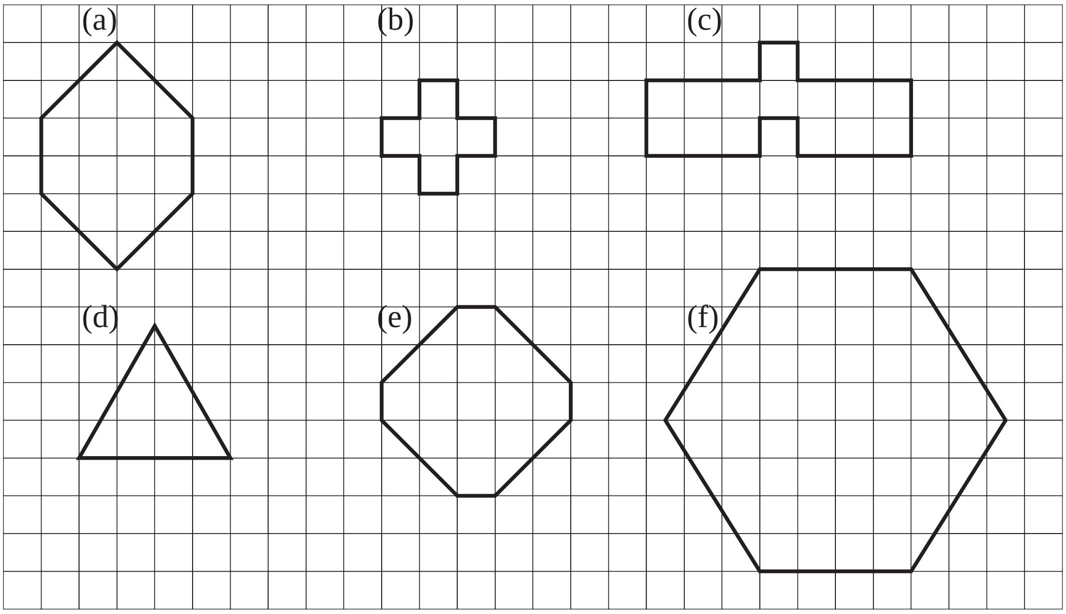
The point where the lines cross is the centre of rotation.



Note: For simple rotations you may be able to spot the centre of rotation without having to use the method shown above. Alternatively, you may be able to find the centre of rotation by experimenting with tracing paper.

Exercises

1. State the order of rotational symmetry of each of the following shapes:



2. Which of the capital letters have rotational symmetry?

3. A rectangle has corners at the points A (2, 4), B (6, 4), C (6, 6) and D (2, 6).
   (a) Draw this rectangle.
   (b) Rotate the rectangle through $90°$ clockwise about the point (0, 0).
   (c) Rotate the rectangle A B C D through $180°$ about the point (0, 0).

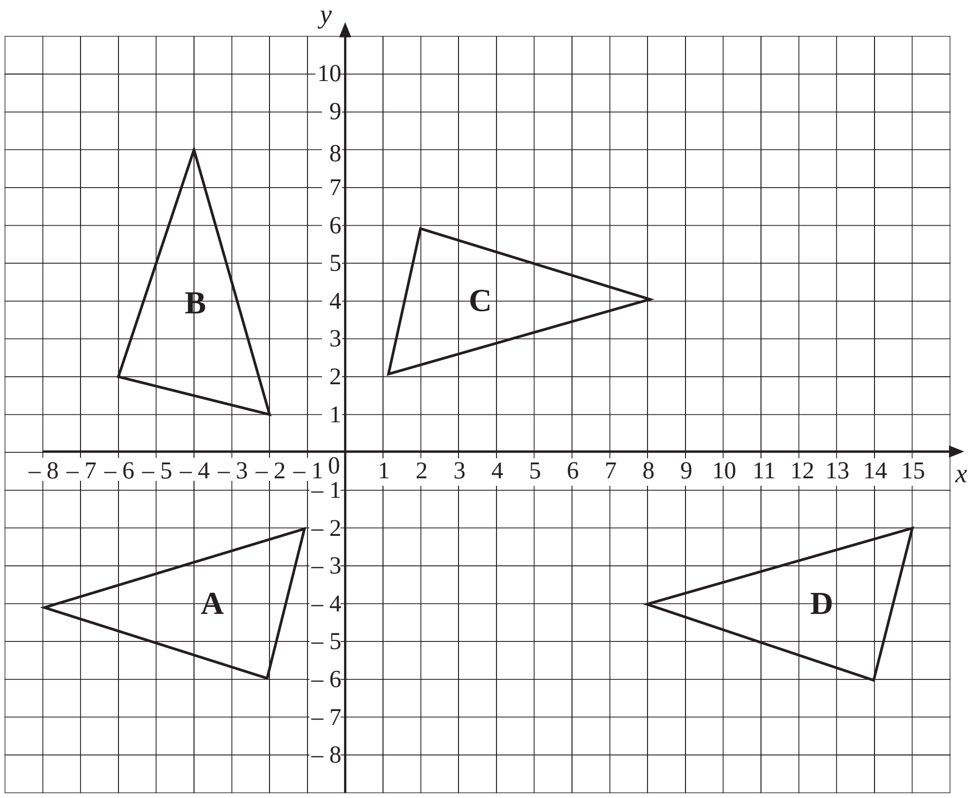
4. Rotate the rectangle formed by joining the points (1, 1), (3, 1), (3, 2) and (1, 2) through $90°$ clockwise about the origin.

5. A triangle has corners at the points with coordinates (4, 7), (3, 2) and (5, 1).

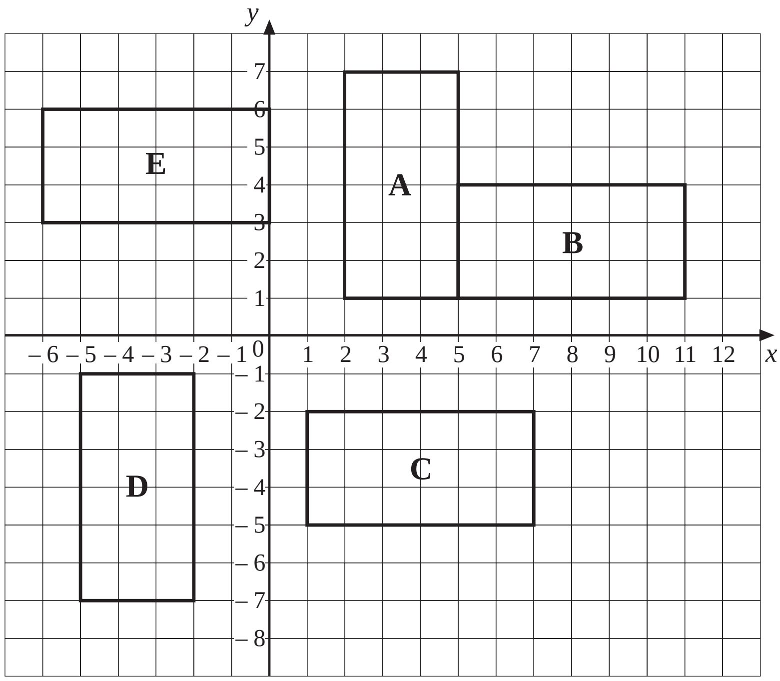
   Determine the coordinates of the triangles that are obtained by rotating the original triangle:
   (a) through $90°$ anticlockwise about (0, 3),
   (b) through $180°$ about (4, 0),
   (c) through $90°$ clockwise about (6, 2).

6. The following diagram shows the triangles A, B C and D.

   Describe the rotation that takes:
   (a) A to B,
   (b) A to C,
   (c) C to B,
   (d) C to D.



7. The following diagram shows the rectangles A, B, C and D.



Describe the rotation that takes:

(a) A to B,

(b) A to C,

(c) A to D,

(d) A to E.

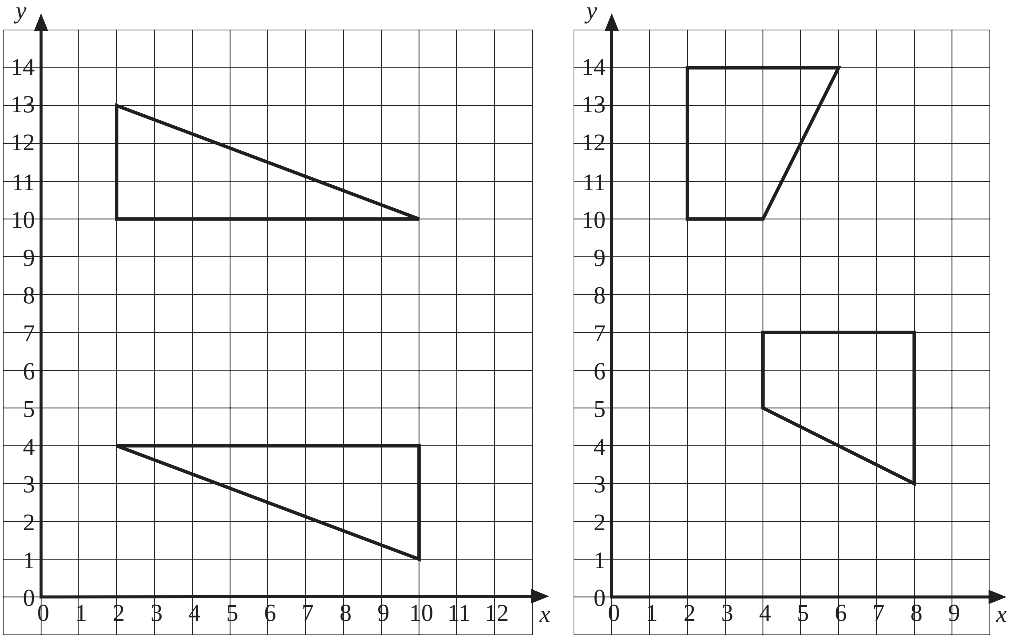
8. The triangle A has corners at the points with coordinates (1, 7), (3, 6) and (2, 4).

   (a) Rotate triangle A through 180 ° about the origin to get triangle B.

   (b) Rotate triangle B clockwise through 90 ° about the point (0, – 4) to get triangle C.

   (c) Write down the coordinates of the corners of triangle C.

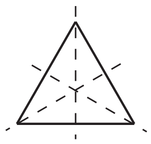
9. The following diagrams show two rotations. Determine the coordinates of the centre of rotation in each case.



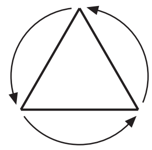
10. A triangle has corners at the points A (4, 2), B (6, 3) and C (5, 7). The triangle is rotated to give the triangle with corners at the points A' (3, – 1), B' (4, – 3) and C' (8, – 2).

    Describe fully this rotation.

11. An equilateral triangle has *3 lines of symmetry.*

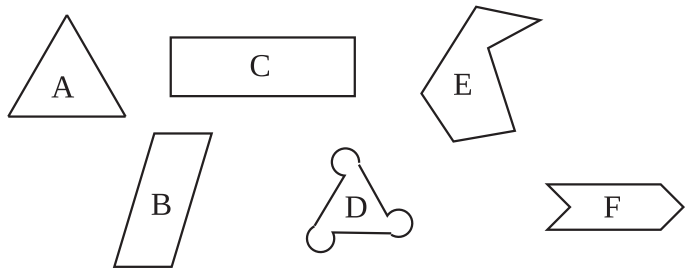
    

    It has *rotational symmetry* of *order 3.*

    Write the letter of each of the following shapes in the correct space in a copy of the table. You may use a mirror or tracing paper to help you.

    The letters for the first two shapes have been written for you.

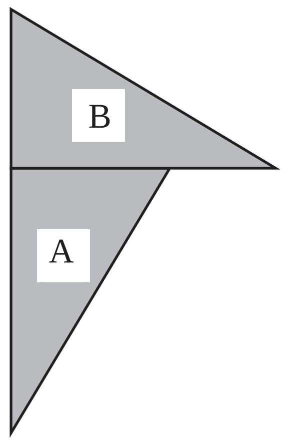


Number of Lines of Symmetry

Order of Rotational Symmetry		0	1	2	3	
	1					
	2	B				
	3				A	

(KS3/99/Ma/Tier 5-7/P1)

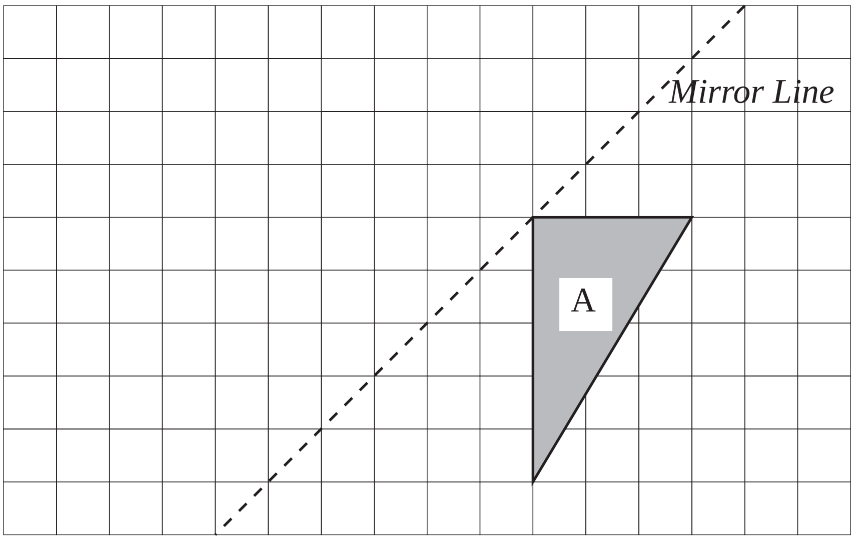
12.



(a) You can rotate triangle A onto triangle B.
Make a copy of the diagram and put a cross on the *centre of rotation.*
You may use tracing paper to help you.

(b) You can *rotate* triangle A onto triangle B.
The rotation is *anti-clockwise.*
What is the *angle* of rotation?

(c) On a copy of the diagram below, *reflect* triangle A in the mirror line. You may use a mirror or tracing paper to help you.



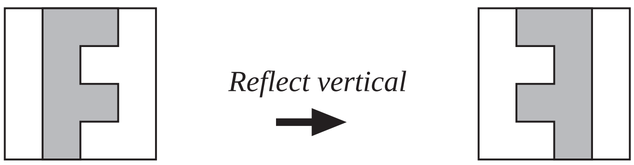
(KS3/99/Ma/Tier 4-6/P2)

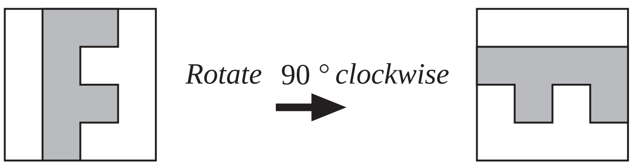
13. Julie has written a computer program to transform pictures of tiles. There are *only two instructions* in her program,

reflect vertical

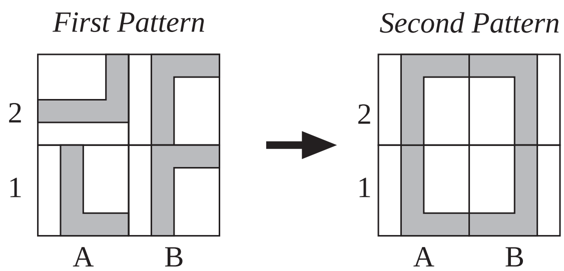
or

rotate 90 ° *clockwise.*





(a) Julie wants to transform the first pattern to the second pattern.



Copy and complete the following instructions to transform the tiles B1 and B2. You must use only *reflect vertical* or *rotate* 90 ° *clockwise.*

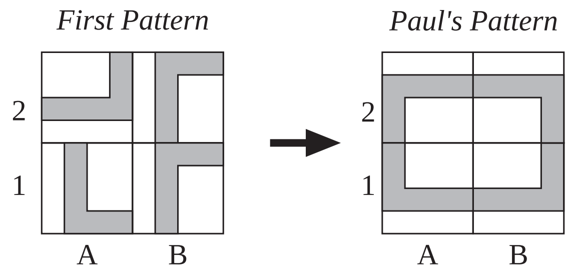
A1 *Tile is in the correct position.*

A2 *Reflect vertical, and then rotate* 90 °*clockwise.*

B1 *Rotate* 90 ° *clockwise and then* ....

B2 ....

(b) Paul starts with the first pattern that was on the screen.



Copy and complete the instructions for the transformations of A2, B1 and B2 to make Paul's pattern. You must use only *reflect vertical* or *rotate* 90 °*clockwise.*

A1 *Reflect vertical, and then rotate* 90 ° *clockwise.*

A2 *Rotate* 90 °*clockwise, and then* ....

B1 ....

B2 ....

(KS3/96/Ma/Tier 4-6/P1)

7.6 Combining Transformations

In this section we combine transformations. We see that sometimes 2 transformations are equivalent to a single transformation.

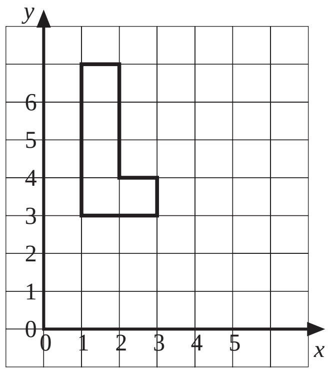
Here we use transformations from the following types:

Translations *Enlargements* *Reflections* *Rotations*

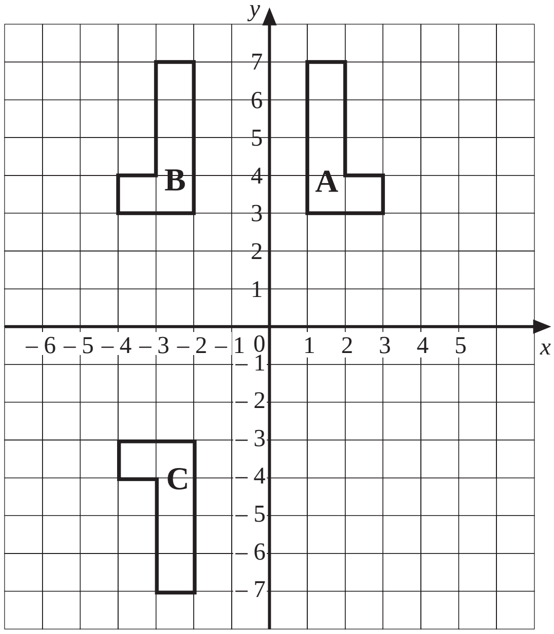
Example 1

The shape shown in the diagram is reflected first in the *y*-axis and its image is then reflected in the *x*-axis.

What *single* transformation would have the same result as these *two* transformations?



Solution



The diagram shows how the original shape A is first reflected to B, and B is then reflected to C.

A rotation of 180 ° about the origin would take A straight to C.

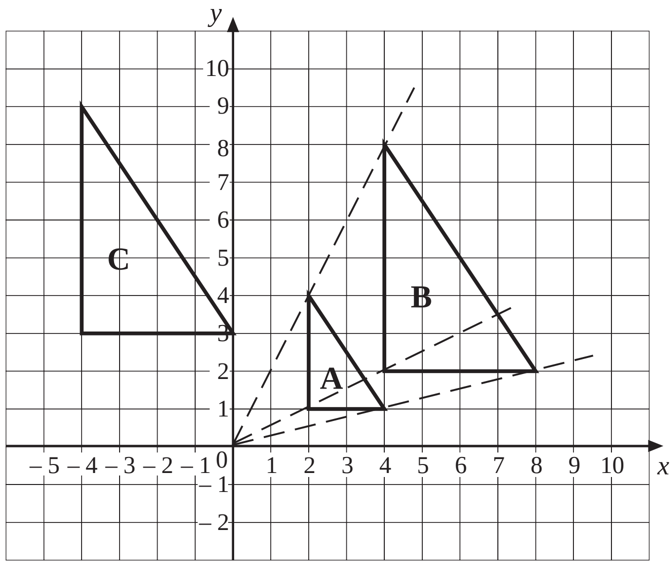
Example 2

A triangle is to be enlarged with scale factor 2, using the origin as the centre of enlargement. Its image is then to be translated along the vector $\begin{pmatrix} -8 \\ 1 \end{pmatrix}$.

The coordinates of the corners of the triangle are (2, 1), (2, 4) and (4, 1).

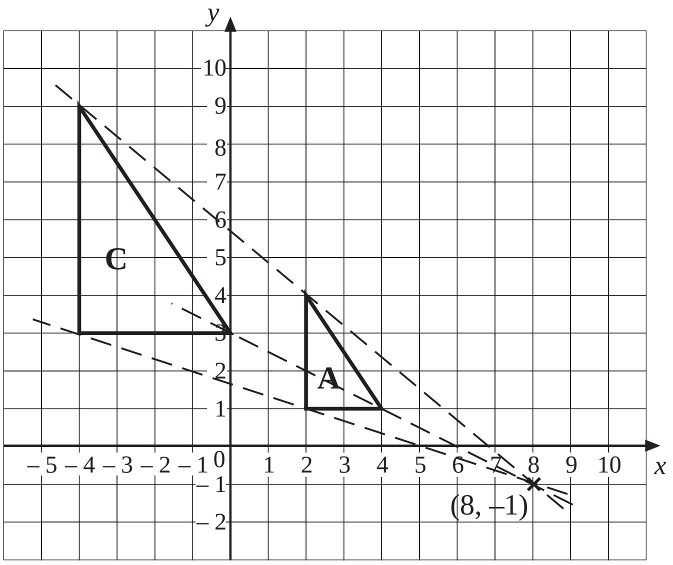
What *single* transformation would have the same result?

Solution



The diagram shows the original triangle, A; the enlargement takes it to B, which is then translated to C.

The triangle A could be enlarged with scale factor 2 to give C.



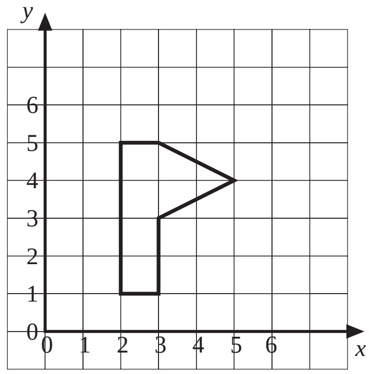
This diagram shows that the centre of enlargement would be the point (8, – 1).

The single transformation that will move triangle A to triangle C is an enlargement, scale factor 2, centre (8, – 1).

Exercises

1. (a) Reflect the shape shown in the *x*-axis and then reflect its image in the *y*-axis.

   (b) What single transformation would have the same result as these two transformations?



2. A rectangle has corners at the points with coordinates (1, 2), (3, 1), (5, 5) and (3, 6). It is first reflected in the *x*-axis and then its image is rotated through 180 ° about the origin.

   Describe how to move the rectangle from its original position to its final position, using only one transformation.

3. A shape is rotated through 180 ° about the origin and then its image is reflected in the *x*-axis.

   (a) Choose a shape and carry out the transformations described above.

   (b) What single transformation would have the same result as the two transformations described above?

4. A triangle has corners at the points with coordinates (2, 2), (3, 6) and (8, 6).

   (a) Draw the triangle and enlarge it with scale factor 2, using the origin as the centre of enlargement.

(b) Translate the enlarged shape along the vector $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$.

(c) Describe fully the enlargement that would produce the final triangle from the original triangle.

5. (a) Draw the triangle with corners at the points (2, 1), (4, 1) and (4, 2).

(b) Reflect this shape in the line $y = x$.

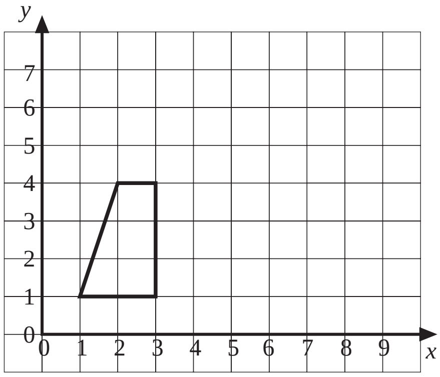
(c) Reflect the new triangle in the y-axis.

(d) What single transformation would have the same result as the two transformations described above?

6. (a) Reflect a shape of your choice in the line $y = x$ and then reflect the image in the line $y = -x$.

(b) Describe a single transformation that would have the same result.

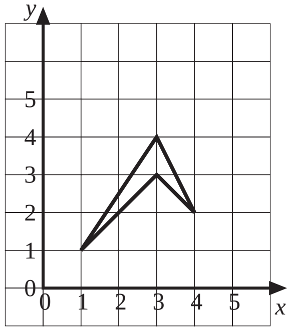
7. The shape shown in the following diagram is to be reflected in the line $x = 4$ and then its image is to be reflected in the line $y = 5$.



(a) Draw a diagram to show how the shape moves.

(b) What single transformation would have the same result?

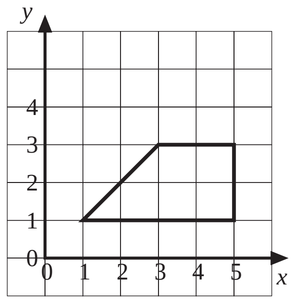
8. The shape shown in the diagram is translated along the vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$.



(a) Draw the final position of the shape.

(b) Describe how the shape could be moved to this position using 2 reflections.

9. The shape shown in the diagram is to be enlarged with scale factor 3 using the point (0, 4) as the centre of enlargement.



(a) Draw the enlarged shape.

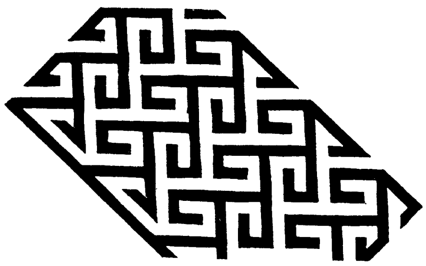
(b) The enlarged shape is translated along the vector $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$. Draw the new position of the shape.

(c) Describe the single enlargement that would have the same result as the two transformations used above.

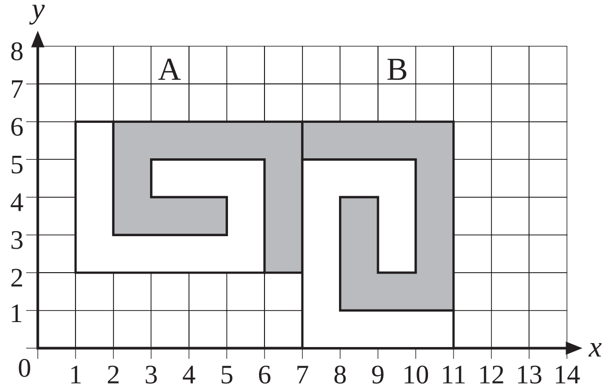
10. A shape is reflected in the line $y = x$, then in the line $y = -x$, and finally in the x-axis.

What single transformation would have the same result?

11. The following design is based on a Celtic pattern.



Part of the pattern is shown below:



The pattern is made of two rectangular blocks, A and B.

Use *two* transformations to map block A onto block B. Your transformations must be either rotations or reflections.

Mark any mirror lines or centres of rotation on a copy of the previous diagram.

Write down instructions for the first and second transformations.

Give coordinates of any centres of rotation, the amount of turn and direction of turn. Give the equations of any lines of reflection.

(KS3/95/Ma/Levels 9-10)

UNIT 7 *Transformations* Activities

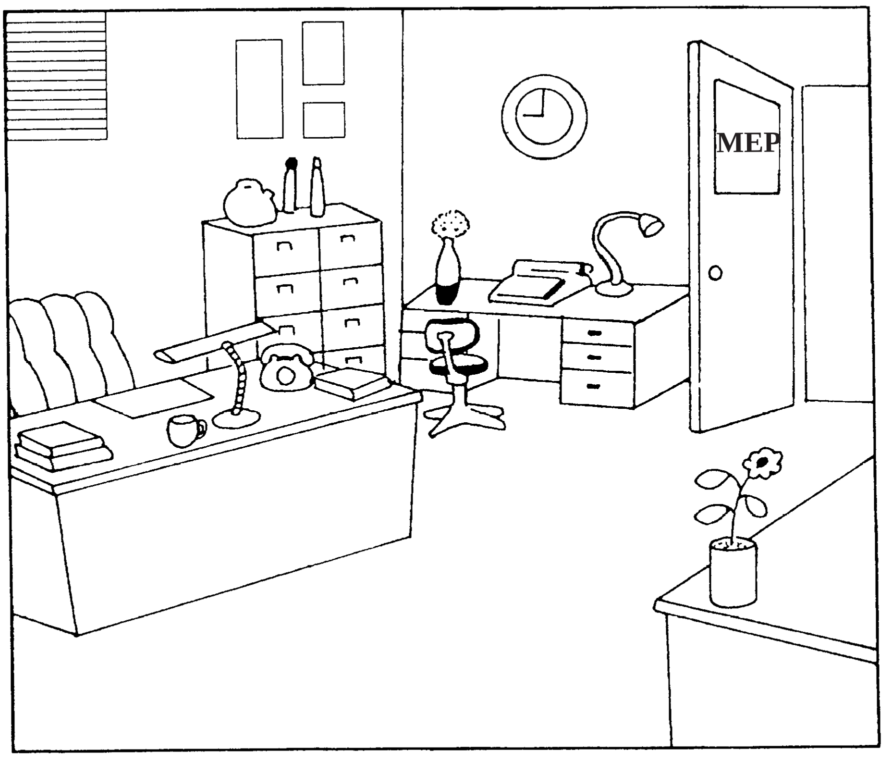
Activities

ACTIVITY 7.1 *Reflections*

Picture 2 should be the reflection of *Picture* 1 in the line AB. Circle the errors.

PICTURE 1

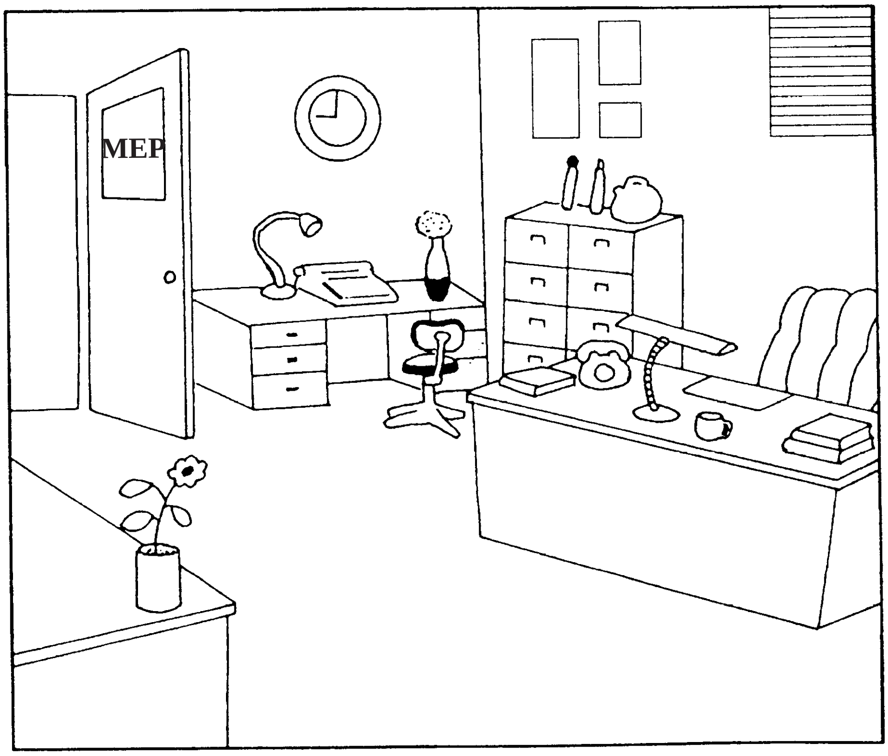
A



B

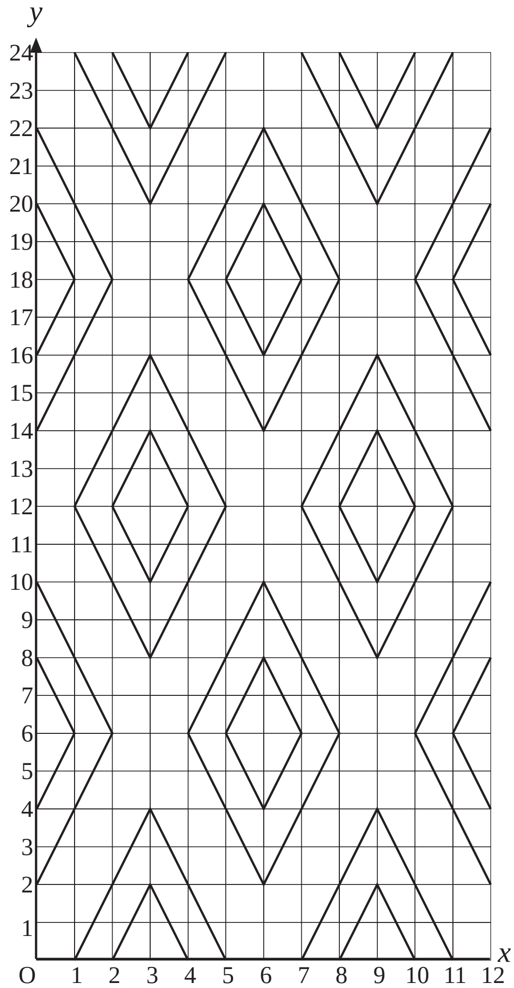
PICTURE 2

A



B

ACTIVITY 7.2 *Wallpaper*



The diagram shows a piece of wallpaper with a repeating pattern.

1. Describe how to obtain this sheet of wallpaper by starting with the line that joins the points with coordinates (2, 0) and (3, 2).

   The origin (O) is at the point with coordinates (0, 0).

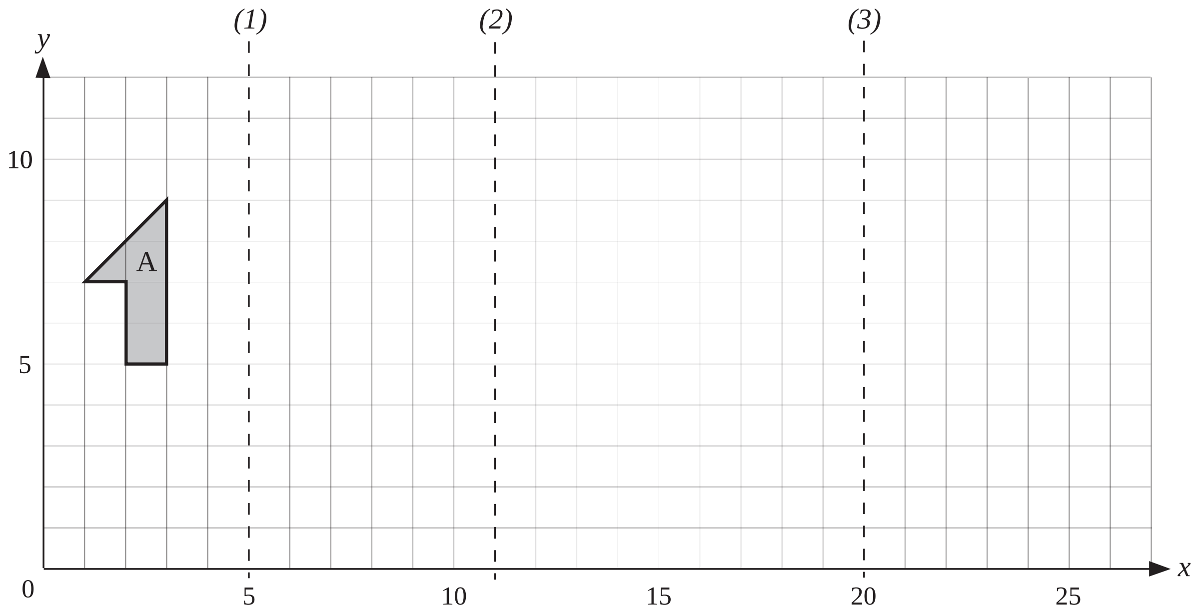
2. How many transformations are needed in total to complete the pattern on the grid

   $0 \le x \le 12, \ \ 0 \le y \le 24$?

2. Design your own wallpaper pattern and describe instructions to obtain it from a single line or shape. Try to use a minimum number of transformations when repeating the pattern.

ACTIVITY 7.3 *Repeated Reflections*

The following diagram shows a shape, A, and mirror lines, 1, 2 and 3. Copy the diagram and then carry out the reflections and answer the question below.



Step 1. Reflect *shape A* in *mirror line 1* and label it B.

Step 2. Reflect *shape B* in *mirror line 2* and label it C.

Step 3. Reflect *shape C* in *mirror line 3* and label it D.

Step 4. *Shape D* can be obtained from *shape A* by just one reflection. Draw the required mirror line on your diagram and label it 4.

What is the equation of this line?

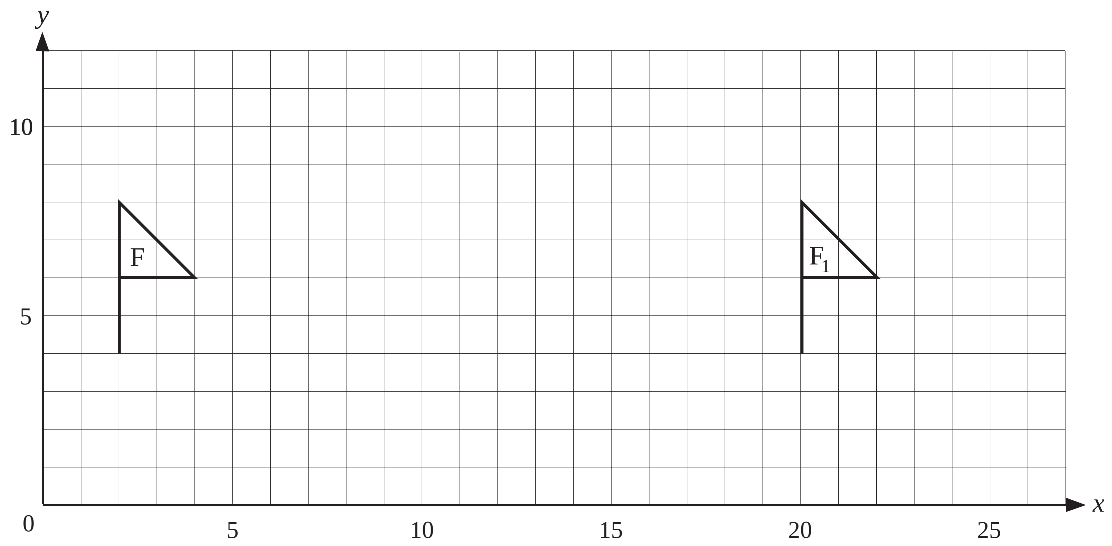
Extension

Repeat steps 1–4 on a new diagram using your own shape.

Does your answer to the question after Step 4 remain the same?

ACTIVITY 7.4 *Transformations*

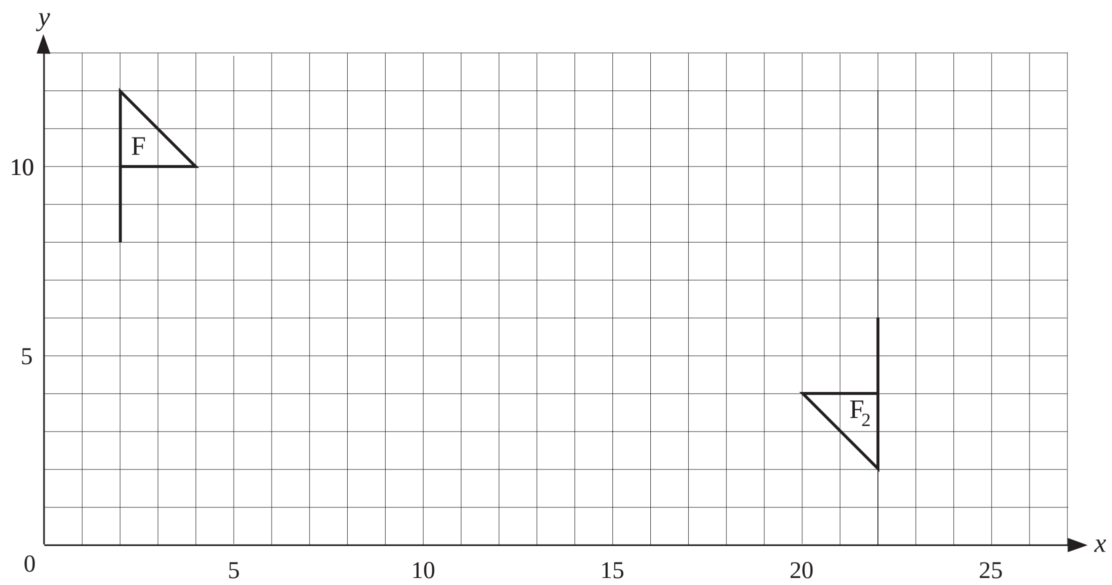
1. (a) F is mapped onto F_1 under a translation. What is the translation?



(b) (i) Describe how F can be mapped onto F_1 using two successive reflections.

(ii) In how many different ways can this be done?

2. (a) F is mapped onto F_2 under a rotation. Describe this rotation.



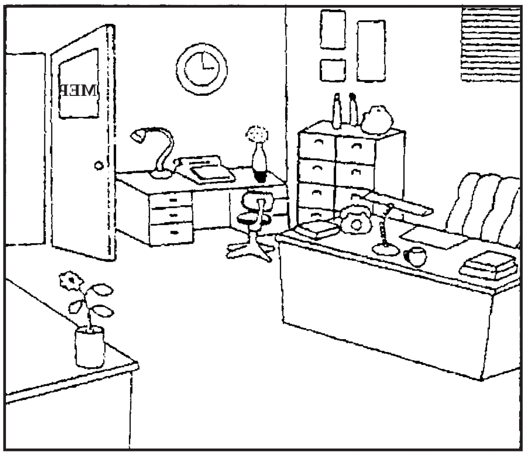
(b) (i) Describe how F can be mapped onto F_2 using two successive reflections.

(ii) Is your answer the *only* way that it can be done?

ACTIVITY 7.1 - 7.3 *Notes and Solutions*

Notes and solutions given only where appropriate.

7.1 Correct reflection is:



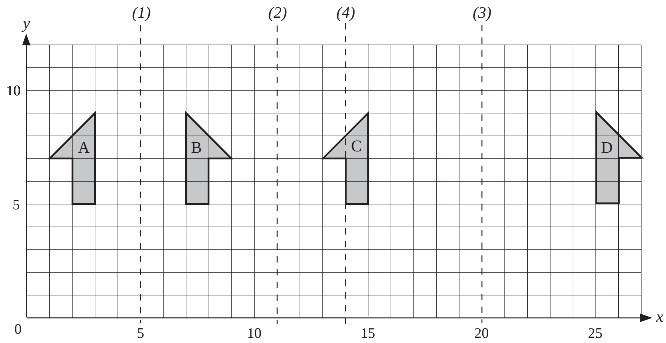
7.2 There are many possible answers.
Here is one of them:

1. Translate line by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
2. Translate line by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
3. Reflect new shape in $x = 3$
4. Reflect new shape in $x = 6$
5. Reflect new shape in $y = 6$
6. Reflect new shape in $y = 12$
7. Translate new shape by $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$
8. Reflect in $y = 6$
9. Reflect in $x = 9$

(NB Ignore any parts of transformed shape that lie outside the wallpaper grid.)

7.3 1, 2, 3 and 4 shown on diagram.

Equation of mirror line is $x = 14$.



Extension
It remains the same; i.e. reflected in line $x = 14$.

ACTIVITY 7.4 *Notes and Solutions*

7.4 1. (a) $\begin{pmatrix} 18 \\ 0 \end{pmatrix}$

(b) (i) e.g. reflect F in line $x = 8$ and then reflect in line $x = 17$.

(ii) Infinite number

2. (a) Rotation, $180°$ about (12, 7)

(b) (i) Reflect in $x = 12$, and in $y = 7$

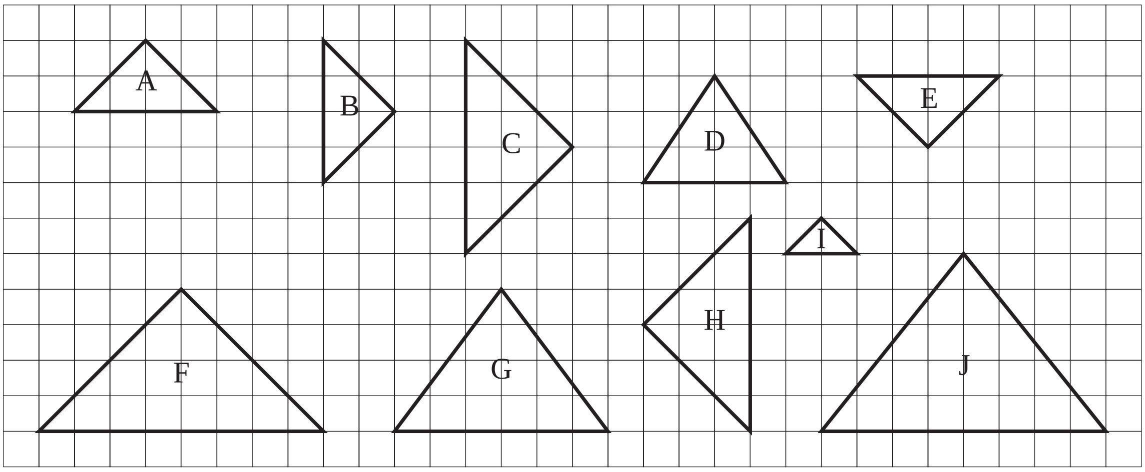
(ii) You can change the order, i.e. reflect in $y = 7$ and then $x = 12$.

UNIT 7 *Transformations* Extra Exercises 7.1

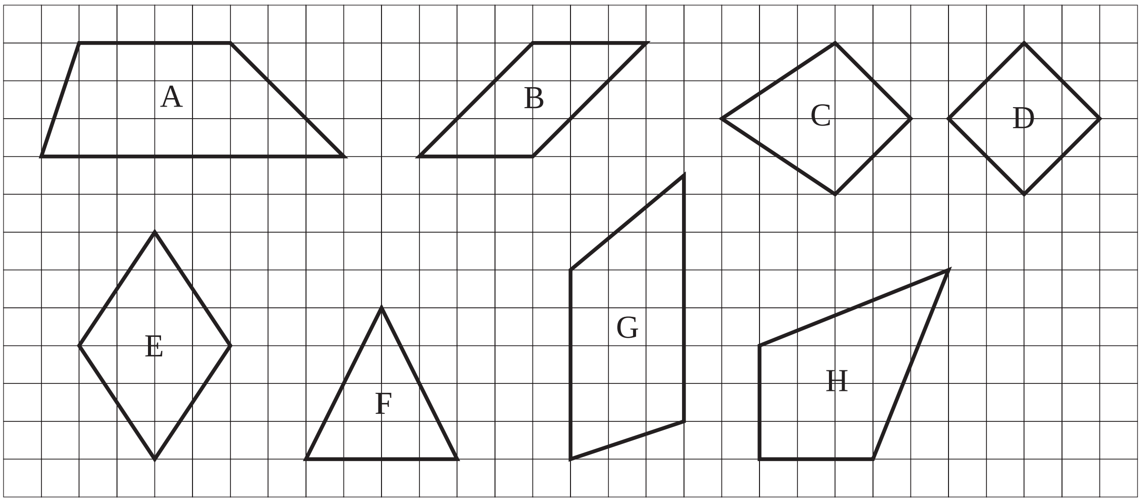
1. Which of the shapes in the following diagram are:

   (a) *similar* to A,

   (b) *congruent* to A ?



2. Write down the name of each of the following shapes:



3. The points A, B, C and D are the corners of a parallelogram. The coordinates of A, B and C are (2, 2), (7, 2) and (9, 6) respectively.

   What are the coordinates of D ?

UNIT 7 *Transformations* Extra Exercises 7.2

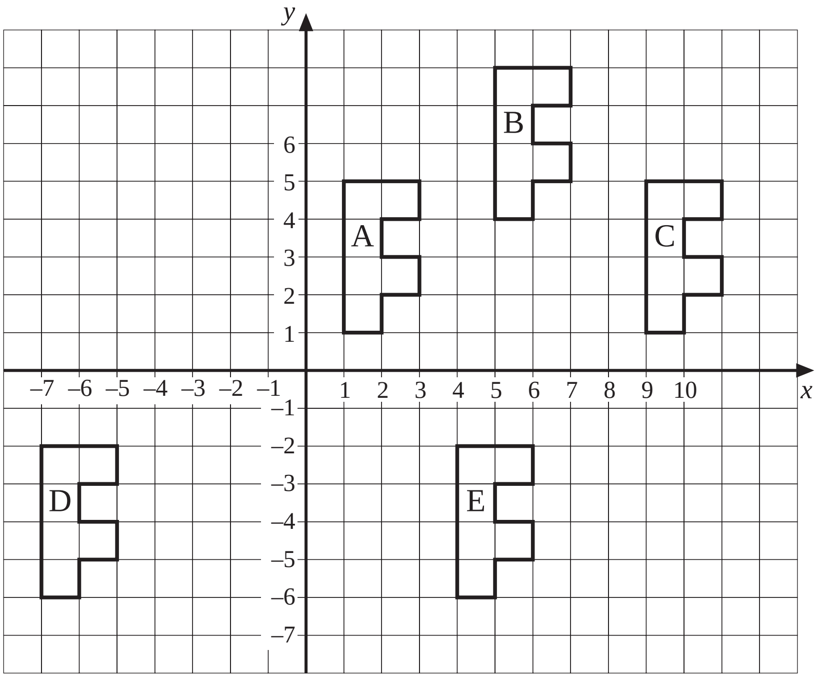
1. Write down the vector that you would use to translate:

   (a) A to B (b) B to C

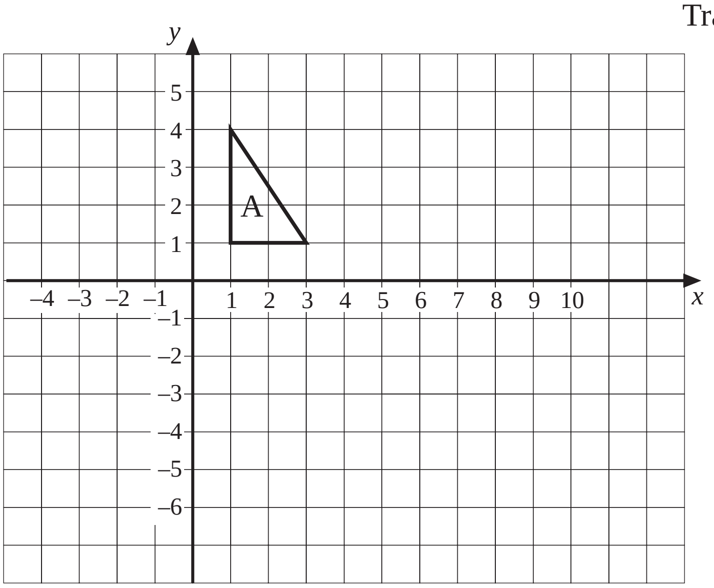
   (c) A to D, (d) C to D,

   (e) B to D, (f) E to A

   on the following diagram.



2. Copy the following diagram.



Translate triangle A by the vectors:

(a) $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$ to obtain B,

(b) $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$ to obtain C,

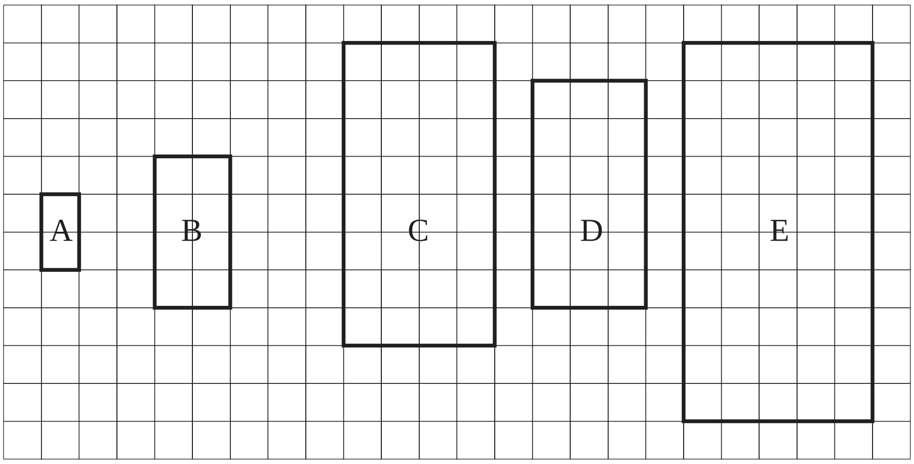
(c) $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$ to obtain D,

(d) $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ to obtain E,

(e) $\begin{pmatrix} 8 \\ -5 \end{pmatrix}$ to obtain F.

UNIT 7 *Transformations* Extra Exercises 7.3

1. The diagram shows several shapes that are enlargements of each other.



What scale factor is used for each of the following enlargements:

(a) A to B, (b) A to C, (c) A to D,

(d) A to E, (e) C to B, (f) C to D ?

2. (a) Draw the triangle that has corners at the point with coordinates (3, 1), (2, 4) and (5, 3).

(b) Enlarge this triangle with scale factor 3 and centre of enlargement (0, 0).

(c) Write down the coordinates of the corners of the enlarged triangle.

3. A triangle has corners at the points with coordinates (2, 11), (2, 8) and (5, 7). It is enlarged to give a triangle with corners at the points with coordinates (4, 16), (4, 10) and (10, 8).

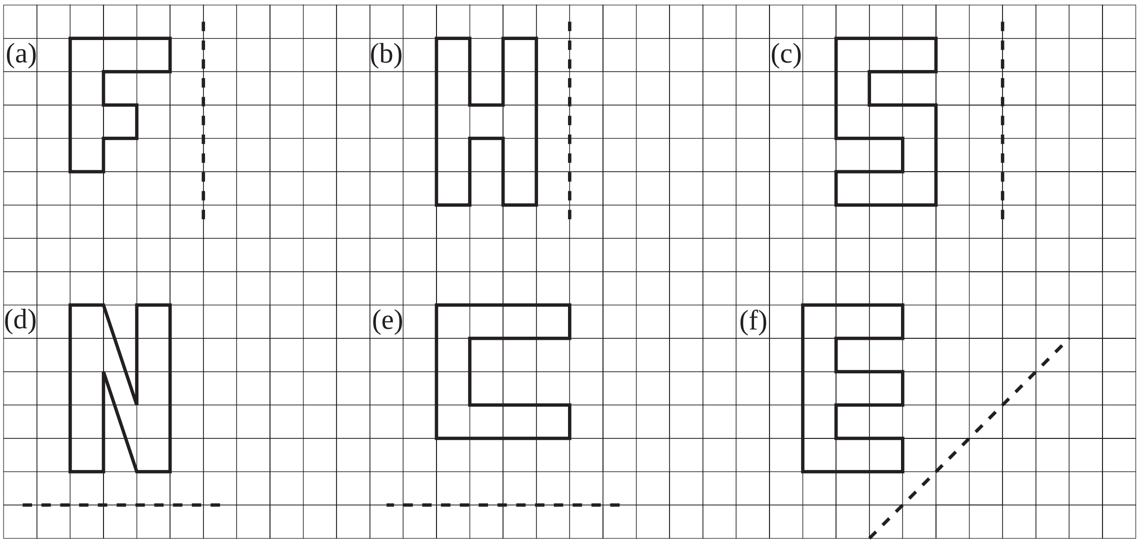
(a) Draw both triangles.

(b) Determine the position of the centre of enlargement.

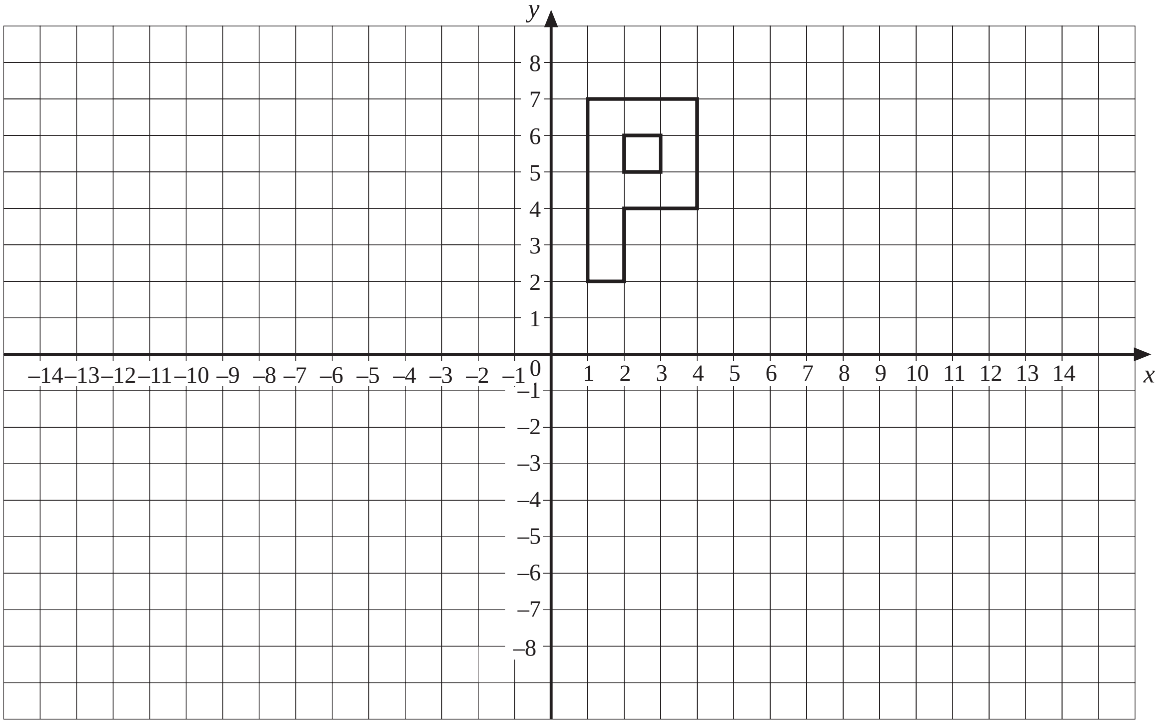
(c) State the scale factor of the enlargement.

UNIT 7 *Transformations* Extra Exercises 7.4

1. Copy each of the following diagrams and draw the reflection of each shape in the mirror line shown.



2. Copy the following diagram.



Reflect the shape in the following lines:

(a) y-axis, (b) x-axis, (c) $x = 7$,

(d) $x = -5$, (e) $y = x$ (f) $y = -x$

UNIT 7 *Transformations* Extra Exercises 7.5

1. (a) Draw the triangle that has corners at the points with coordinates (2, 3), (5, 3) and (2, 7). Label it A.

   (b) Rotate A through $90°$ clockwise around (0, 0) to obtain B.

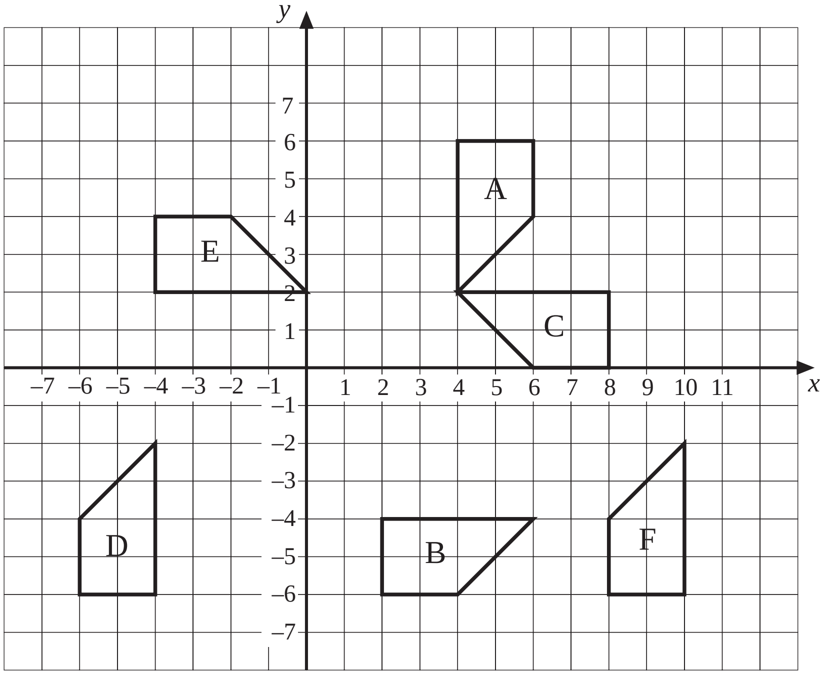
   (c) Rotate A through $180°$ around (0, 0) to obtain C.

   (d) Rotate A through $180°$ around (0, 4) to obtain D.

   (e) Rotate A through $90°$ clockwise around (6, 3) to obtain E.

   (f) Rotate A through $90°$ anticlockwise around (9, 2) to obtain F.

2. The following diagram shows the shapes A, B, C, D, E and F that have been obtained by rotating the shape A. Describe each rotation fully.



UNIT 7 *Transformations* Extra Exercises 7.6

1. (a) Draw the triangle with corners at the points with coordinates (1, 5), (3, 5) and (3, 2).

   (b) Reflect the triangle in the line $x = 3$.

   (c) Reflect the triangle in the line $y = 5$.

   (d) Rotate the shape through $180°$ around the point (3, 5).

   (e) What is the name of the shape you obtain?

2. (a) Draw the rectangle A that has corners at the points with coordinates (3, 1), (6, 4), (4, 6) and (1, 3).

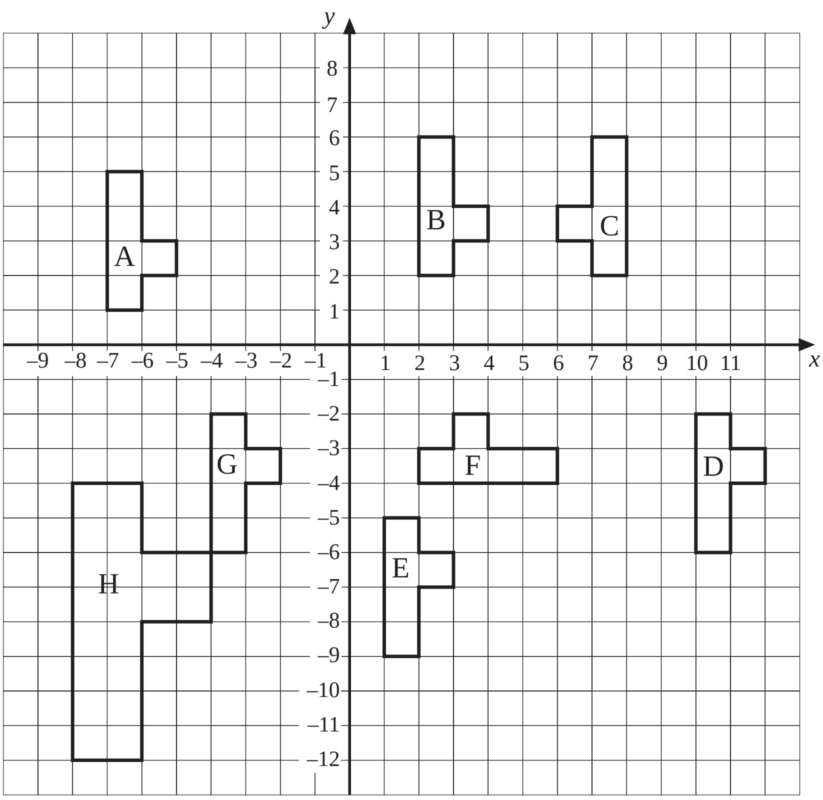
   (b) Reflect the rectangle A in the line $x = 6$ to obtain the rectangle B.

   (c) Reflect the rectangles A and B in the line $y = 6$ to obtain the rectangles C and D.

   (d) Translate all 4 rectangles using the vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

   (e) Rotate all 8 rectangles through $180°$ about the point (11, 1).

3. The shape A is moved to the shape H by a series of transformations, in order, as shown on the diagram. Describe each transformation.



Extra Exercises 7.1 Answers

1. (a) C, F, H, I (b) B, E

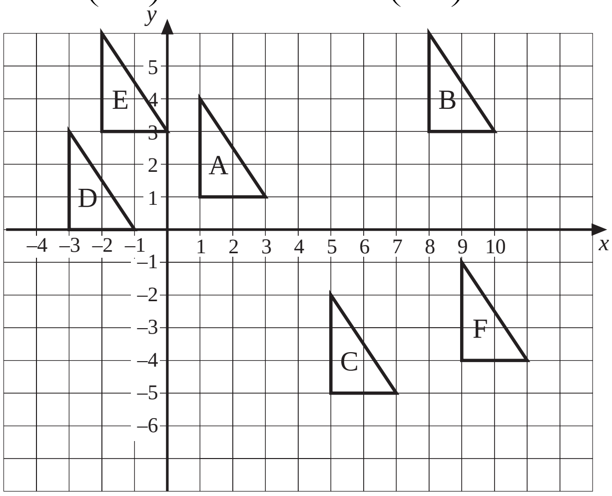
2. A - Trapezium; B - Parallelogram; C - Kite; D - Square; E - Rhombus;
   F - Isosceles triangle; G - Trapezium; H - Kite

3. (4, 6)

Extra Exercises 7.2 Answers

1. (a) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ (c) $\begin{pmatrix} -8 \\ -7 \end{pmatrix}$

   (d) $\begin{pmatrix} -16 \\ -7 \end{pmatrix}$ (e) $\begin{pmatrix} -12 \\ -10 \end{pmatrix}$ (f) $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$

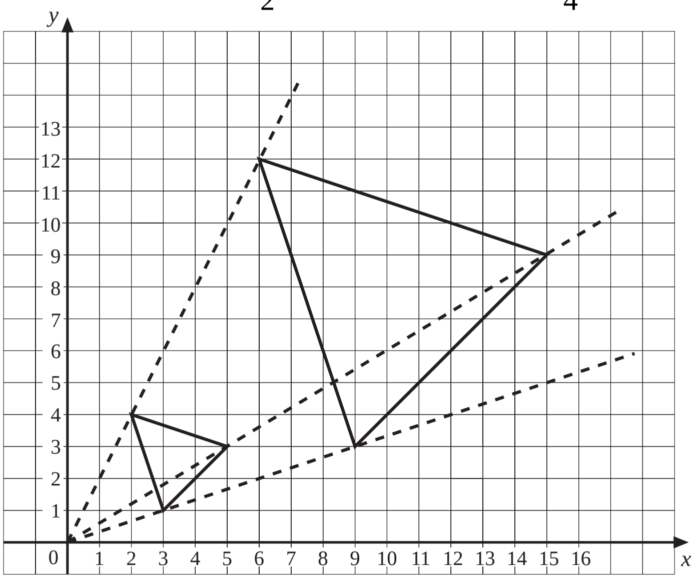
2. 

Extra Exercises 7.3, Questions 1-2 Answers

1. (a) 2 (b) 4 (c) 3

   (d) 5 (e) $\frac{1}{2}$ (f) $\frac{3}{4}$

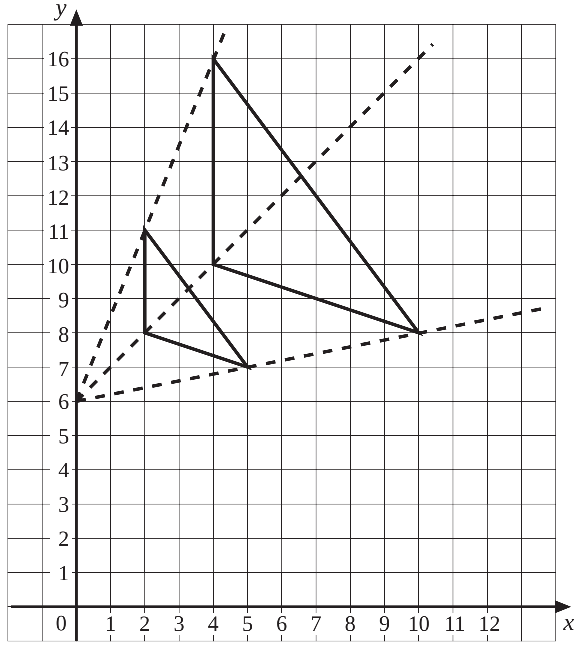
2. (a), (b)



(c) (9, 3), (6, 12) and (15, 9)

Extra Exercises 7.3 Question 3 Answers

3. (a)

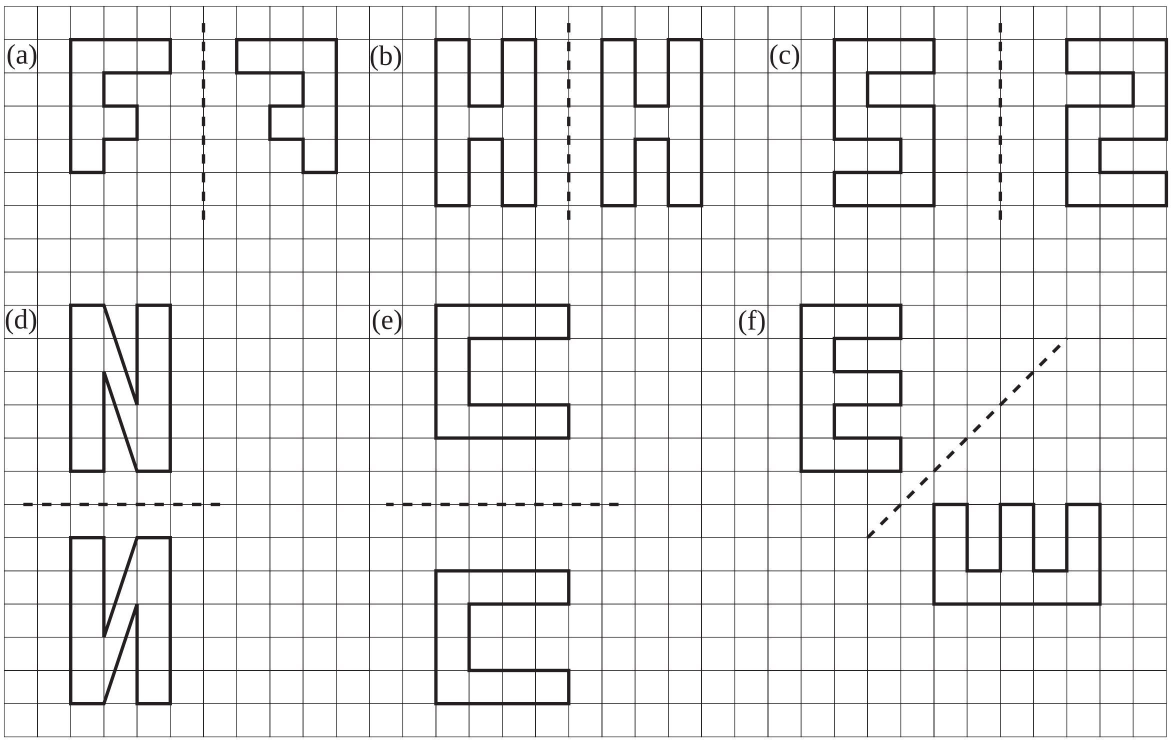


(b) (0, 6)

(c) 2

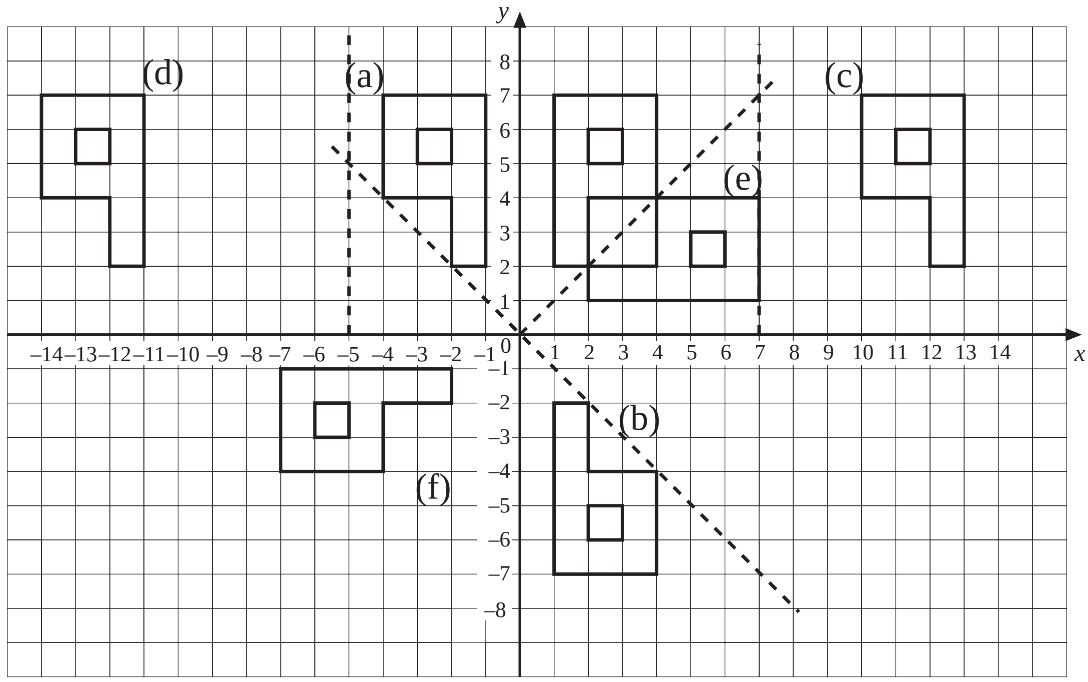
Extra Exercises 7.4 Question 1 Answers

1.



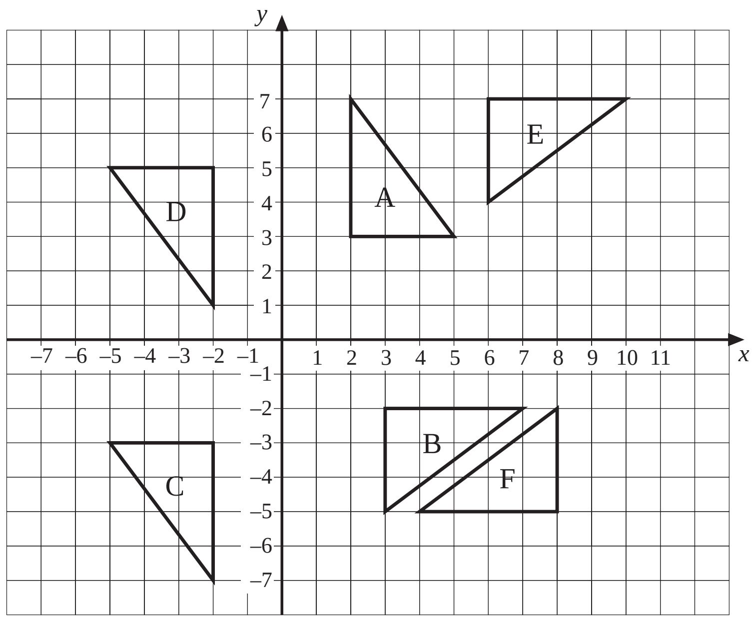
Extra Exercises 7.4 Question 2 Answers

2.



Extra Exercises 7.5 Answers

1.



2. A $\rightarrow$ B : 90 ° clockwise around (0, 0)

A $\rightarrow$ C : 90 ° clockwise around (4, 2)

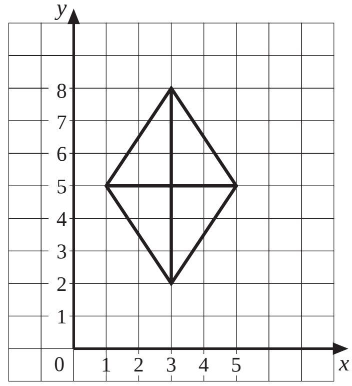
A $\rightarrow$ D : 180 ° around (0, 0)

A $\rightarrow$ E : 90 ° anticlockwise around (2, 0)

A $\rightarrow$ F : 180 ° around (7, 0)

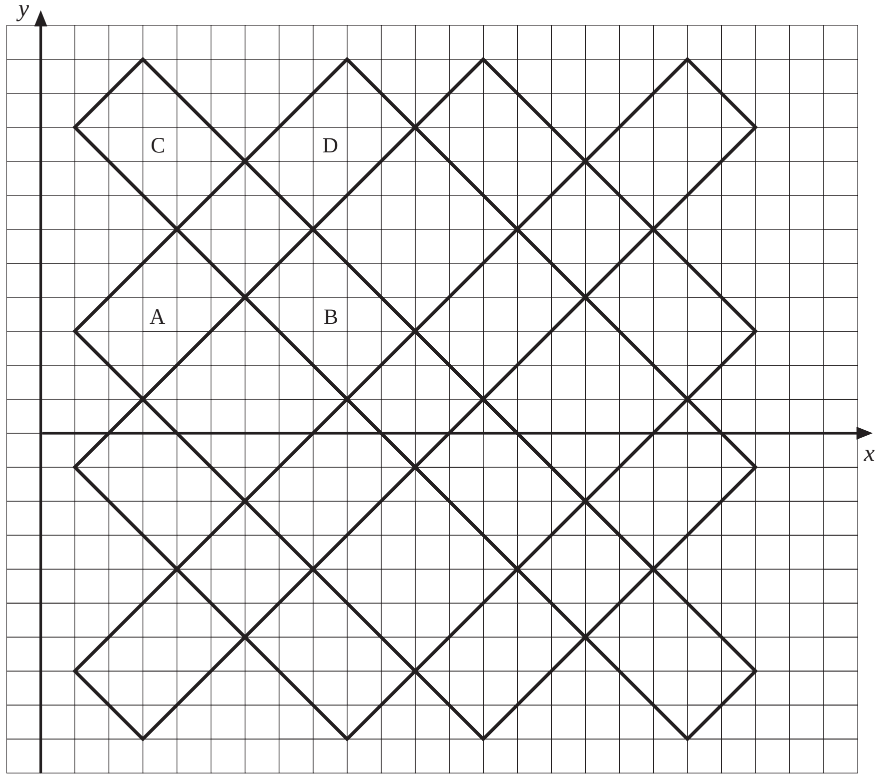
Extra Exercises 7.6 Answers

1.



(e) a rhombus

2.



3. A → B : translation $\begin{pmatrix} 9 \\ 1 \end{pmatrix}$

B → C : reflection in $x = 5$

C → D : rotation, 180° around (9, 0)

D → E : translation $\begin{pmatrix} -9 \\ -3 \end{pmatrix}$

E → F : rotation, 90° anticlockwise around (1, – 4)

F → G : rotation, 90° clockwise around (0, 0)

G → H : enlargement, scale factor 2, centre (0, 0)

UNIT 7 *Transformations* Lesson Plans **St**

These are based on 45/50 minute lessons.

Lesson No.	*Suggested Plan*	*References*
1.	**Shapes**	
	Names and Properties of 2-D Shapes	OS 7.1
	Exercises	PB 7.1, Q1
	Review answers	
	'Congruent' or 'similar'	OS 7.2
	Exercises	PB 7.1, Q2
	Review answers	
	Exercises	PB 7.1, Q5
	Review answers	
	Set homework	PB 7.1, Q6 and Q7
2.	**Translations**	
	Discuss homework	
	Introduction	OS 7.3
	Exercises	PB 7.2, Q1
	Review answers	
	Exercises	PB 7.2, Q2
	Review answers	
	Exercises	PB 7.2, Q3
	Review answers	
	Set homework	PB 7.2, Q4 and Q6
3.	**Enlargements**	
	Discuss homework	
	Introduction to concept	OS 7.4
	Exercises	PB 7.3, Q1
	Review answers	
	Exercises	PB 7.3, Q2
	Review answers	
	Set homework	PB 7.3, Q11
4.	**Reflections**	
	Discuss homework	
	Introduction to concept	OS 7.7
	Exercises	PB 7.4, Q1
	Review answers	
	Exercises	PB 7.4, Q2
	Review answers	
	Activity	Activity 7.1
	Set homework	PB 7.4, Q11 and Q12

UNIT 7 *Transformations* Lesson Plans **St**

Lesson No.	*Suggested Plan*	*References*
5.	**Rotations**	
	Discuss homework	
	Introduction - rotational symmetry	PB 7.5, Example 1
	Exercises	PB 7.5, Q1 and Q2
	Review answers	
	Rotations (including centre of rotation)	OS 7.9
	Exercises	PB 7.5, Q3
	Review answers	
	Mental Test	M 7.1
	Review answers	
	Set homework	PB 7.5, Q4 and Q12
6.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 7.1
7.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 7 *Transformations* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
1.	**Shapes**	
	Names and Properties of 2-D Shapes	OS 7.1
	Exercises	PB 7.1, Q1
	Review answers	
	'Congruent' or 'similar'	OS 7.2
	Exercises	PB 7.1, Q2
	Review answers	
	Exercises	PB 7.1, Q5
	Review answers	
	Set homework	PB 7.1, Q4, Q6 and Q7
2.	**Translations**	
	Discuss homework	
	Introduction	OS 7.3
	Exercises	PB 7.2, Q1
	Review answers	
	Exercises	PB 7.2, Q2
	Review answers	
	Exercises	PB 7.2, Q6
	Review answers	
	Set homework	PB 7.2, Q4, Q7 and Q9
3.	**Enlargements**	
	Discuss homework	
	Introduction to concept	OS 7.4
	Exercises	PB 7.3, Q1
	Review answers	
	Centre of enlargement	OS 7.5
	Exercises	PB 7.3, Q6
	Review answers	
	Finding centre of enlargement	OS 7.6
	Exercises	PB 7.3, Q7
	Review answers	
	Set homework	PB 7.3, Q5 and Q11

UNIT 7 *Transformations* Lesson Plans **A**

Lesson No.	*Suggested Plan*	*References*
4.	**Reflections**	
	Discuss homework	
	Recap concept	OS 7.7
	Exercises	PB 7.4, Q2 and Q3
	Review answers	
	Equation of mirror line	OS 7.8
	Exercises	PB 7.4, Q4 and Q5
	Review answers	
	Activity	Activity 7.1
	Review answers	
	Set homework	PB 7.4, Q11 and Q12
5.	**Rotations**	
	Discuss homework	
	Introduction - rotational symmetry	PB 7.5, Example 1
	Exercises	PB 7.5, Q1 and Q2
	Review answers	
	Rotations (including centre of rotation)	OS 7.9
	Exercises	PB 7.5, Q3
	Review answers	
	Activity	Activity 7.2 or Activity 7.3
	Set homework	PB 7.5, Q11, Q12 and Q13
6.	**Combining Transformations**	
	Discuss homework	
	Introduction	OS 7.11
	Exercises	PB 7.6, Q1
	Review answers	
	Mental Test	M 7.2
	Review answers	
	Activity	Activity 7.3 or Activity 7.4
	Set homework	Complete Activity 7.3 or Activity 7.4
7.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 7.2
8.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 7 *Transformations* Lesson Plans **E**

Lesson No.	*Suggested Plan*	*References*
1.	**Translations**	
	Introduction	OS 7.3
	Exercises	PB 7.2, Q2
	Review answers	
	Exercises	PB 7.2, Q7
	Review answers	
	Set homework	PB 7.2, Q9 and Q10
2.	**Enlargements**	
	Discuss homework	
	Introduction to concept	OS 7.4
	Exercises	PB 7.3, Q1
	Review answers	
	Centre of enlargement	OS 7.5
	Exercises	PB 7.3, Q5
	Review answers	
	Finding the centre of enlargement	OS 7.6
	Exercises	PB 7.3, Q7
	Review answers	
	Set homework	PB 7.3, Q10 and Q11
3.	**Reflections**	
	Discuss homework	
	Introduction to concept	OS 7.7 and OS 7.8
	Exercises	PB 7.4, Q3
	Review answers	
	Exercises	PB 7.4, Q5
	Review answers	
	Activity	Activity 7.2 or Activity 7.3
	Set homework	Complete Activity 7.2 or Activity 7.3 and PB 7.4, Q13 and Q14
4.	**Rotations**	
	Discuss homework	
	Introduction	OS 7.9
	Exercises	PB 7.5, Q6
	Review answers	
	Describing rotations	OS 7.10
	Exercises	PB 7.5, Q11
	Review answers	
	Mental Test	M 7.3
	Review answers	
	Set homework	PB 7.5, Q9 and Q13

UNIT 7 *Transformations* Lesson Plans **E**

Lesson No.	*Suggested Plan*	*References*
5.	**Combining Translations**	
	Discuss homework	
	Introduction	OS 7.11
	Activity	Activity 7.4
	Review answers	
	Exercises	PB 7.6, Q7 and Q8
	Review answers	
	Set homework	PB 7.6, Q11
6.	**Revision Test**	
	Discuss homework	
	Revision Test	RT 7.3
7.	**Recap**	
	Give back marked tests	
	Go over test questions interactively	
	Revise topics	

UNIT 7 Transformations Mental Tests

M 7.1 Standard Route *(no calculator)*

Look at the *Information Sheet* for this test.

1. Which shape do you obtain if you reflect A in the x-axis? (I)
2. Which shape do you obtain if you reflect A in the y-axis? (B)
3. Which shape do you obtain if you translate A by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$? (C)
4. What vector is needed to translate G onto H ? ($\begin{pmatrix} 5 \\ 1 \end{pmatrix}$)
5. What vector is needed to translate B onto D ? ($\begin{pmatrix} 10 \\ 0 \end{pmatrix}$)
6. Shape A is rotated through 180 ° about the point (0, 0). Which shape do you obtain? (H)
7. What vector is needed to translate D onto J ? ($\begin{pmatrix} -2 \\ -10 \end{pmatrix}$)
8. Which shapes are enlargements of D ? (E, F, K)
9. Which shape do you obtain if you reflect H in the y-axis? (I)
10. Which shape do you obtain if you reflect H in the x-axis? (B)

UNIT 7 Transformations — Mental Tests

M 7.2 Academic Route *(no calculator)*

Look at the *Information Sheet* for this test.

1. The shape D is enlarged to give E. What is the scale factor? (3)

2. The shape G is translated onto H. What is the vector for this translation? $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$

3. The shape A is reflected onto D. What is the equation of the mirror line? ($x = 5$)

4. The shape I is rotated onto J. What is the angle of rotation? (180 °)

5. The shape C is translated to A. What is the vector for this translation? $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$

6. Through what angle is the shape E rotated to obtain F ? (90 ° clockwise)

7. When H is reflected in the *x*-axis, which shape do you obtain? (B)

8. When A is reflected in the *y*-axis, which shape do you obtain? (B)

9. What type of transformation takes shape H onto shape A ? (Rotation)

10. What type of transformation takes shape B onto shape J ? (Translation)

UNIT 7 Transformations Mental Tests

M 7.3 Academic Route *(no calculator)*

Look at the *Information Sheet* for this test.

1. The shape A is reflected onto the shape D. What is the equation of the mirror line? ($x = 5$)

2. The shape B is translated onto J. What is the vector for this translation? ($\begin{pmatrix} 8 \\ -10 \end{pmatrix}$)

3. The shape E is rotated onto F.
   (a) What are the coordinates of the centre of rotation? ((15, – 3))
   (b) What is the angle of rotation? ($90°$ clockwise)

4. The shape D is enlarged onto E.
   (a) What is the scale factor? (3)
   (b) What are the coordinates of the centre of enlargement? ((5, 4))

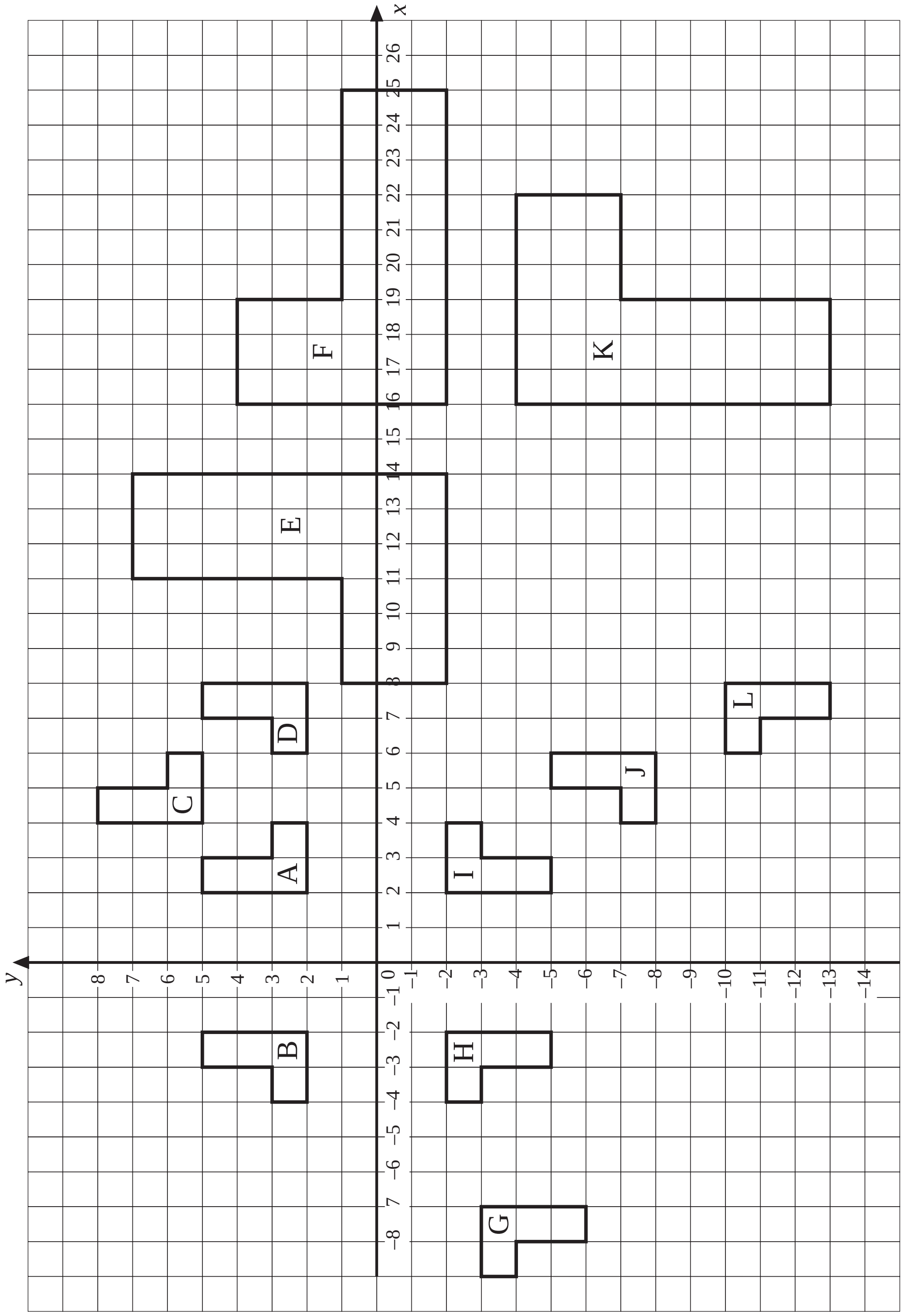
5. The shape I is rotated onto J.
   (a) What is the angle of rotation? ($180°$)
   (b) What are the coordinates of the centre of rotation? ((4, – 5))

6. The shape D is reflected onto L. What is the equation of the mirror line? ($y = -4$)

7. What vector is needed to translate shape H onto shape G ? ($\begin{pmatrix} -5 \\ -1 \end{pmatrix}$)

UNIT 7 Transformations Mental Tests

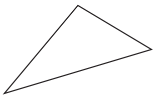
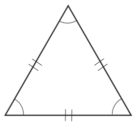
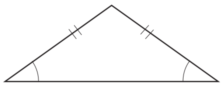
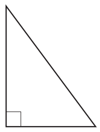
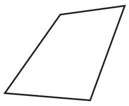
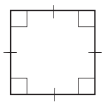
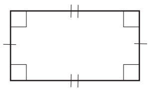
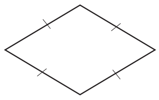
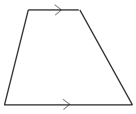
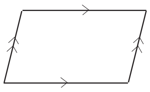
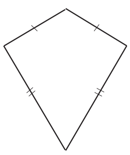
Information Sheet



UNIT 7 *Transformations* Overhead Slides

Overhead Slides

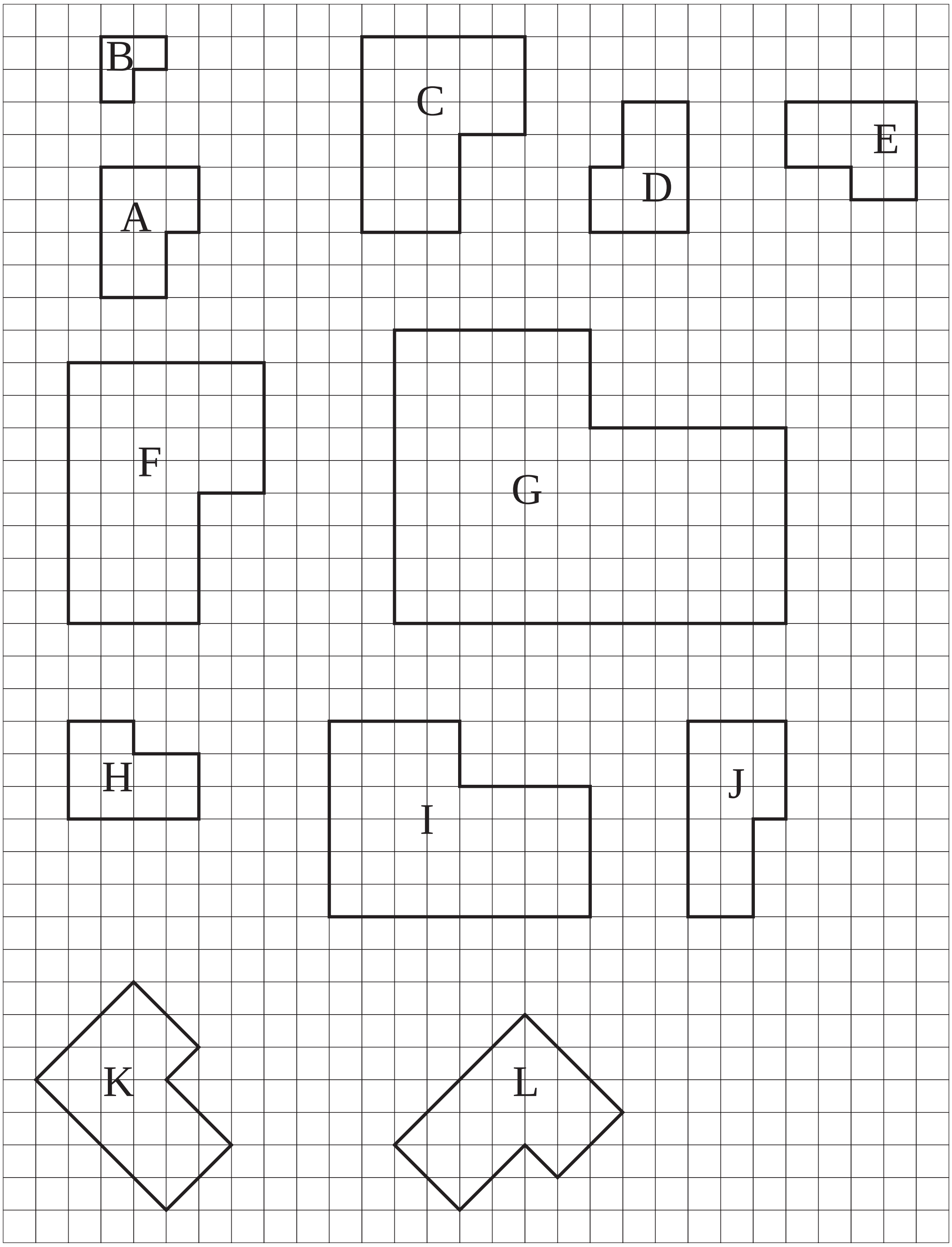
OS 7.1 *Shapes*

NAME	*ILLUSTRATION*	*NOTES*
Triangle		3 straight sides
Equilateral Triangle		3 equal sides and 3 equal angles (= 60 °)
Isosceles Triangle		2 equal sides and 2 equal angles
Right-angled Triangle		One angle = 90 °
Quadrilateral		4 straight sides
Square		4 equal sides and 4 right angles
Rectangle		Opposite sides equal and 4 right angles
Rhombus		4 equal sides; opposite sides parallel
Trapezium		One pair of opposite sides parallel
Parallelogram		Both pairs of opposite sides equal and parallel
Kite		Two pairs of adjacent sides equal

OS 7.2 *Congruent or Similar?*

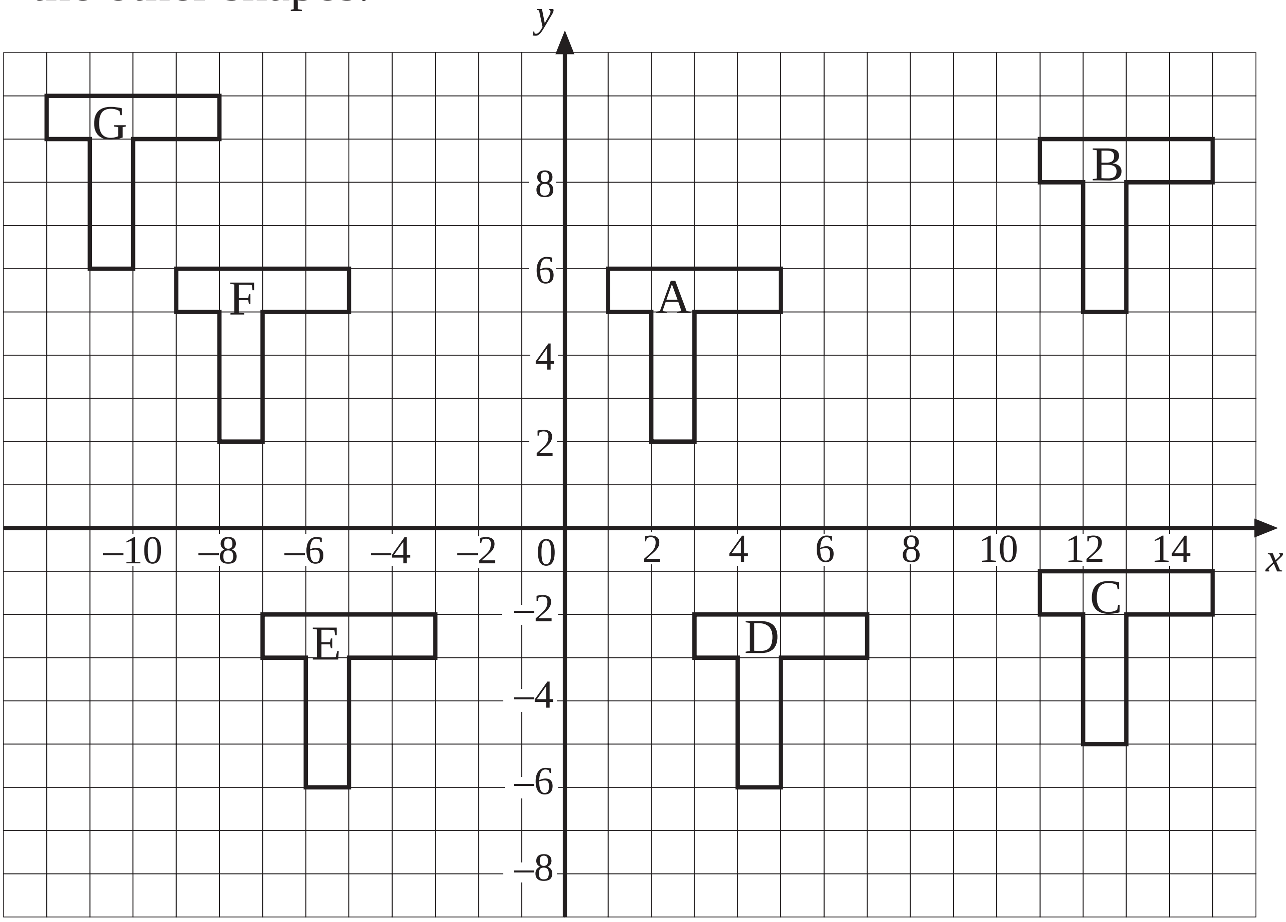
1. Which of the following shapes are *congruent* to A ?

2. Which of the following shapes are *similar* to A ?



OS 7.3 *Translations*

Describe the translations that will take shape A to each of the other shapes.



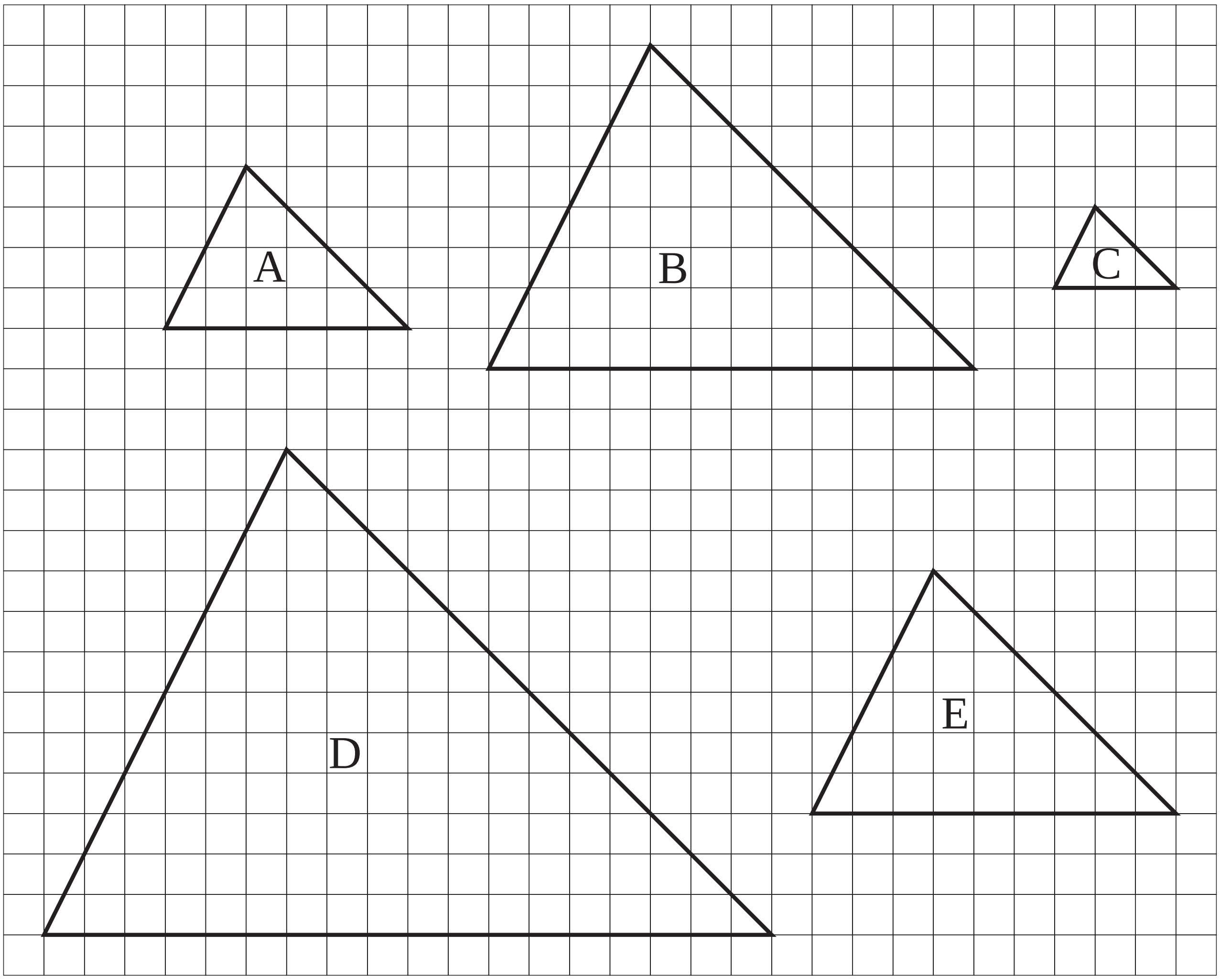
A → B $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ A → C $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$

A → D $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ A → E $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$

A → F $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ A → G $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$

OS 7.4 *Enlargements: Scale Factor*

The triangle A has been enlarged to give the other triangles.



A $\rightarrow$ B Scale factor ☐ A $\rightarrow$ C Scale factor ☐

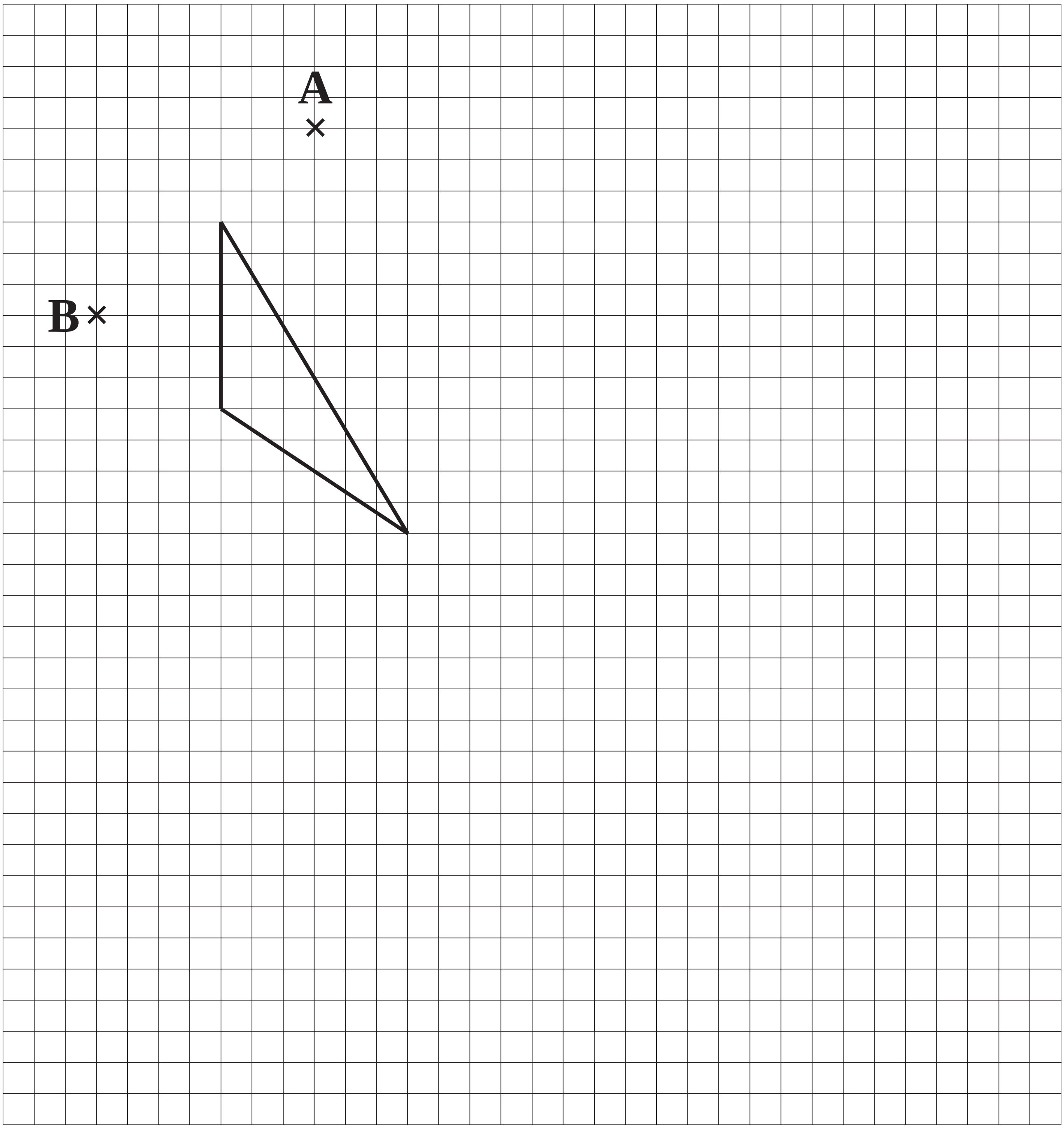
A $\rightarrow$ D Scale factor ☐ A $\rightarrow$ E Scale factor ☐

OS 7.5 *Centre of Enlargement*

Enlarge the triangle shown with:

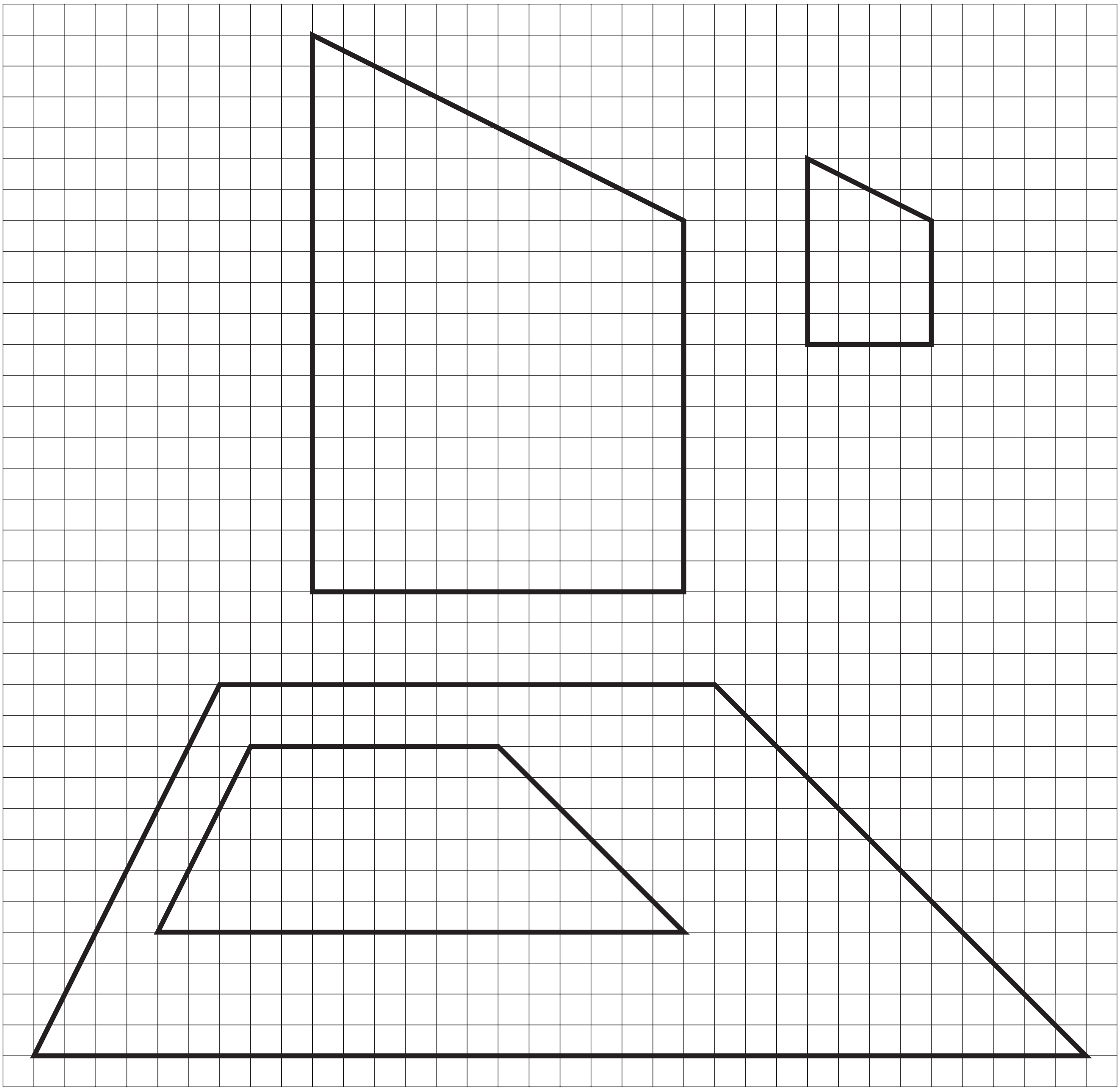
(a) centre of enlargement A and scale factor 2,

(b) centre of enlargement B and scale factor 3.



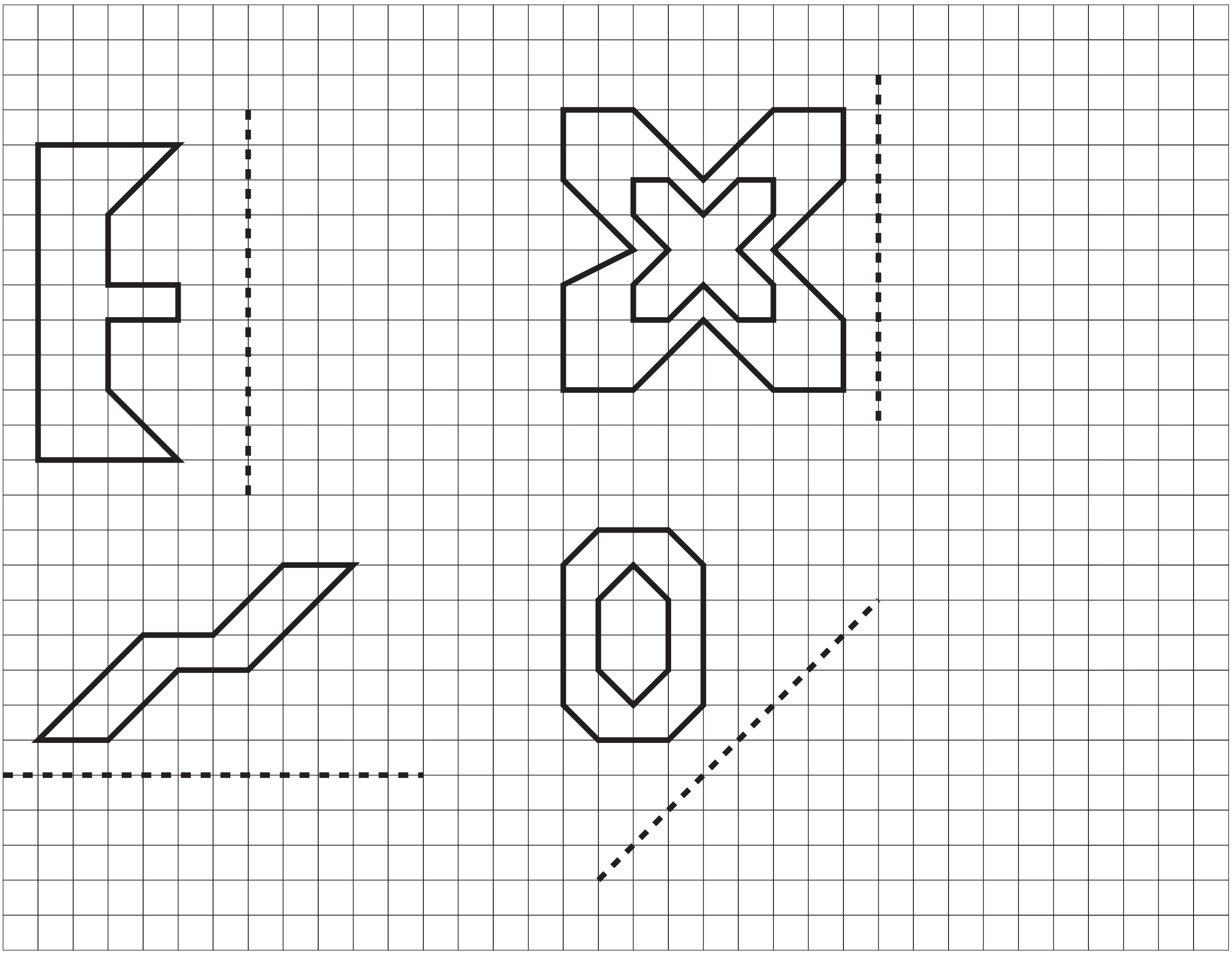
OS 7.6 *Finding the Centre of Enlargement*

Determine the centre of enlargement for each of the enlargements shown and also find the scale factor:



OS 7.7 *Reflections*

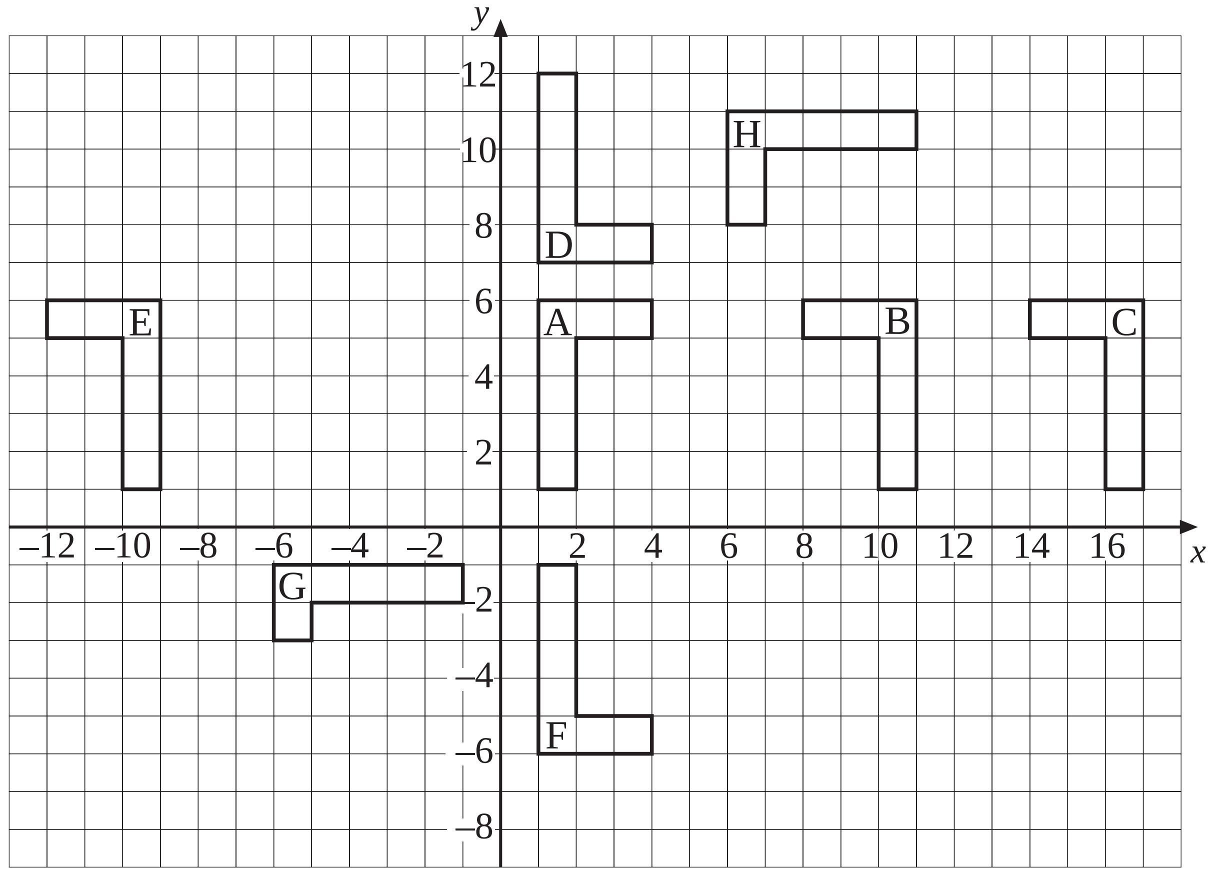
Draw the reflection of each shape in the mirror line shown:



OS 7.8 *Reflections: Equation of Mirror Line*

The diagram shows several reflections of the shape A.

Write down the equation of the mirror line for each reflection.



A $\rightarrow$ B

A $\rightarrow$ C

A $\rightarrow$ D

A $\rightarrow$ E

A $\rightarrow$ F

A $\rightarrow$ G

A $\rightarrow$ H

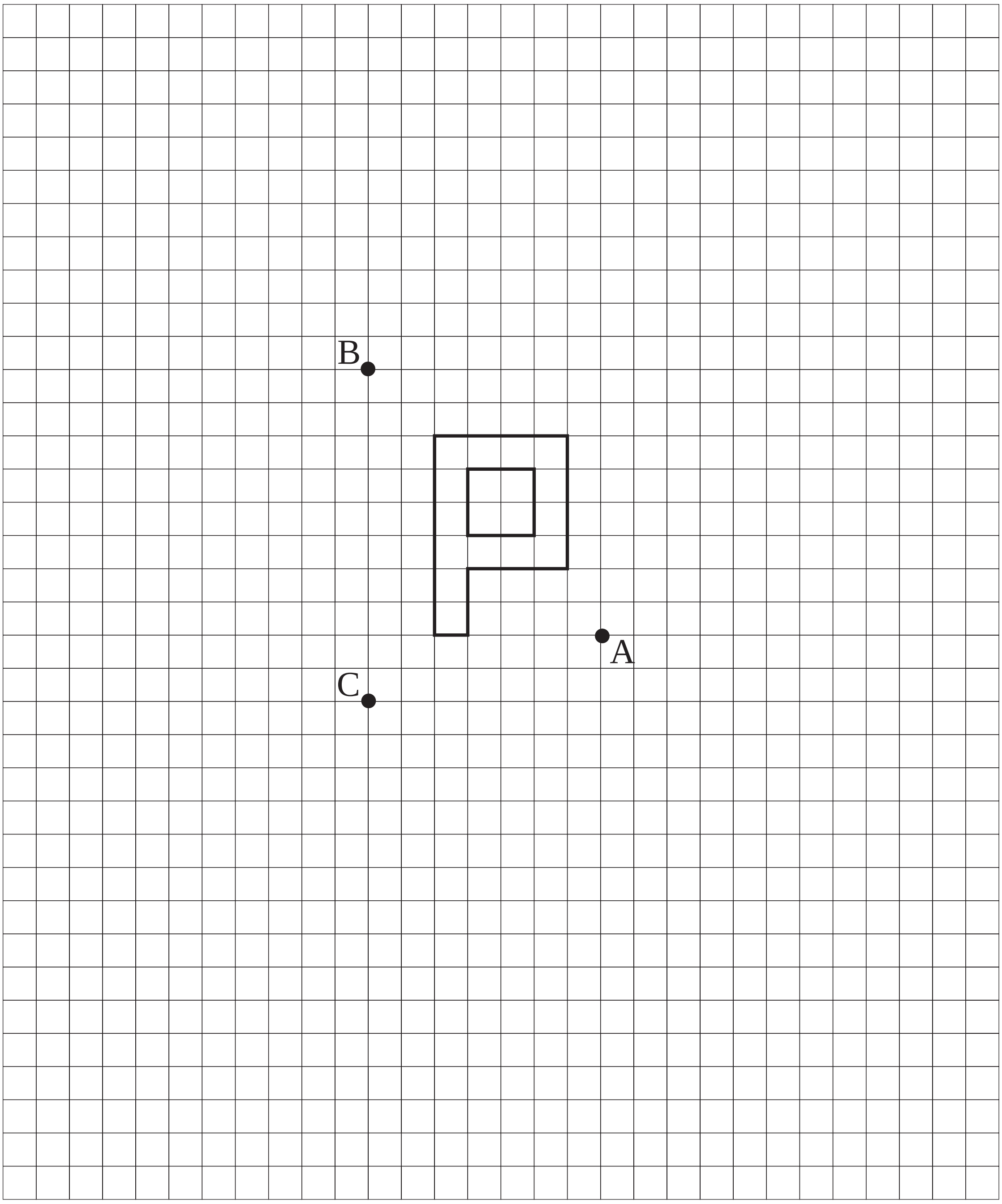
OS 7.9 *Rotations*

Rotate the shape shown through,

(a) 90 ° clockwise around the point A,

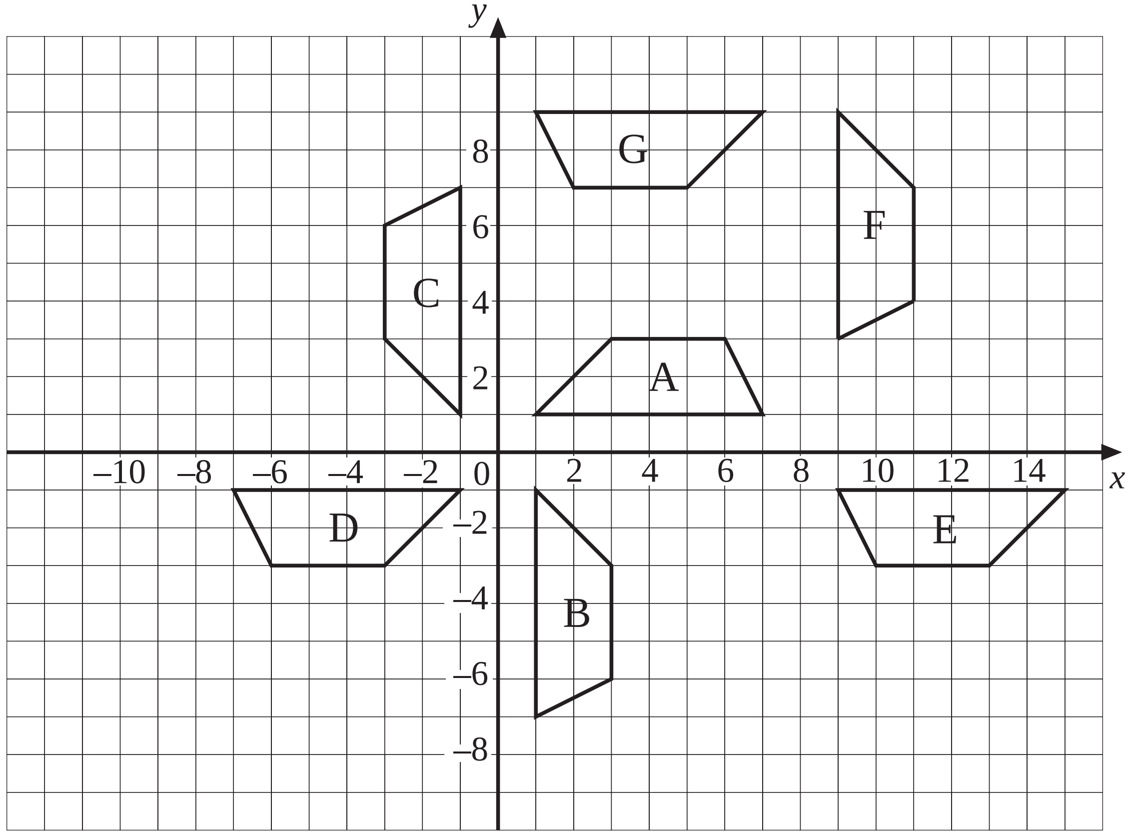
(b) 90 ° anticlockwise around the point B,

(c) 180 ° around the point C.



OS 7.10 *Describing Rotations*

The shape A is rotated to give the other shapes in the diagram. Describe each rotation.



A → B ....

A → C ....

A → D ....

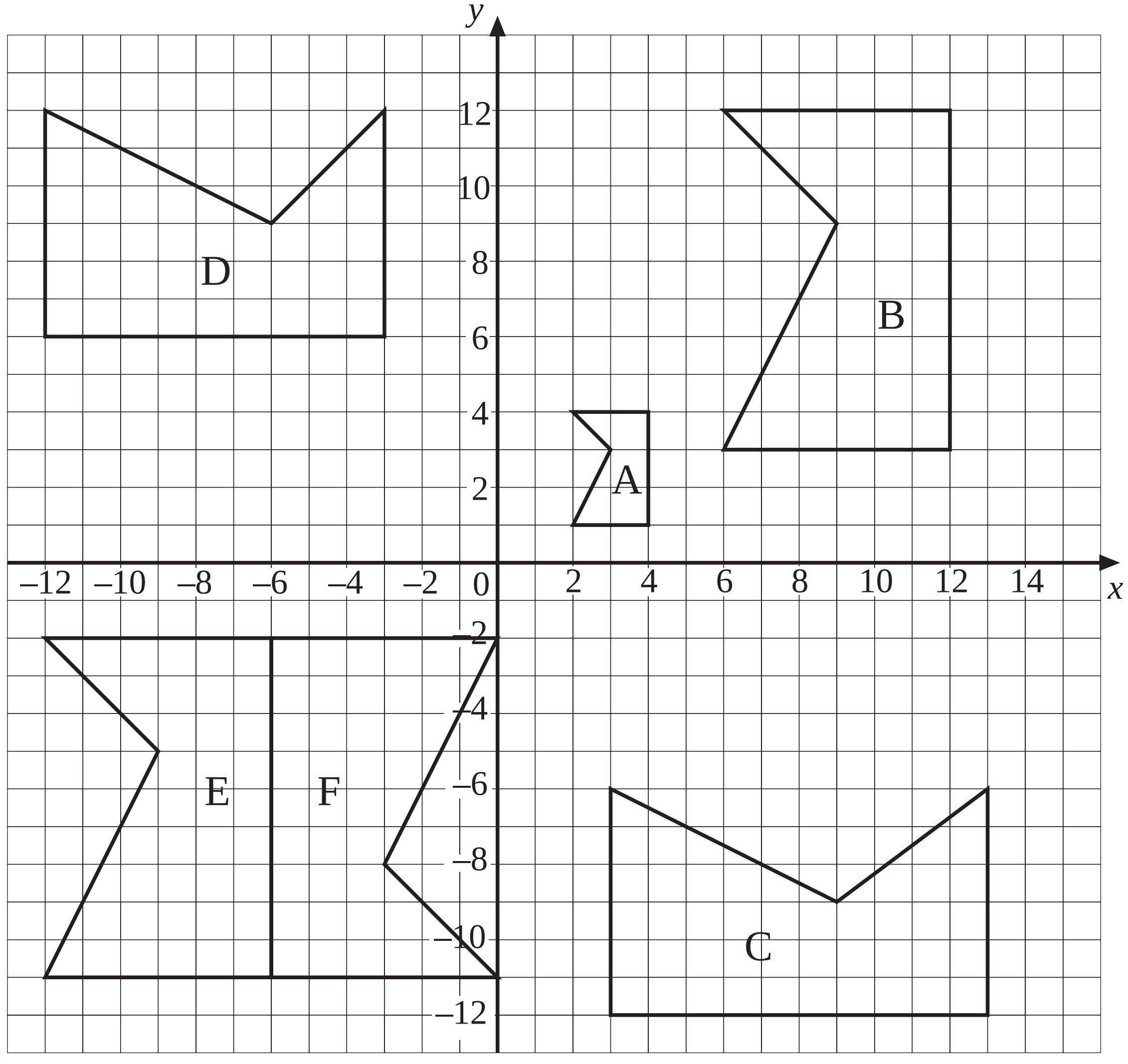
A → E ....

A → F ....

A → G ....

OS 7.11 *Combined Transformations*

The shape A moves to the shape F by a number of transformations. Describe each transformation fully.



A → B ....

B → C ....

C → D ....

D → E ....

E → F ....

Practice Book *UNIT 7 Transformations* Answers

7.1 Shapes

1. (a) Rectangle or square (b) Trapezium (c) Square, rhombus or kite

2. (a) C and H (b) B, C, H and I

3.

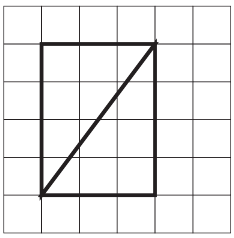
Length (cm)	*Width* (cm)	*Area* (cm^2)
1	19	19
2	18	36
3	17	51
4	16	64
5	15	75
6	14	84
7	13	91
8	12	96
9	11	99
10	10	100

Length (cm)	*Width* (cm)	*Area* (cm^2)
9	11	99
9.1	10.9	99.19
9.2	10.8	99.36
9.3	10.7	99.51
9.4	10.6	99.64
9.5	10.5	99.75
9.6	10.4	99.84
9.7	10.3	99.91
9.8	10.2	99.96
9.9	10.1	99.99
10	10	100

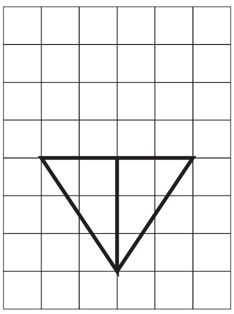
The first table gives the integer possibilities, but pupils should be encourged to justify the correct maximum area by looking at rectangles close in size to a square of side 10 cm, as in the second table.

The shape that gives the maximum area is a square of side 10 cm.

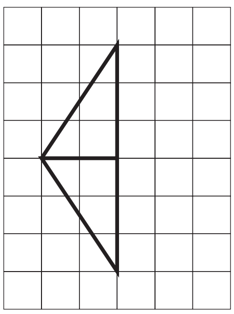
4. (a)



(b)

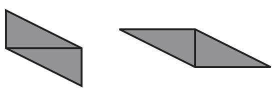


(c)

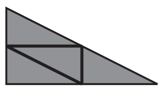


N.B. The answers for parts (b) and (c) are interchangeable.

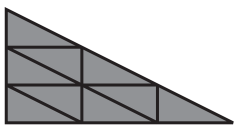
5. (a) and (b) In either order



(c)

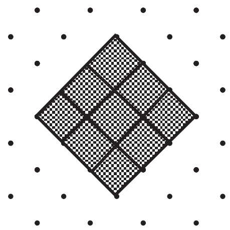


(d) Any suitable combination of 9, 16, 25, 36, ... triangles to make a larger triangle, e.g.

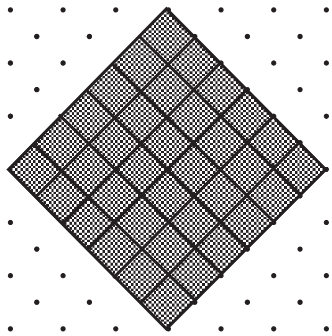


7.1 Answers

6. (a)

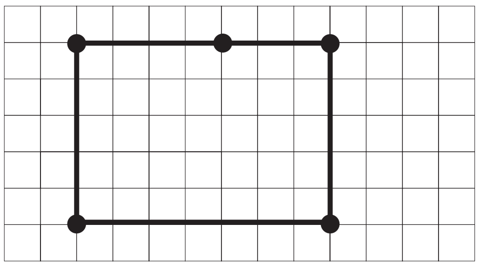


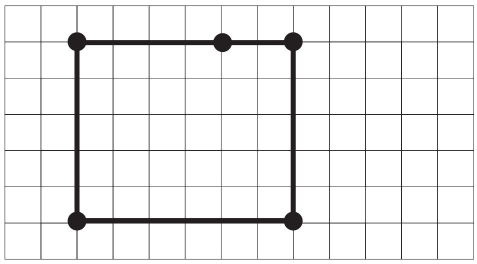
(b) A suitable combination of 16, 25, 36, ... square tiles. In the case illustrated, there are 36 tiles:



(c) Any three square numbers from the list 16, 25, 36, 49, 64, 81, 100, ... excluding the number of tiles given as the answer to part (b).

7. (a) There are two possible combinations using the rods 5, 5, 7 and 4 + 3 or 5, 5, 6, and 4 + 2.





(c) The possible solution are given in the following table:

Rods Used				*Rectangle Size*
5	5	8 + 3	7 + 4	5 × 11
5	5	8 + 2	7 + 3	5 × 10
5	5	8 + 2	6 + 4	5 × 10
5	5	7 + 3	6 + 4	5 × 10
5	5	7 + 2	6 + 3	5 × 9
8	6 + 2	7	4 + 3	8 × 7
8	5 + 3	7	5 + 2	8 × 7
8	5 + 3	6	4 + 2	8 × 6

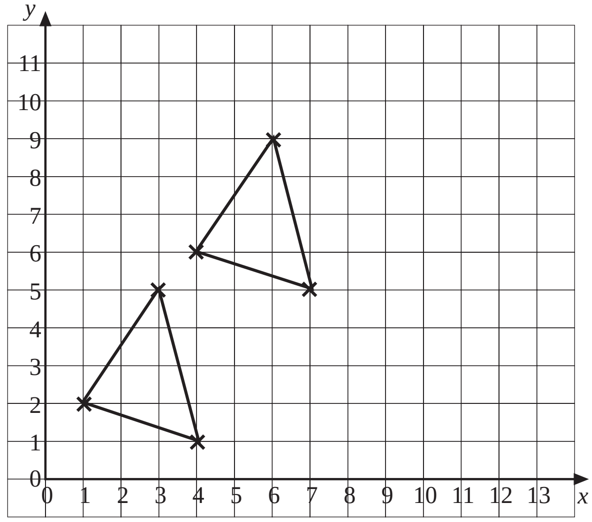
(d) The total length of all the rods is 40 cm so the square must have sides of length 10 cm. There is only one combination here:

5 + 5, 6 + 4, 7 + 3 and 8 + 2

7.2 **Answers**

7.2 Translations

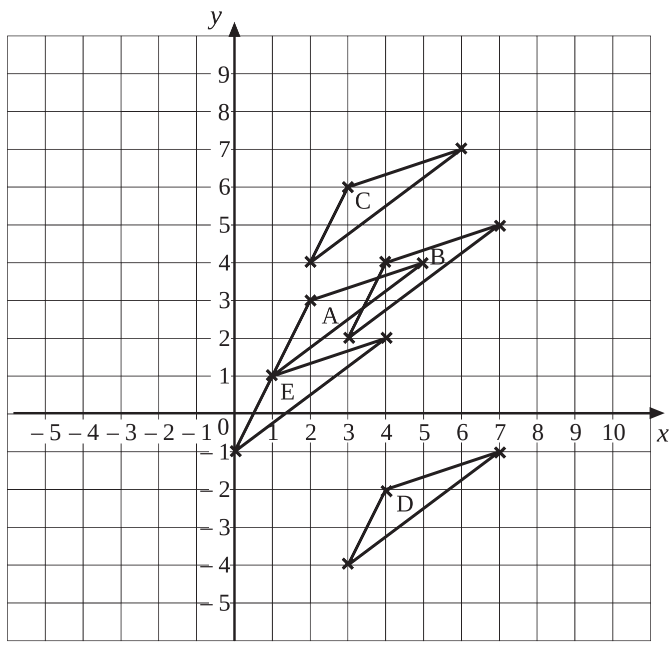
1. (a) and (b)



(c) (7, 5), (6, 9), (4, 6)

2. (a) $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$ (b) $\begin{pmatrix} 11 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ (d) $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$ (e) $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$

3. (a) and (b)



4. A to B $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ A to C $\begin{pmatrix} 5 \\ -11 \end{pmatrix}$ A to D $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$

A to E $\begin{pmatrix} -15 \\ -7 \end{pmatrix}$ A to F $\begin{pmatrix} -11 \\ -3 \end{pmatrix}$ A to G $\begin{pmatrix} -12 \\ 4 \end{pmatrix}$

5. (a) $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ (b) (11, 4)

7.2 Answers

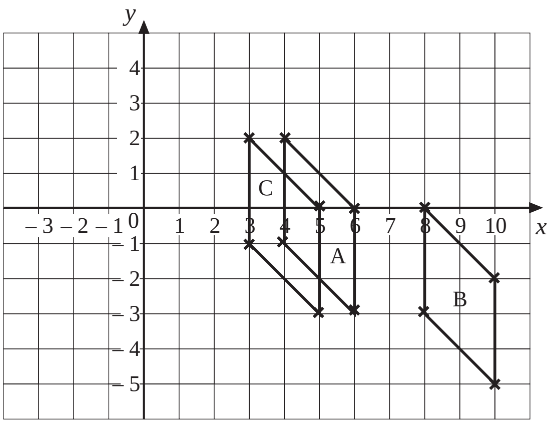
6. (a) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ (c) $\begin{pmatrix} 12 \\ -1 \end{pmatrix}$

The relationship is $\begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$, i.e. for the top numbers (the x movement), $6 + 6 = 12$

and for the bottom numbers (the y movement), $3 + (-4) = -1$.

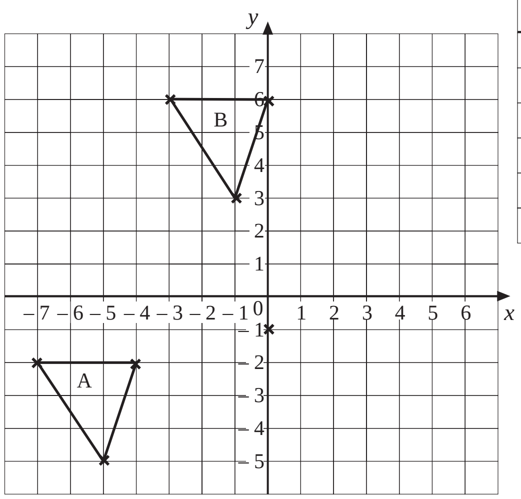
7. (a) Parallelogram

(b)



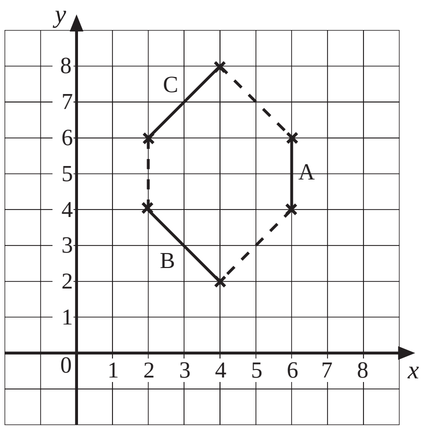
(c) Translation by the vector $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

8. (a)
(b)



(c) $\begin{pmatrix} -4 \\ -8 \end{pmatrix}$

9. (a)



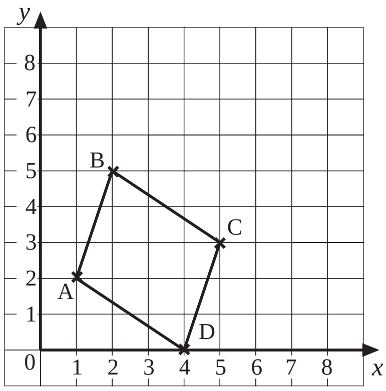
(b) A $\rightarrow \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

B $\rightarrow \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

C $\rightarrow \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

7.2 Answers

10. (a)



(b) A B to D C $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

A D to B C $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

7.3 Enlargements

1. (a) 2 (b) 2 (c) 6

   (d) 4 (e) $1\frac{1}{2}$ (f) $\frac{1}{2}$

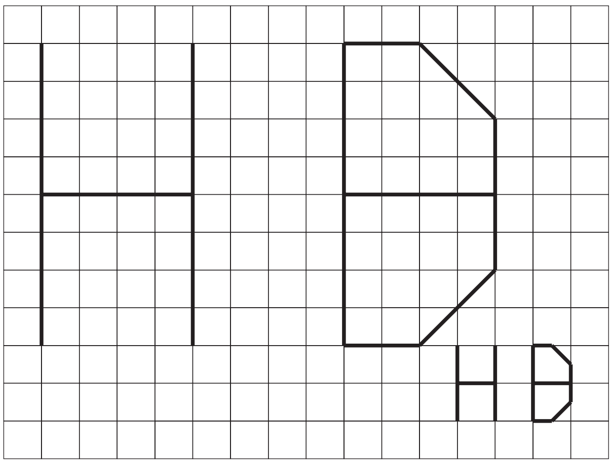
2. (a) An accurately drawn 2 cm × 4 cm rectangle.

   (b) Accurately drawn 4 cm × 8 cm, 6 cm × 12 cm, 8 cm × 16 cm and 1 cm ×2 cm rectangles.

3. (a) An accurately drawn triangle with sides 3 cm, 4 cm and 5.5 cm.

   (b) Accurately drawn triangles with sides 6 cm, 8 cm and 11 cm and 9 cm, 12 cm and 16.5 cm.

4.



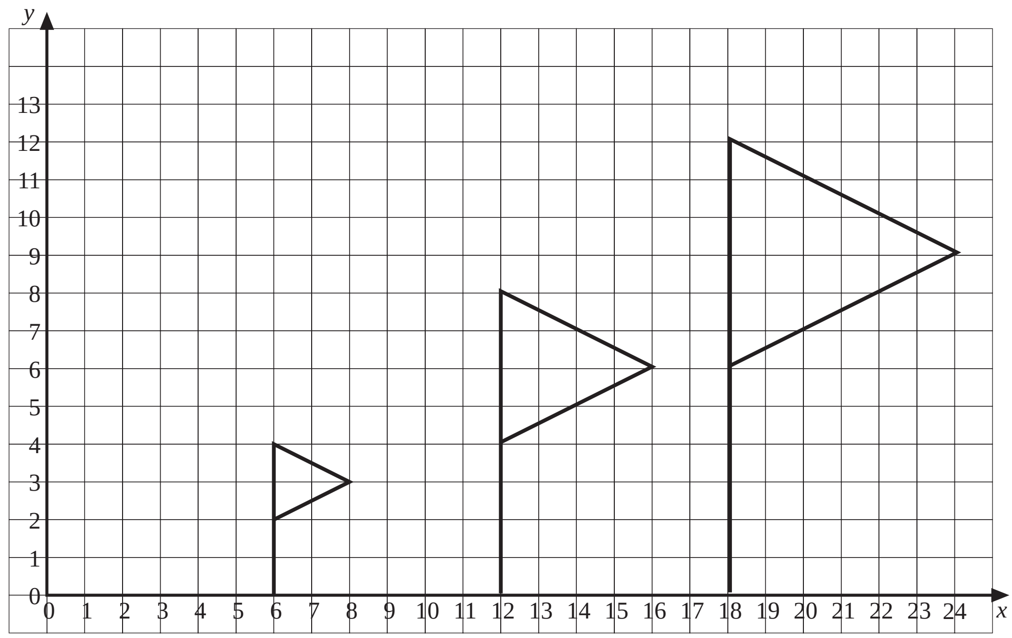
Scale factor 2

Scale factor $\frac{1}{2}$

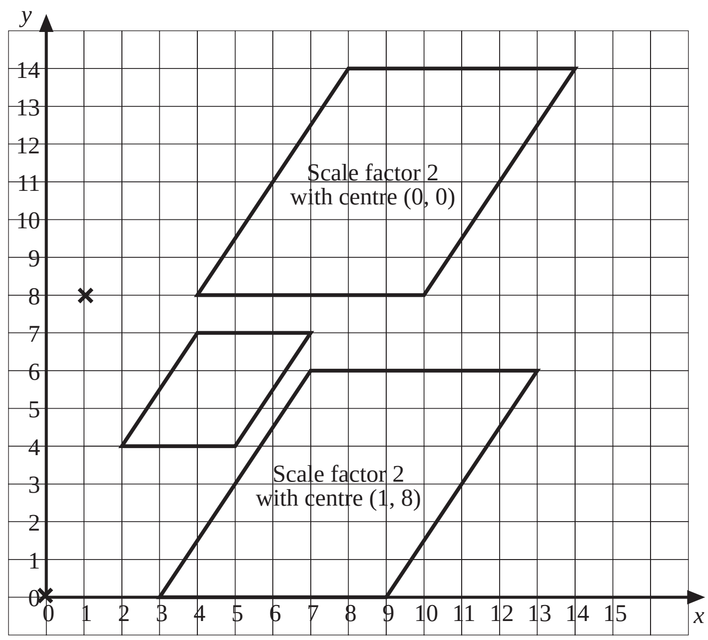
7.3 # Answers

5. (a)

(b)

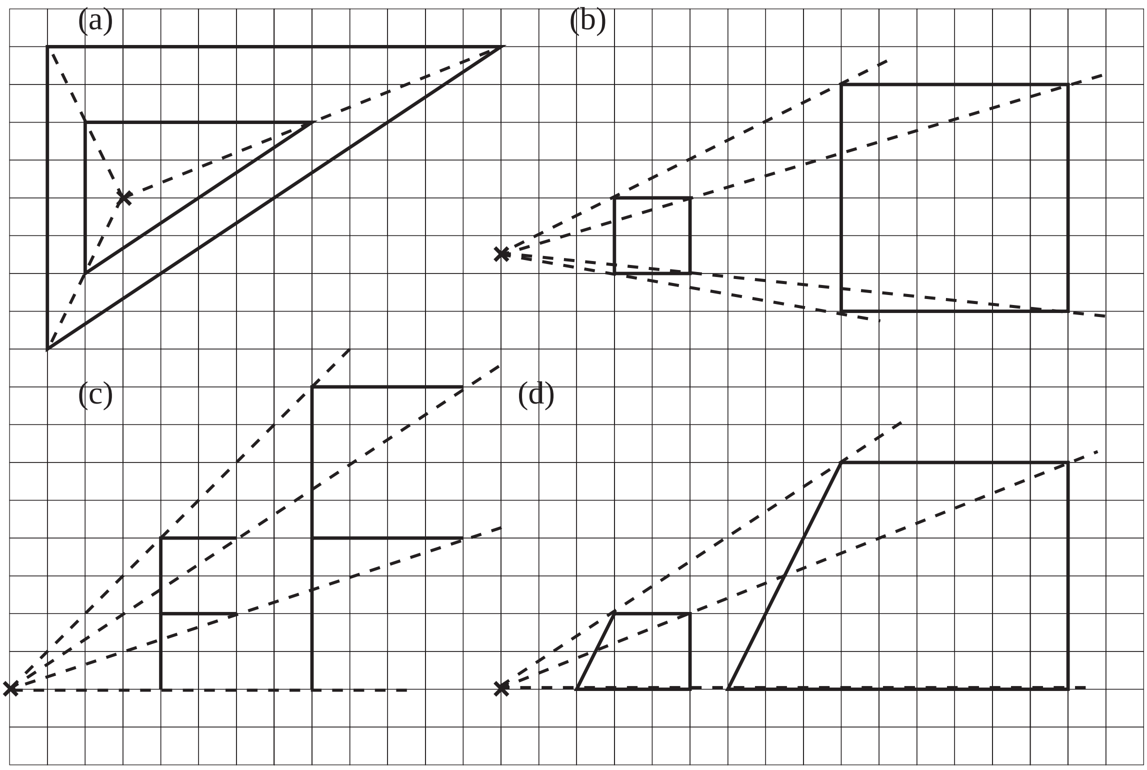


6.

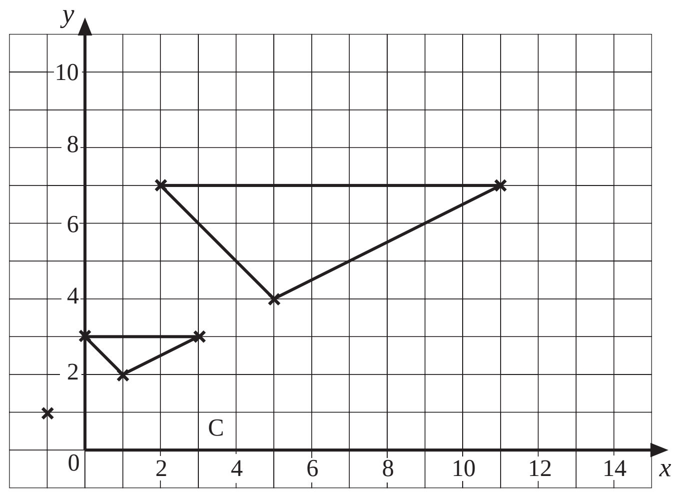


7.3 Answers

7.



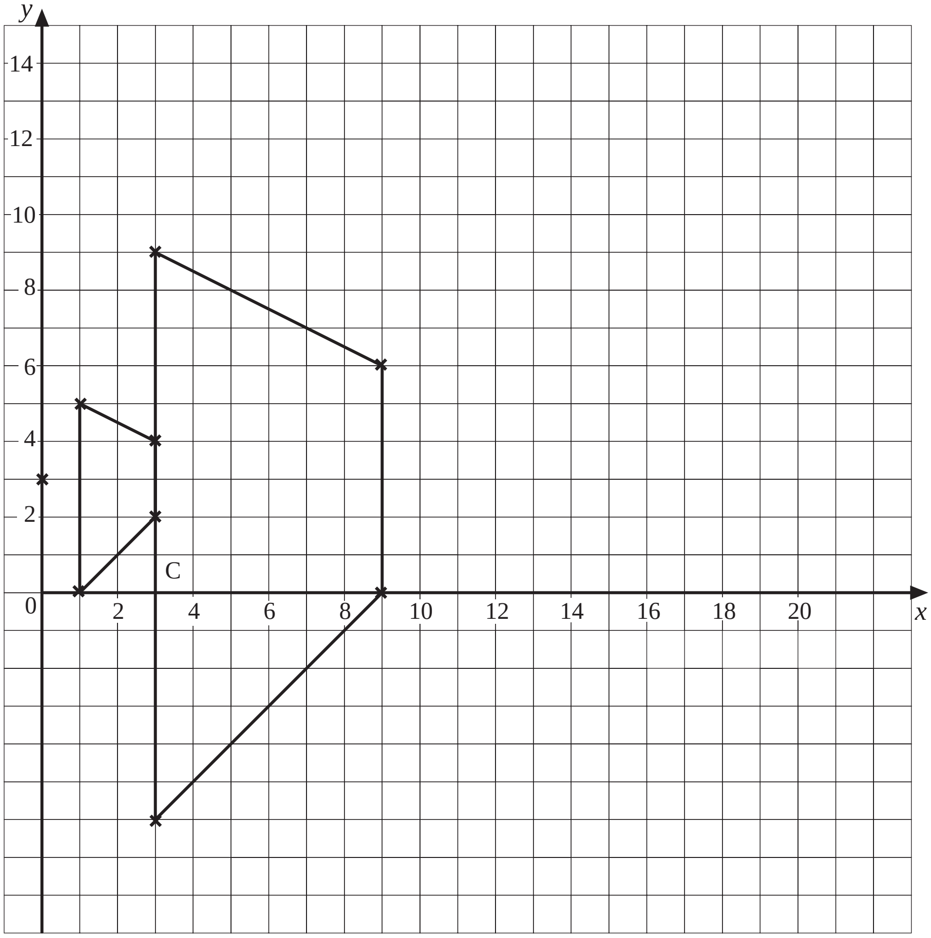
8.



Scale factor = 3.
Centre of enlargement is at (– 1, 1).

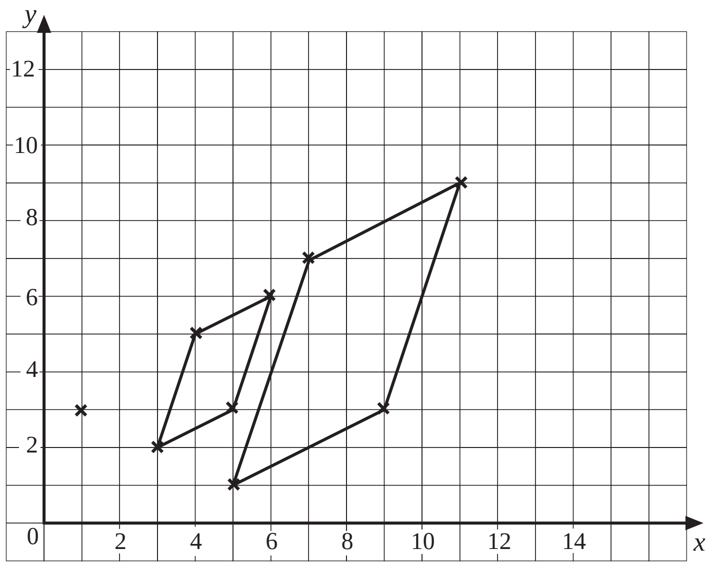
7.3 Answers

9.



Corners of enlarged trapezium are at (3, – 6), (3, 9), (9, 6) and (9, 0).

10.



11. (a) The scale factor for heights must be at most $24 \div 6.5 = 3.692$ (to 3 d.p.);

the scale factor for widths must be at most $12 \div 4 = 3$,

so the maximum scale factor Jill can use is 3.

7.3 **Answers**

(b) The scale factor for heights must be at most $2.7 \div 6.5 = 0.415$ (to 3 d.p.);
the scale factor for widths must be at most $2.7 \div 4 = 0.675$,
so the maximum scale factor Jill can use is 0.415 (to 3 d.p.).

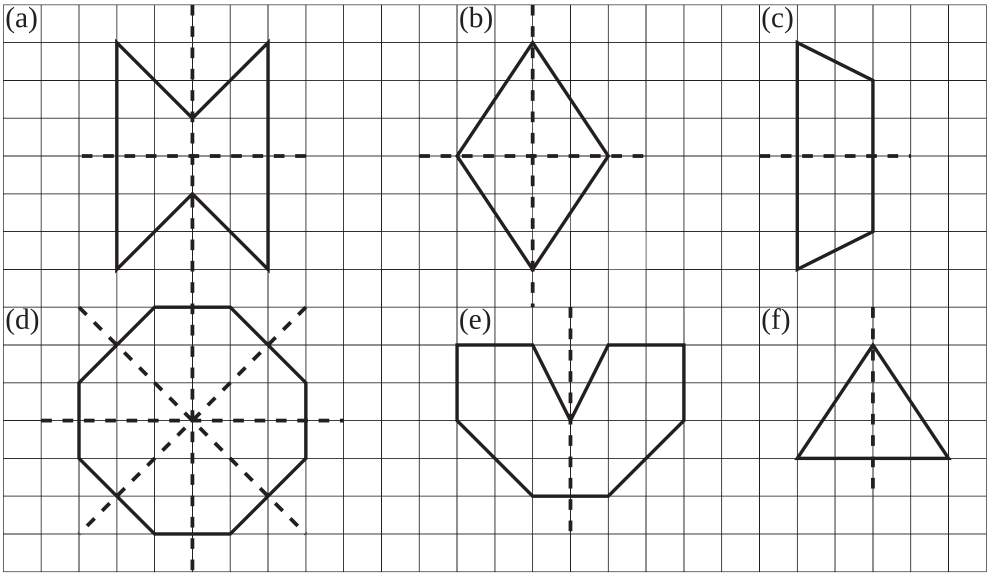
(c) The perimeter $= (\pi \times 6.6) + (2 \times 6.6)$

$= 33.93451151$ cm

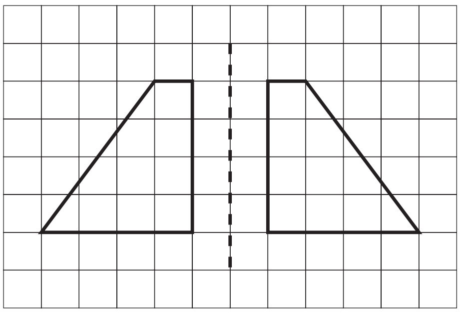
$= 33.93$ cm (to 2 d.p.)

7.4 Reflections

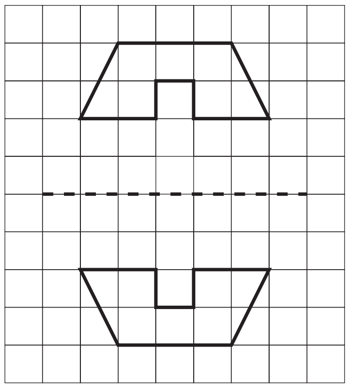
1.



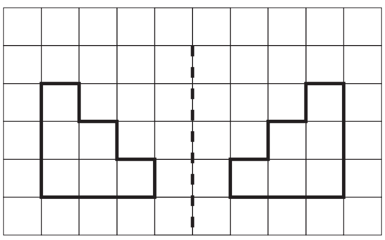
2. (a)



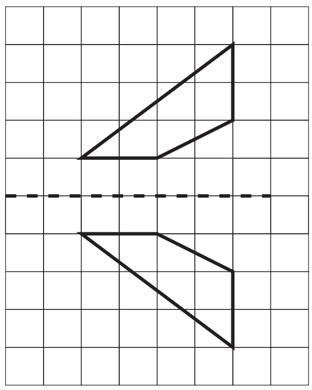
(b)



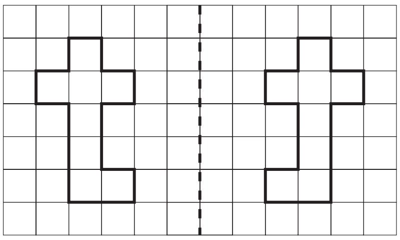
(c)

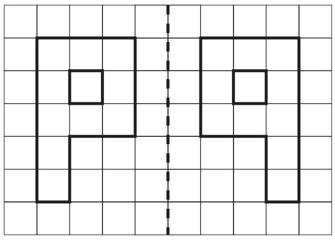


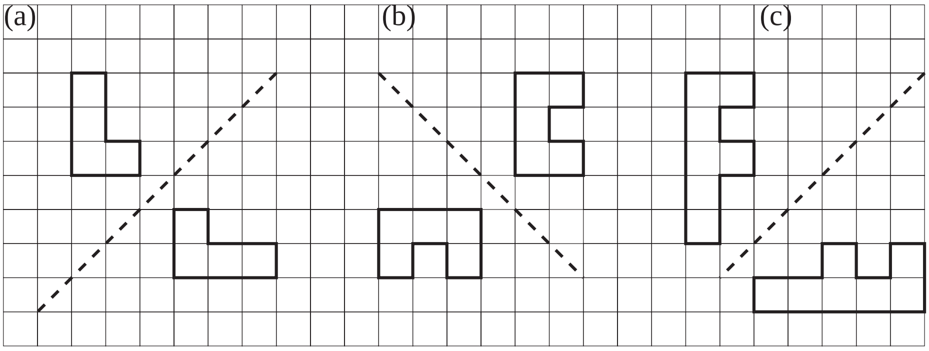
(d)



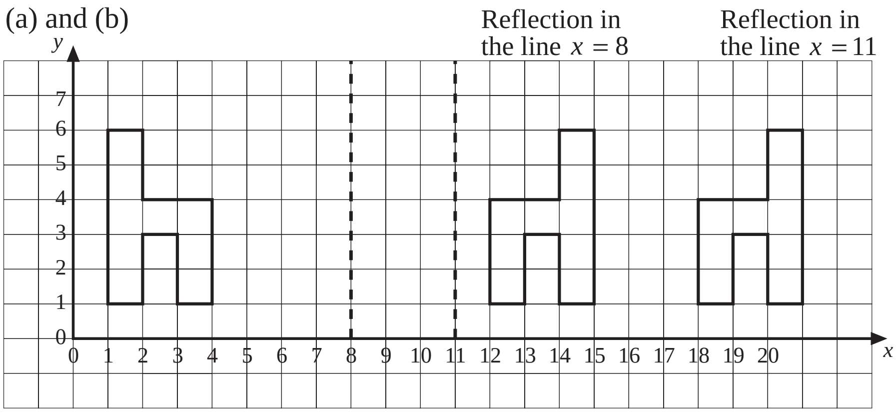
7.4 **Answers**

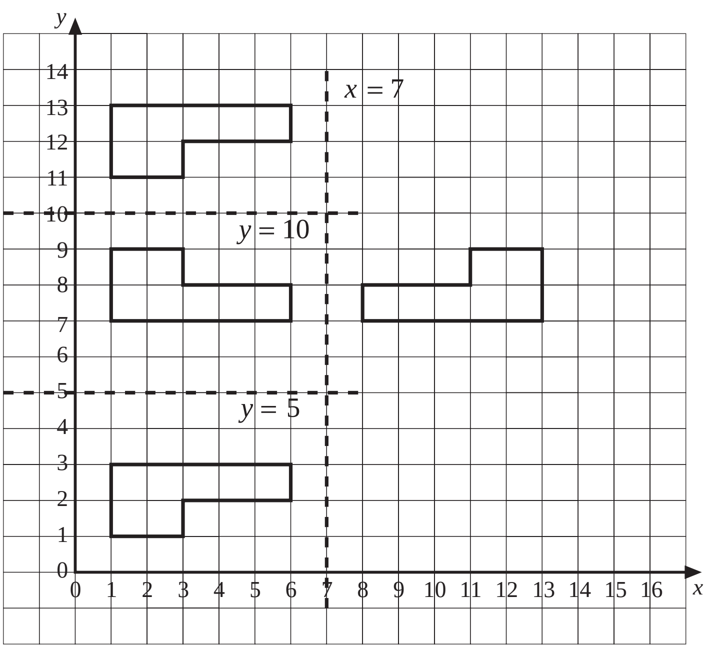
(e) 

(f) 

3. 

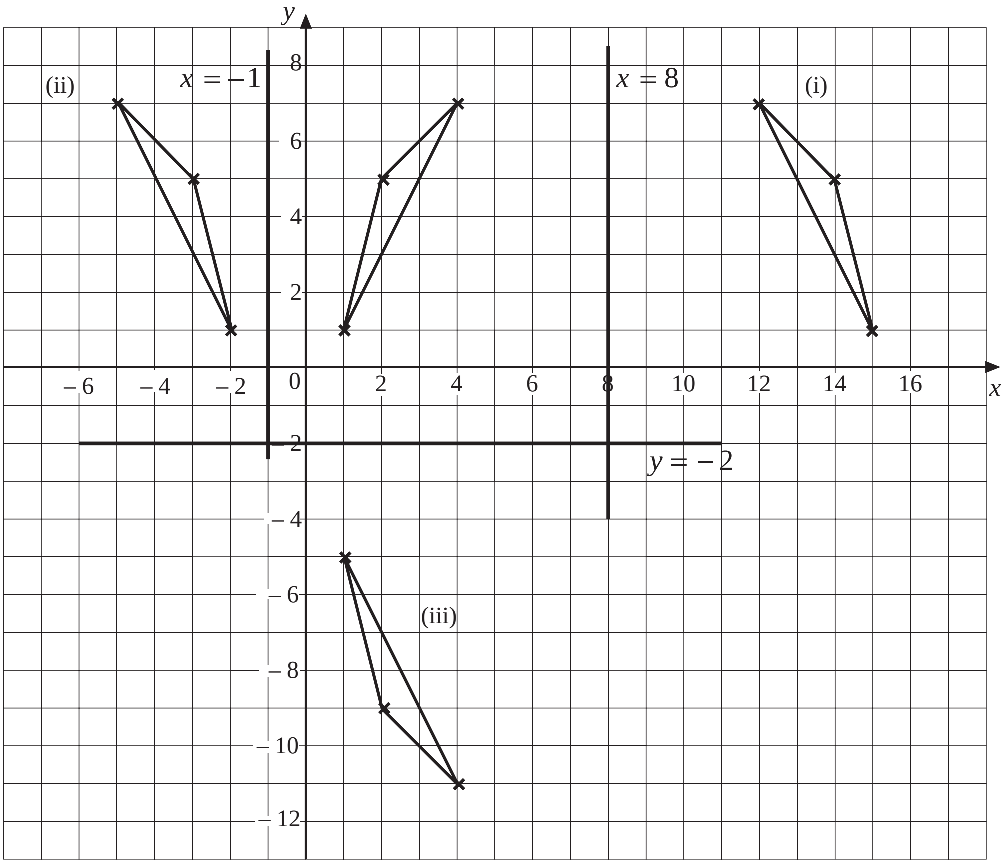
4. (a) and (b)



5. 

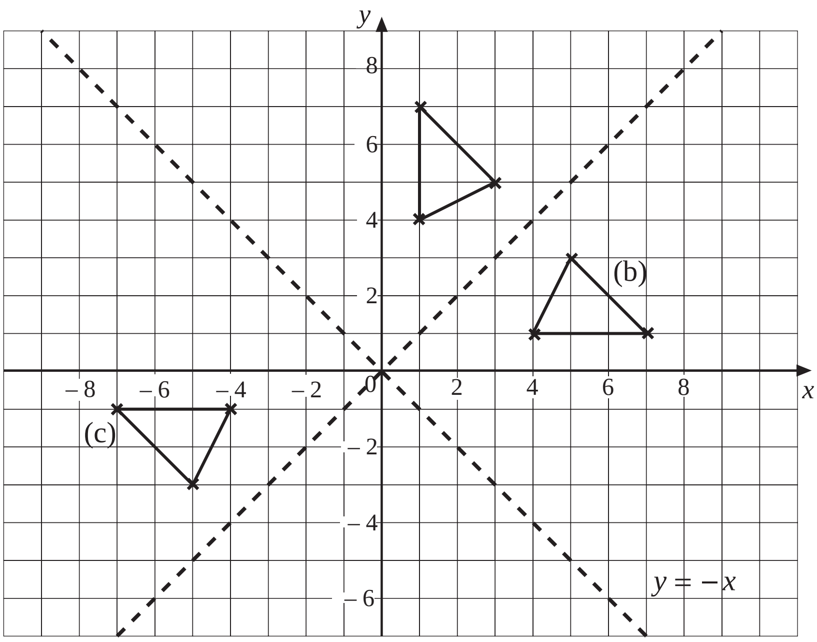
7.4 Answers

6. (a) and (b)



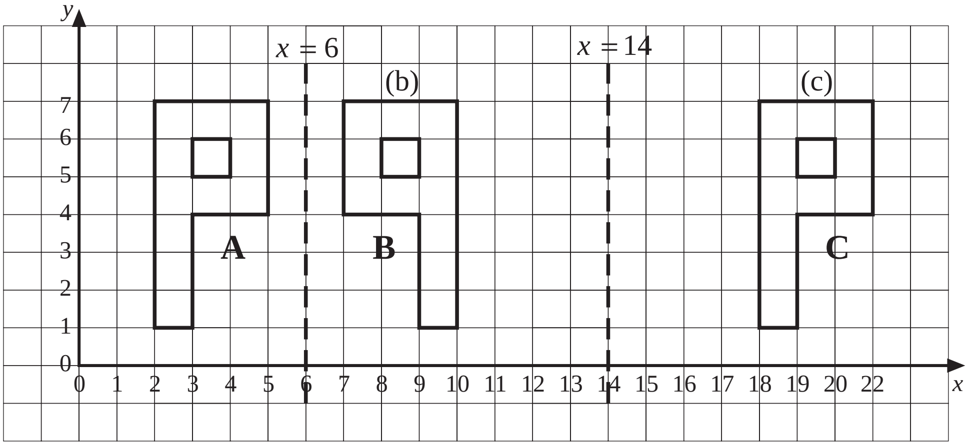
7. (a) $x = 5$ (b) $x = 9$ (c) $x = 10$
(d) $x = 16$ (e) $x = 21$ (f) $x = 14$

8. (a)



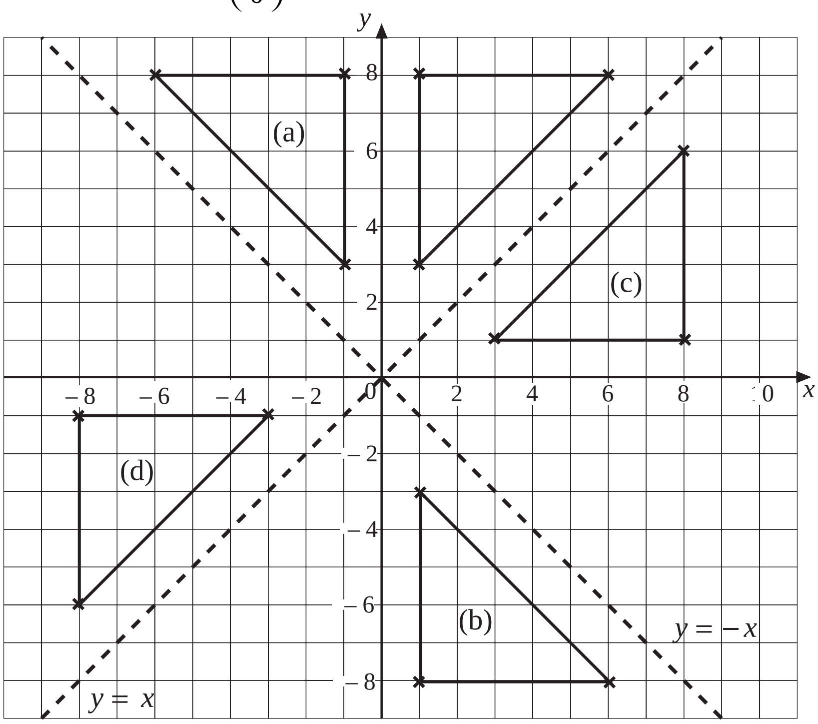
7.4 Answers

9. (a)

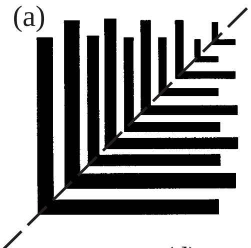


(d) Translation $\begin{pmatrix} 16 \\ 0 \end{pmatrix}$

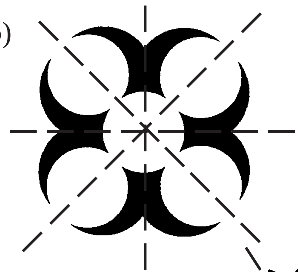
10.



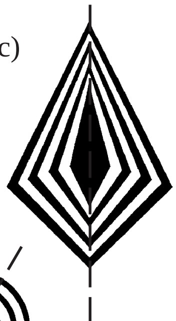
11. (a)



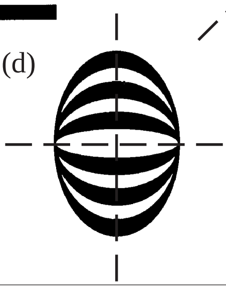
(b)

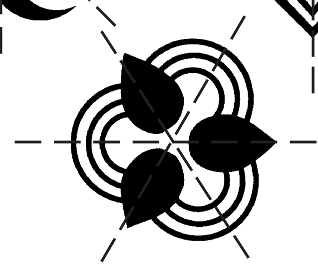


(c)



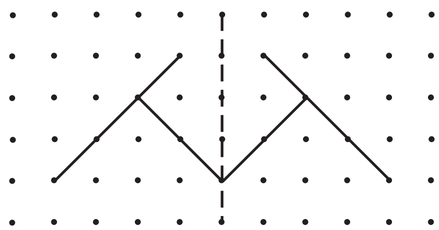
(d)



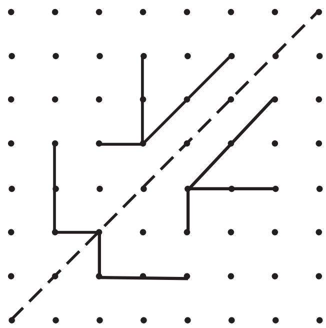


7.4 Answers

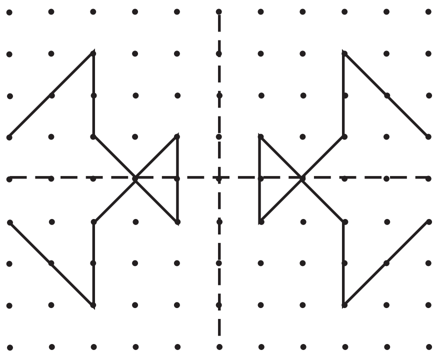
12. (a)



(b)



(c)



13. (a) $x = y$ (b) $14\frac{1}{2}$

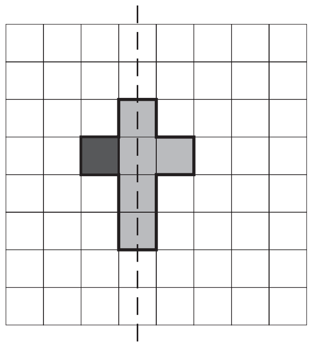
(c) (10, 12) is above the line because its *y*-coordinate is greater than its *x*-coordinate.

(d) Any *x*-coordinate less than 15, e.g. (13, 15).

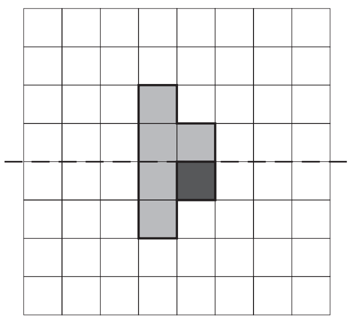
(e) Coordinates reversed

(f) (13, 20)

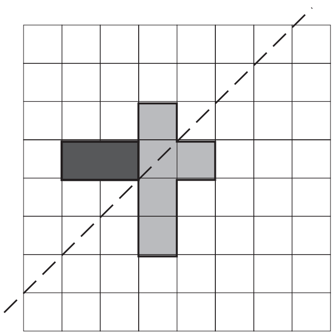
14. (a)



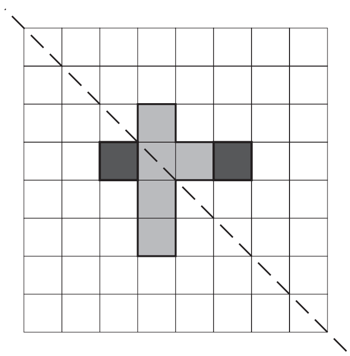
(b)



(c)



(d)



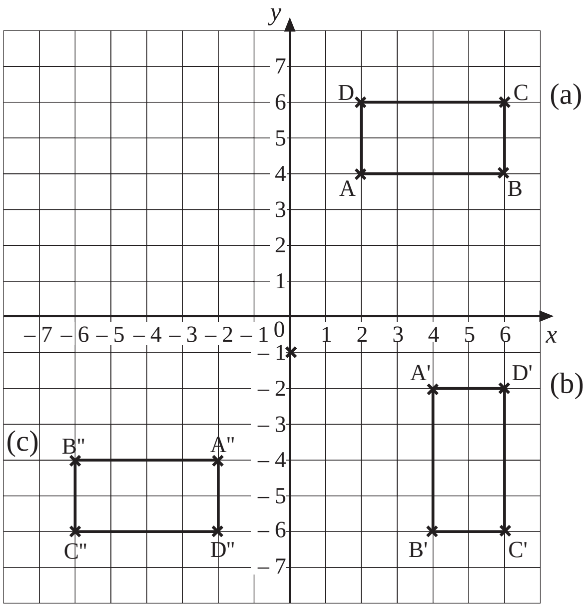
7.5 **Answers**

7.5 Rotations

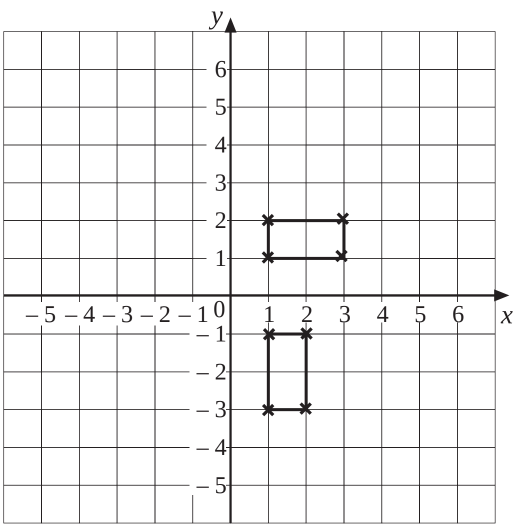
1. (a) 2 (b) 4 (c) 1
   (d) 3 (e) 4 (f) 6

2. H, I, N, O, S, X, Z (depending on how the letters are printed or written)

3.

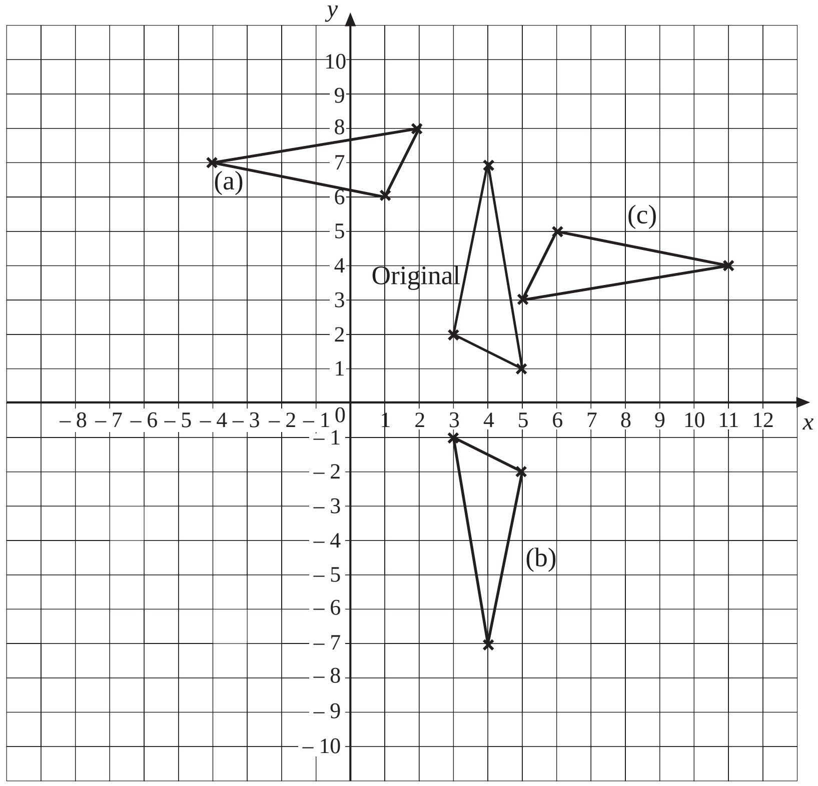


4.



7.5 Answers

5.



(a) (1, 6), (2, 8), (– 4, 7)

(b) (3, – 1), (4, – 7), (5, – 2)

(c) (5, 3), (11, 4), (6, 5)

6. (a) Rotation through $90°$ clockwise about the origin (0, 0)

(b) Rotation through $180°$ about the origin (0, 0)

(c) Rotation through $90°$ anticlockwise about the origin (0, 0)

(d) Rotation through $180°$ about (8, 0).

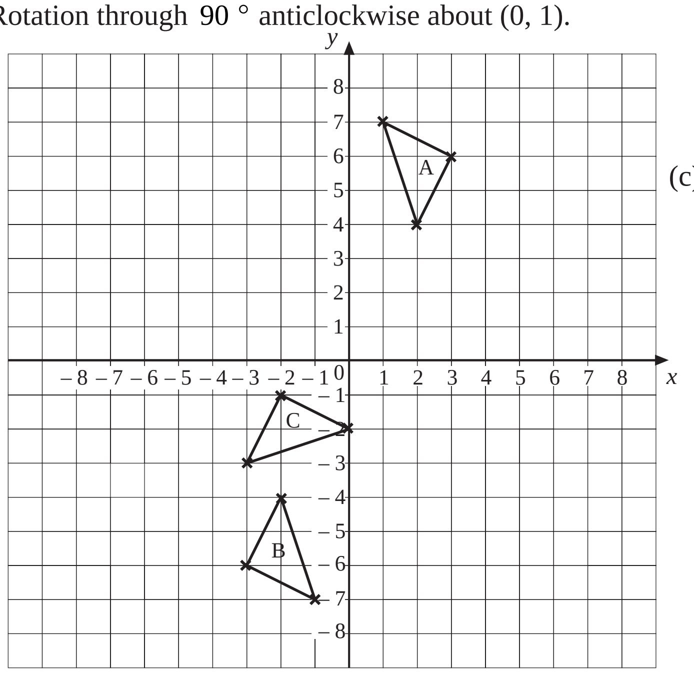
7. (a) Rotation through $90°$ clockwise about (5, 1)

(b) Rotation through $90°$ clockwise about the origin (0, 0)

(c) Rotation through $180°$ about the origin (0, 0)

(d) Rotation through $90°$ anticlockwise about (0, 1).

8. (a), (b)

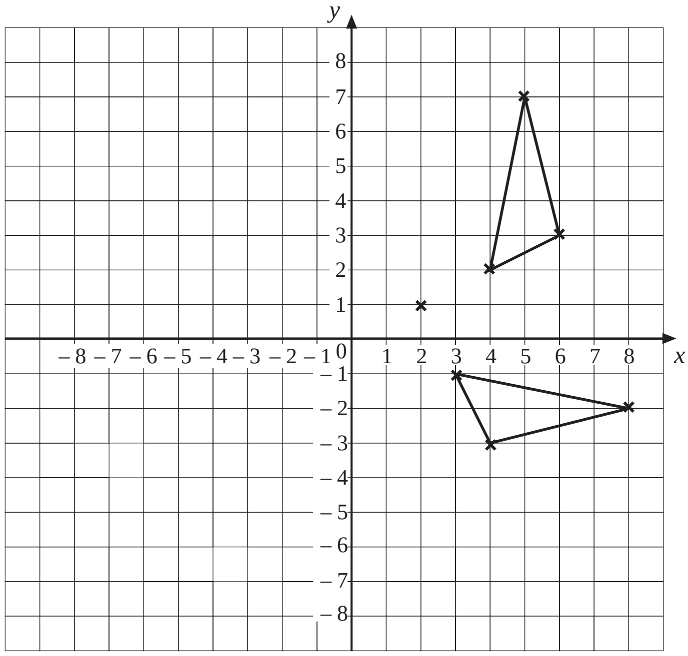


(c) Corners of triangle C have coordinates (– 3, – 3), (– 2, – 1), (0, – 2)

7.5 Answers

9. (a) (6, 7) (b) (1.5, 7.5)

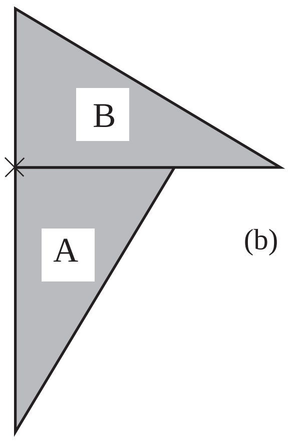
10. Rotation clockwise through $90°$ about the point (2, 1) (as diagram).



11. *Lines of Symmetry*

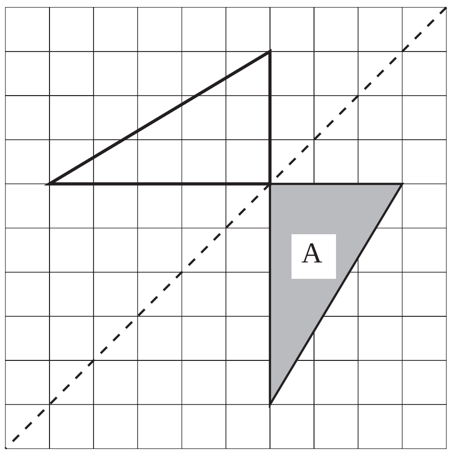
Order of Rotational Symmetry		0	1	2	3
	1	E	F		
	2	B		C	
	3	D			A

12. (a)



(b) $90°$

(c)



13. (a) B1 Rotate $90°$ clockwise and then rotate $90°$ clockwise again.

B2 Reflect vertical

(b) A2 Rotate $90°$ clockwise and then rotate $90°$ clockwise again.

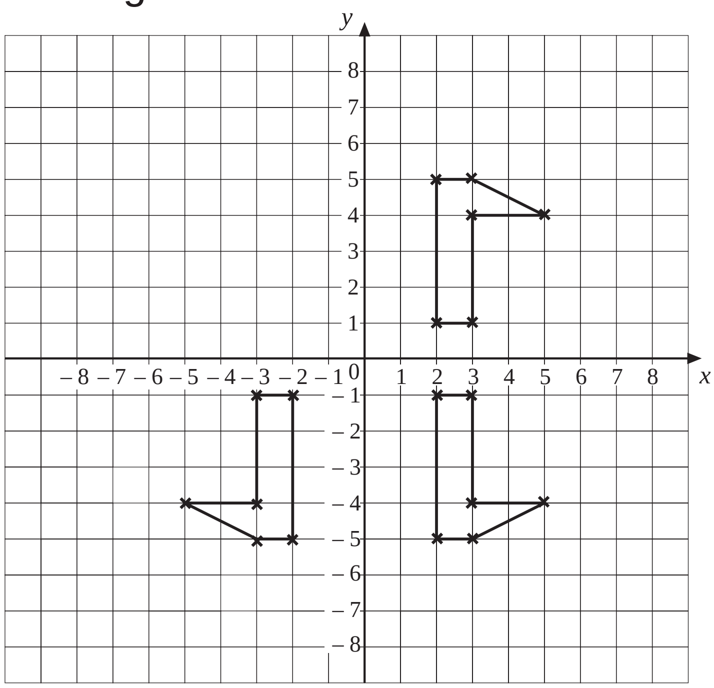
B1 Reflect vertical and then rotate $90°$ clockwise.

B2 Rotate $90°$clockwise.

7.6 **Answers**

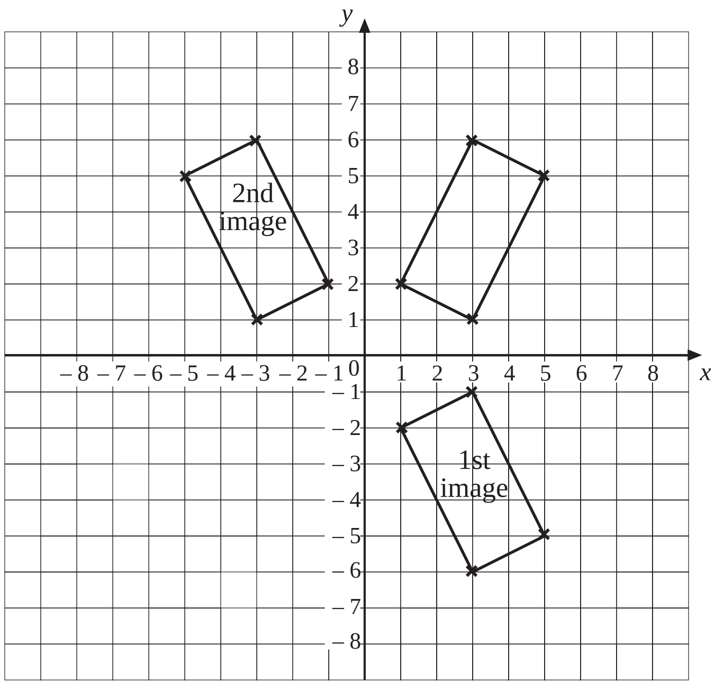
7.6 Combining Transformations

1. (a)



(b) Rotation of 180 ° about the origin.

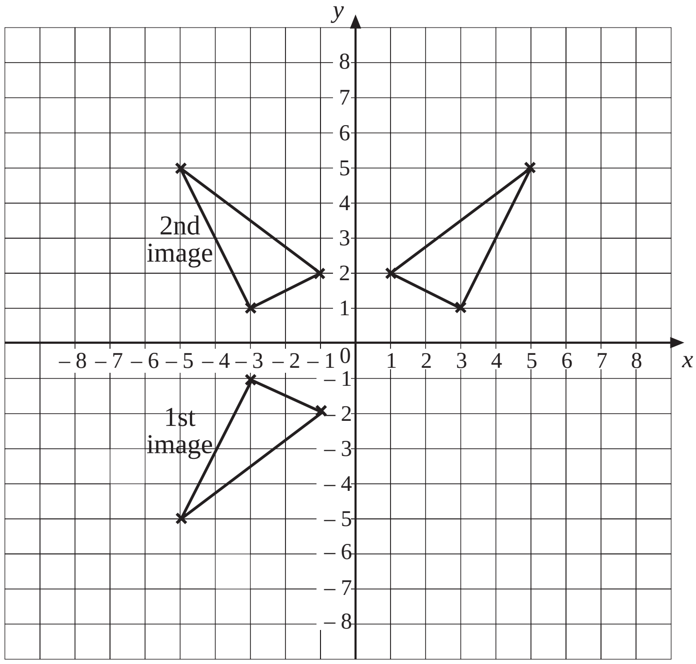
2. (a)



(b) The single transformation is a reflection in the *y*-axis.

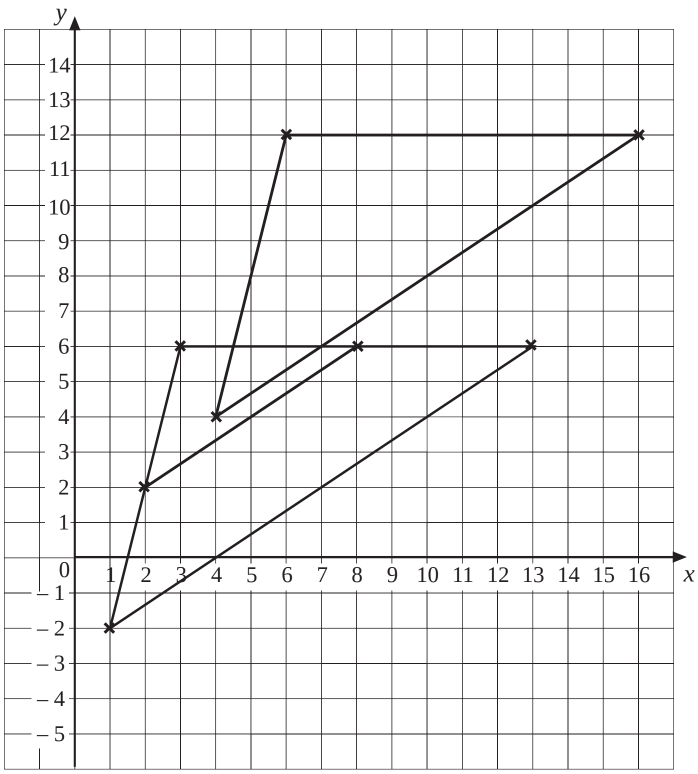
7.6 Answers

3. (a)



(b) The single transformation is a reflection in the *y*-axis.

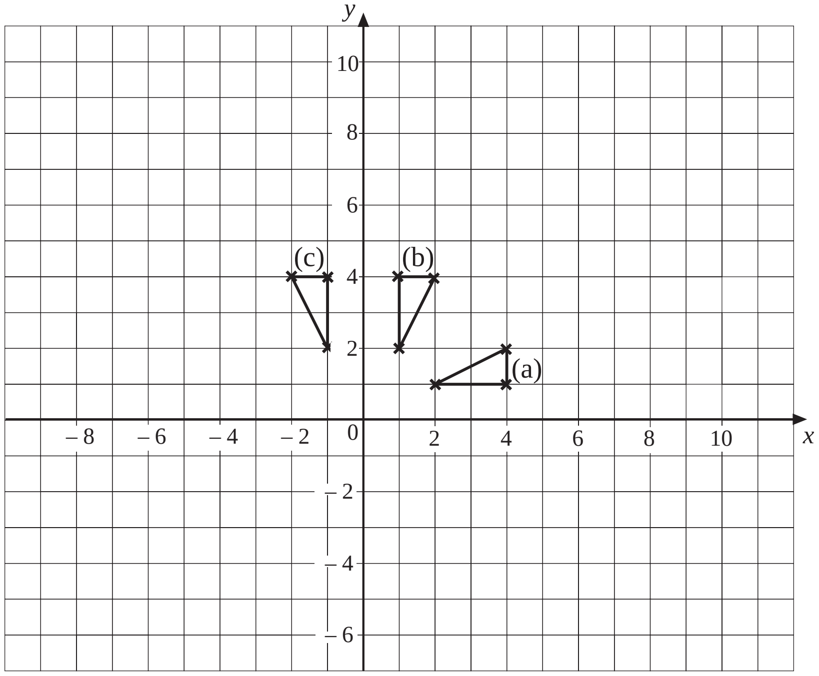
4. (a), (b)



(c) Enlargement, centre (3, 6), scale factor 2.

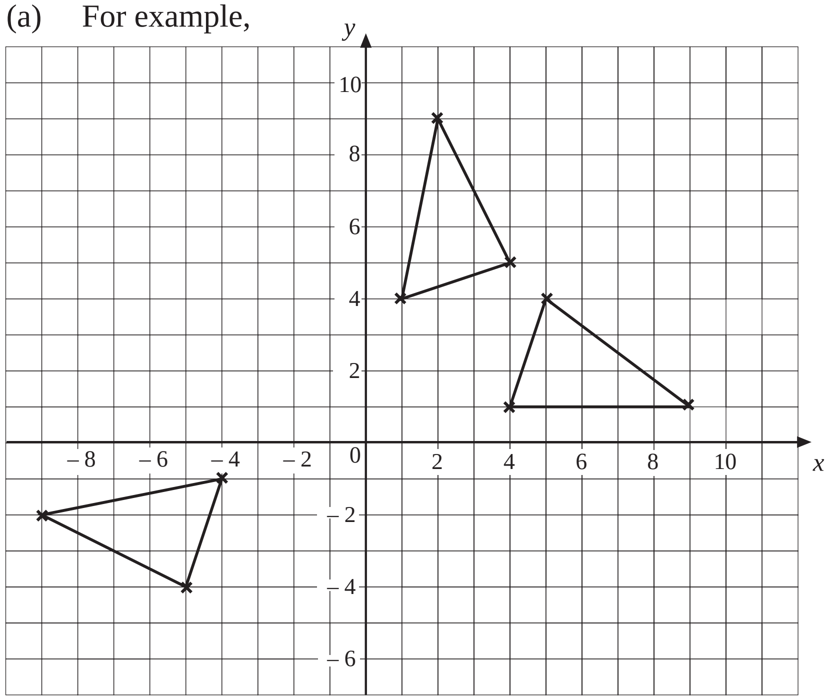
7.6 Answers

5.



(d) 90 ° anticlockwise rotation about the origin.

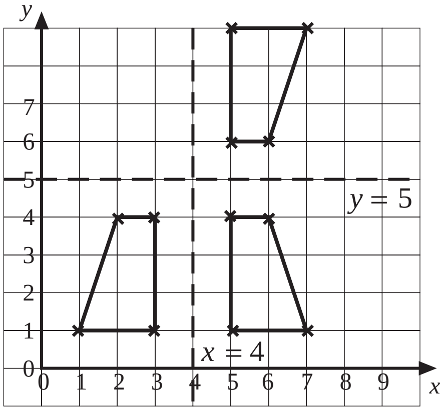
6. (a) For example,



(b) 180 ° anticlockwise rotation about the origin.

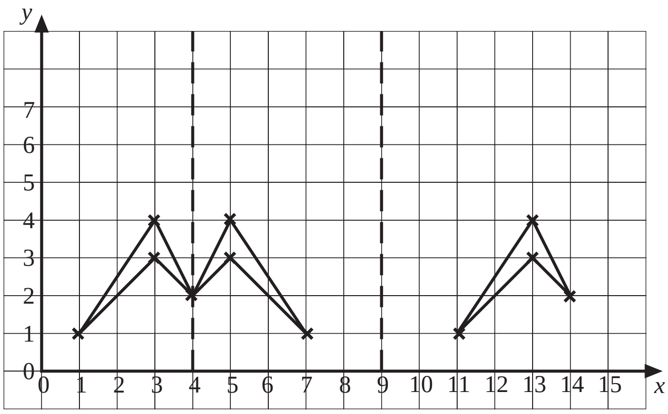
7.6 **Answers**

7. (a)



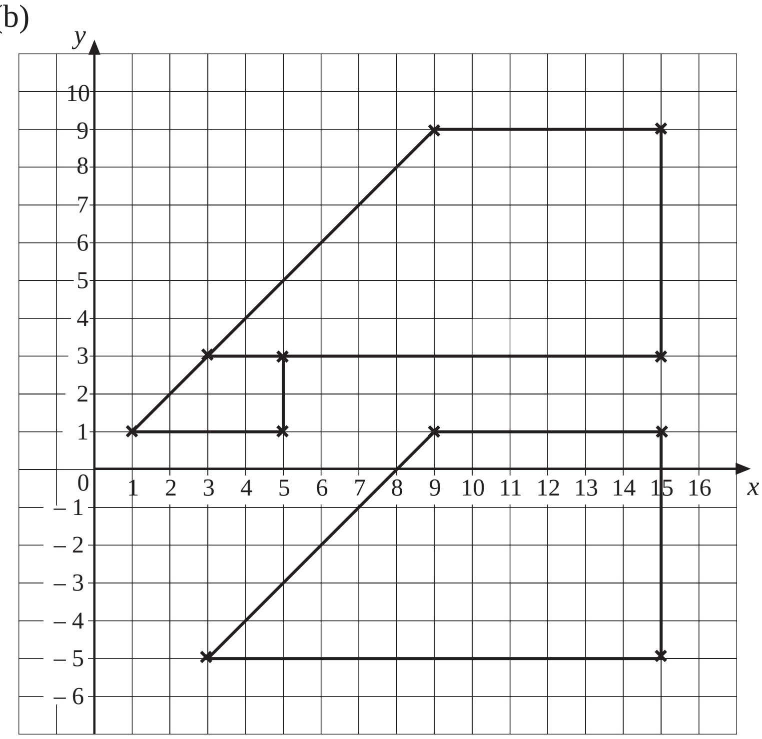
(b) 180 ° rotation about (4, 5).

8. (a)



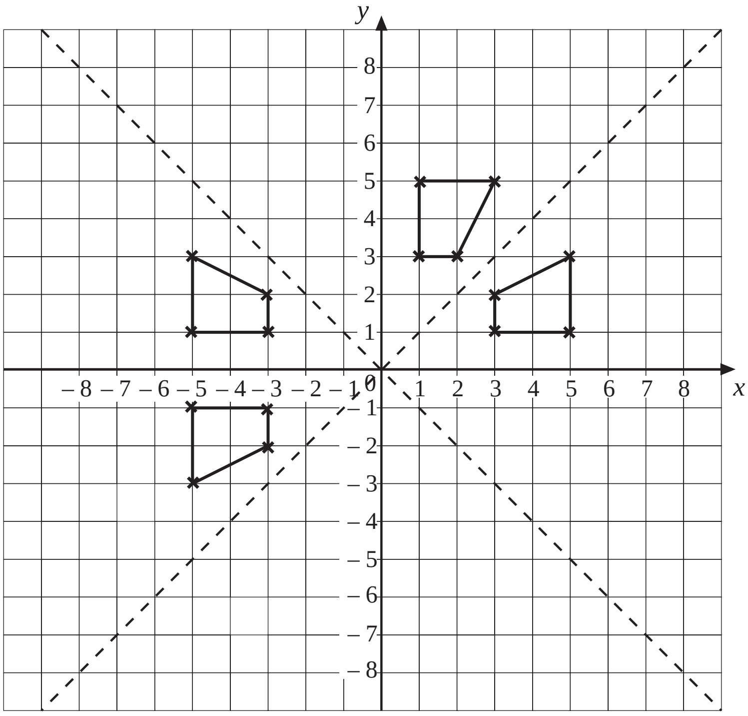
(b) There are many ways of doing this; one of the easiest is to reflect the original chevron in the line $x = 4$ and then to reflect its image in the line $x = 9$, as in the diagram.

9. (a), (b)



(c) Enlargement, scale factor 3, centre (4, 0).

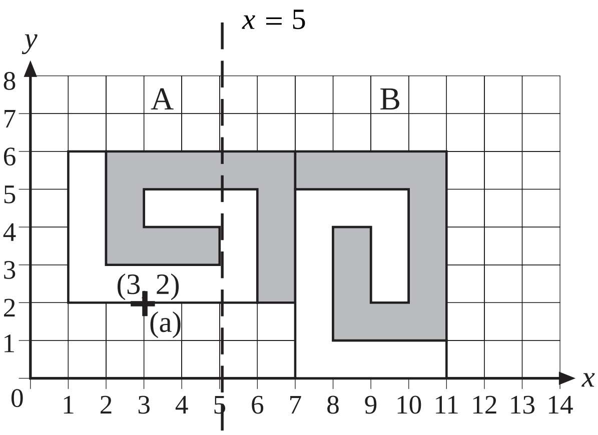
7.6 Answers

10. 

Single transformation is a reflection in the y-axis.

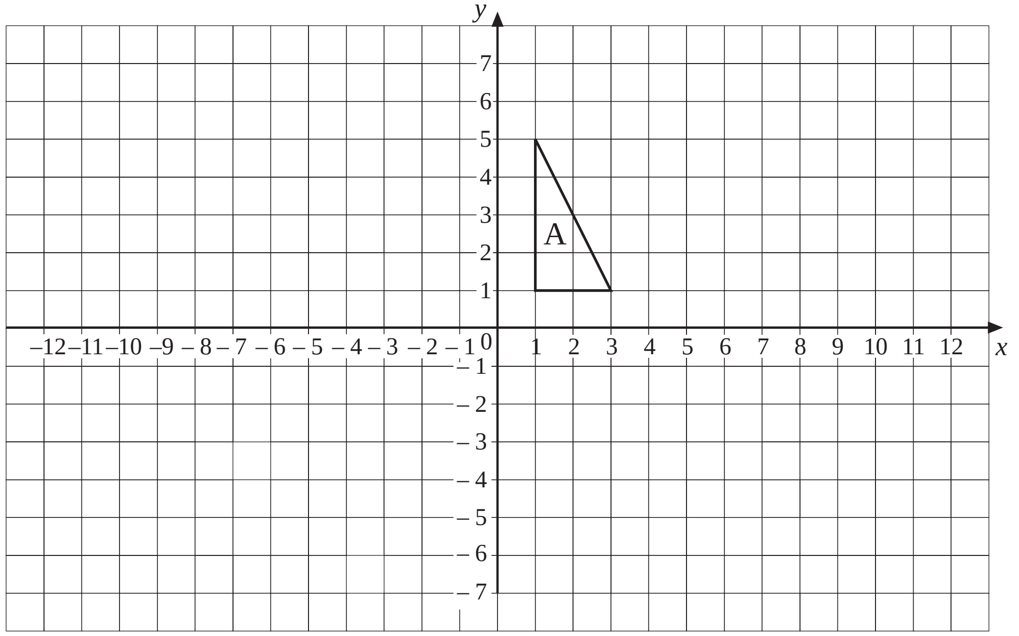
11. There are 5 possible combinations, for any suitable value of k:

- rotate through $90°$ anticlockwise about $(k, k-1)$ then reflect in the line $x = k + 2$, e.g. rotate $90°$ anticlockwise about (3, 2) then reflect in the line $x = 5$,
- reflect in the line $y = k - 3$ then rotate through $90°$ anticlockwise about $(k, k-1)$,
- rotate through $90°$ anticlockwise about $(k, k-5)$ then reflect in the line $y = k - 3$.
- reflect in the line $x = k - 2$ then rotate through $90°$ clockwise about $(k, k-5)$,
- reflect in the line $y = k - x$ then rotate through $180°$ about $\left(\frac{k+5}{2}, \frac{k+5}{2} - 3\right)$.



UNIT 7 *Transformations* **Revision Test 7.1** (Standard)

1. Copy the following diagram.

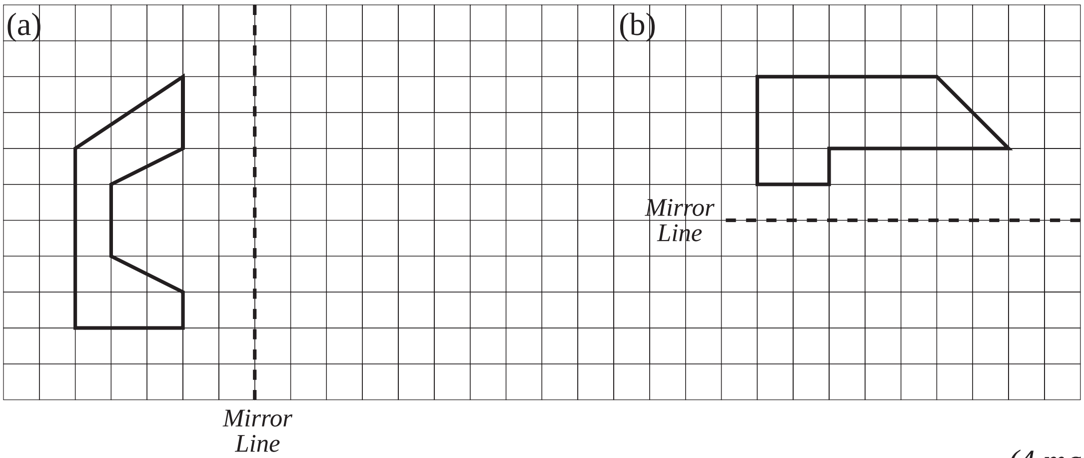


Translate the triangle A by each of the following vectors:

(a) $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$ (d) $\begin{pmatrix} -9 \\ -4 \end{pmatrix}$

(8 marks)

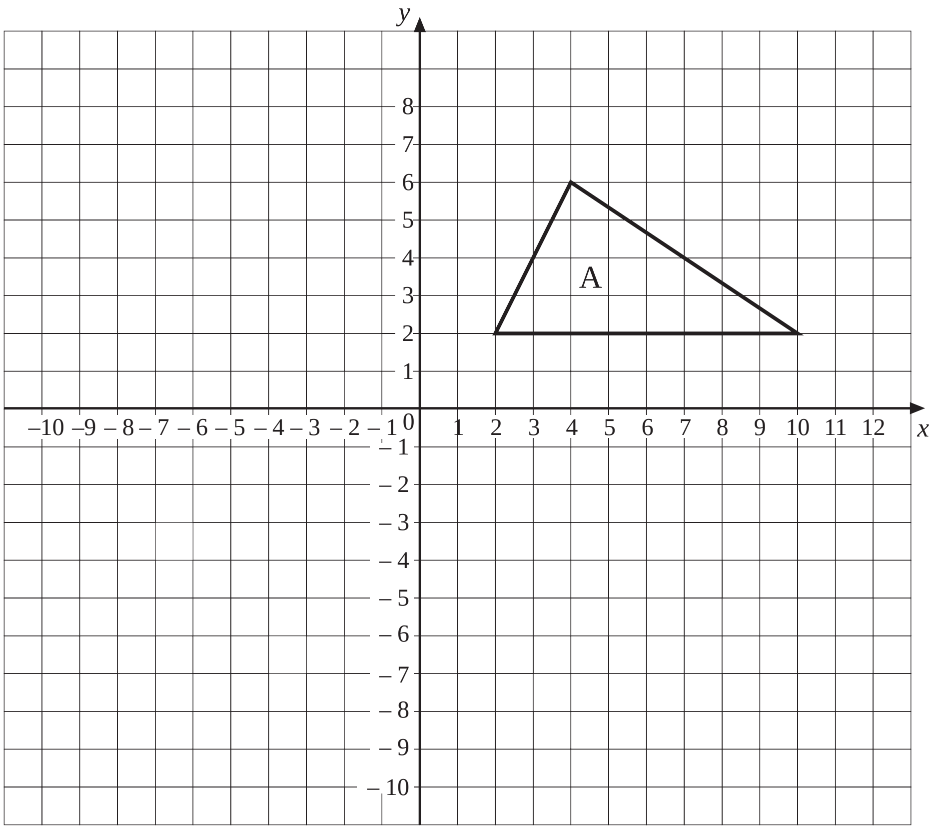
2. Copy each of the following shapes and draw its reflection in the mirror line shown:



(4 marks)

Revision Test 7.1 (Standard)

3. Copy the following diagram.



Rotate the triangle A through:

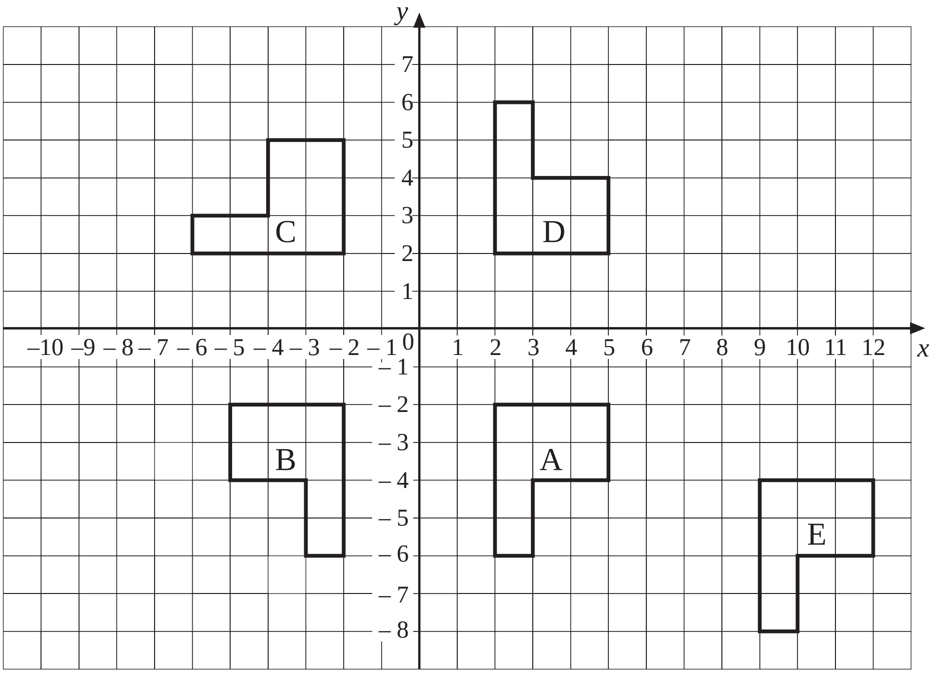
(a) 90 ° clockwise around (0, 0),

(b) 180 ° around (0, 0).

(4 marks)

Revision Test 7.1 (Standard)

4. The diagram shows some transformations of the shape A.



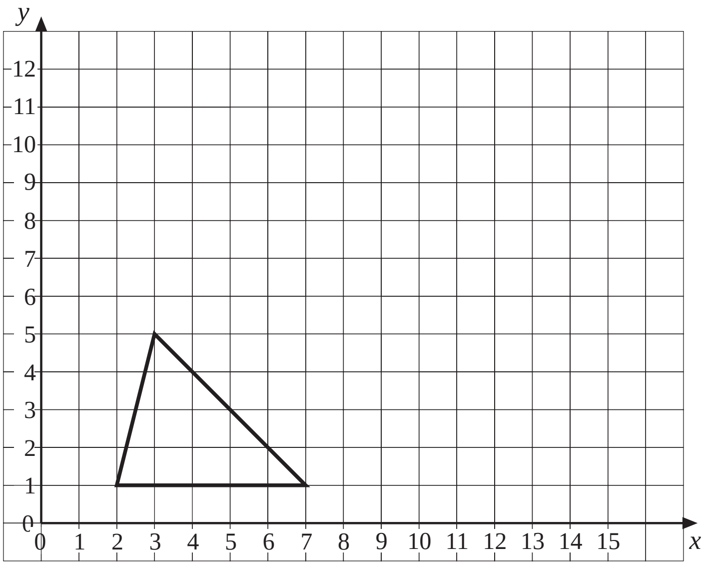
Describe the transformation that moves:

(a) A to B, (b) A to D,

(c) A to E, (d) D to C.

(10 marks)

5. Copy the diagram below. Enlarge the triangle shown with scale factor 2, using (0, 0) as the centre of enlargement.



(4 marks)

UNIT 7 *Transformations* **Revision Test 7.2 (Academic)**

1. The triangle A has corners at the points with coordinates (3, 7), (2, 4) and (4, – 2).
   (a) Draw the triangle.
   (b) Translate A using the vector $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$ to obtain B.
   (c) Translate B using the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ to obtain C.
   (d) What vector would you use to translate A straight to C ?

(7 marks)

2. The triangle A has corners at the points with coordinates (2, 2), (1, 7) and (5, 3).
   (a) Draw the triangle.
   (b) Reflect the triangle A in the *y*-axis and label it B.
   (c) Reflect the triangle A in the line $x = 8$ and label it C.
   (d) Reflect the triangle B in the line $y = 1$ and label it D.

(7 marks)

3. A quadrilateral has corners at the points with coordinates (4, 2), (3, 6), (2, 5) and (1, 1).
   (a) Draw the quadrilateral.
   (b) Rotate the quadrilateral through $90°$ clockwise around the point with coordinates (0, 0) and label it B.
   (c) Rotate the quadrilateral through $180°$ about the point with coordinates (5, 2) and label it C.

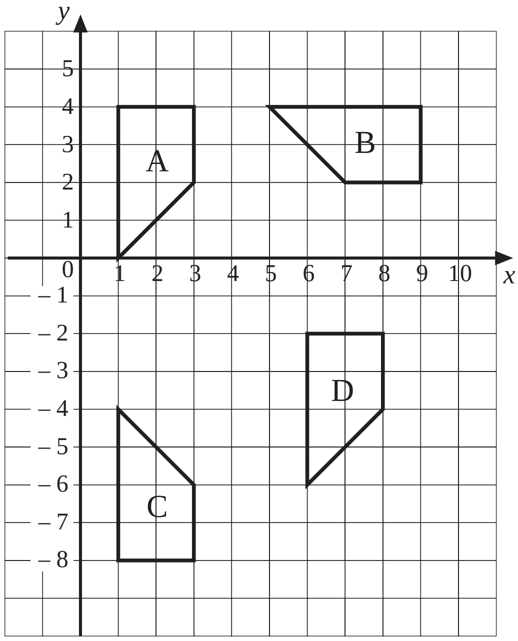
(5 marks)

4. (a) Draw the triangle which has corners at the points with coordinates (2, 2), (3, 5) and (4, 0).
   (b) Enlarge the triangle using the point with coordinates (0, 2) as the centre of enlargement and with scale factor 3.

(3 marks)

Revision Test 7.2 (Academic)

5. On the following diagram the shape A is transformed to give the shapes B, C and D.



Describe the transformations that move:

(a) A to B,

(b) A to C ,

(c) A to D.

(8 marks)

UNIT 7 *Transformations* Revision Test 7.3 (Express)

1. The triangle A has corners at the points with coordinates (4, 0), (4, 7) and (2, 4).

   (a) Draw the triangle A.

   (b) Reflect the triangle A in the line $x = 5$ to obtain triangle B.

   (c) Reflect triangle B in the *y*-axis to obtain triangle C.

   (d) Describe the transformation that would take triangle A directly to triangle C.

(5 marks)

2. The quadrilateral A has corners at the points with coordinates (2, 3), (1, 5), (2, 7) and (3, 6).

   (a) Draw the quadrilateral A.

   (b) Reflect A in the line $y = x$ to obtain B.

   (c) Reflect A in the line $y = -x$ to obtain C.

   (d) Describe how to obtain B from C.

(5 marks)

3. The pentagon A has corners at the points with coordinates (0, 1), (2, 1), (4, 3), (4, 5) and (2, 5).

   (a) Draw the pentagon A.

   (b) Rotate A through $180°$ around the point (5, 0) to obtain B.

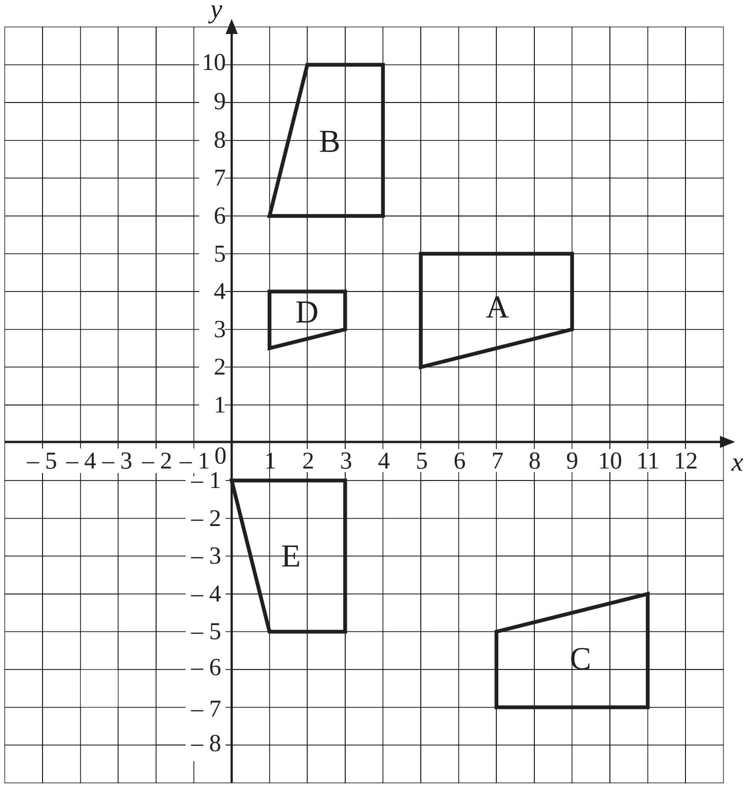
   (c) Rotate B through $90°$ anticlockwise around the point (5, – 6) to obtain C.

   (d) Describe the transformation that takes C back to A.

(8 marks)

Revision Test 7.3 (Express)

4. The diagram shows the shape A and some transformations of this shape.



Describe the transformation that moves:

(a) A to B,

(b) A to C,

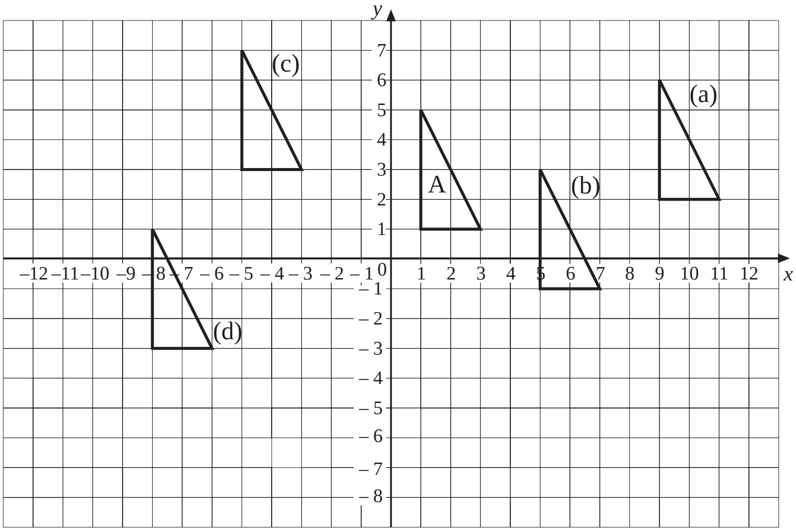
(c) A to D,

(d) C to E.

(12 marks)

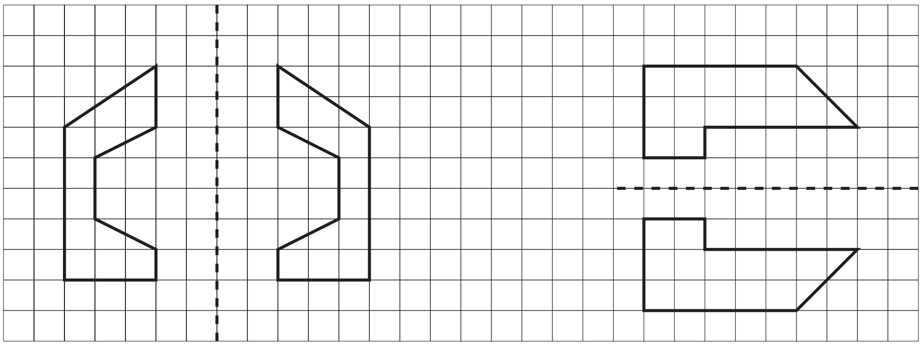
Revision Test 7.1 (Standard) Answers

1.



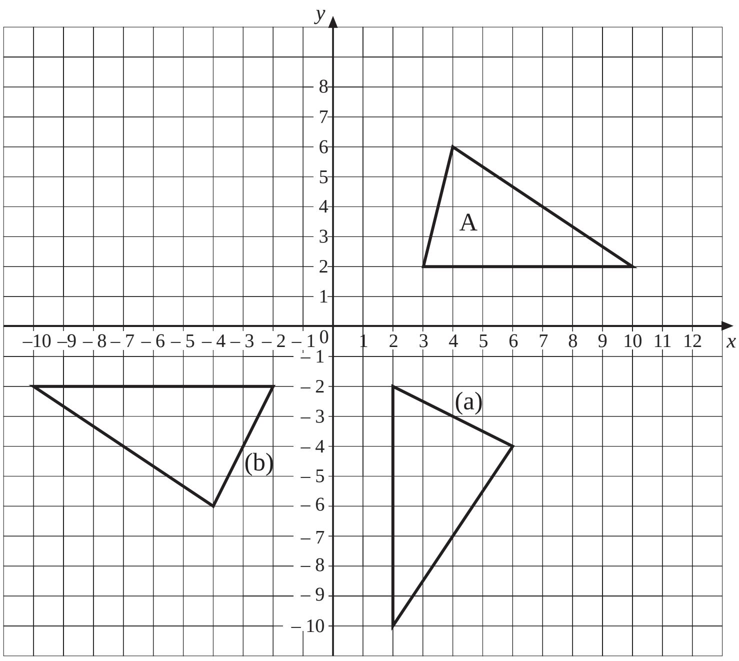
B2
B2
B2
B2 *(8 marks)*

2.



B2
B2
(4 marks)

3.



B2
B2 *(4 marks)*

Revision Test 7.1 (Standard) ANSWERS

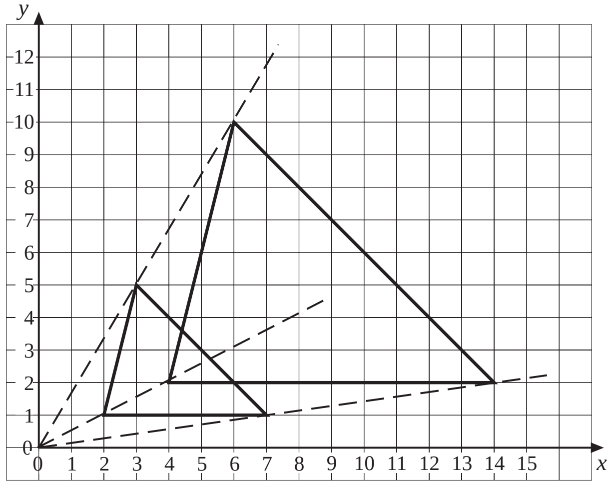
4. (a) Reflection in y-axis — B1 B1

(b) Reflection in x-axis — B1 B1

(c) Translation by $\begin{pmatrix} 7 \\ -2 \end{pmatrix}$ — B1 B1

(d) Rotation $90°$ anticlockwise about (0, 0) — B1 B1 B1 B1 *(10 marks)*

5.

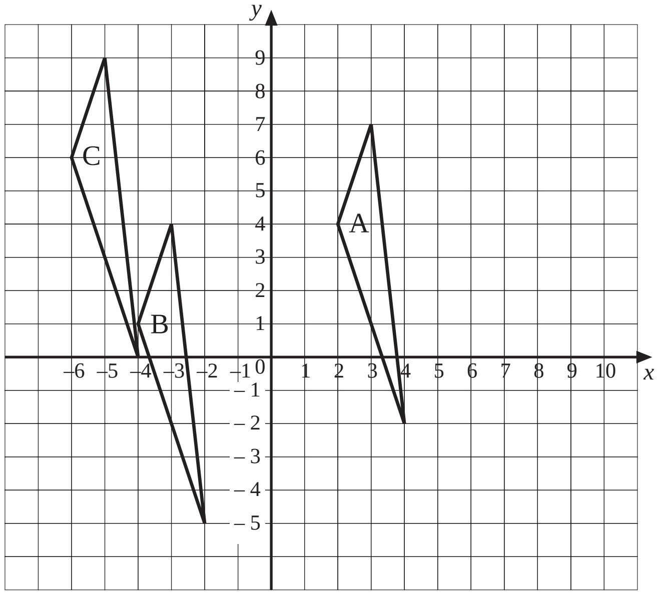


B4 *(4 marks)*

(TOTAL MARKS 30)

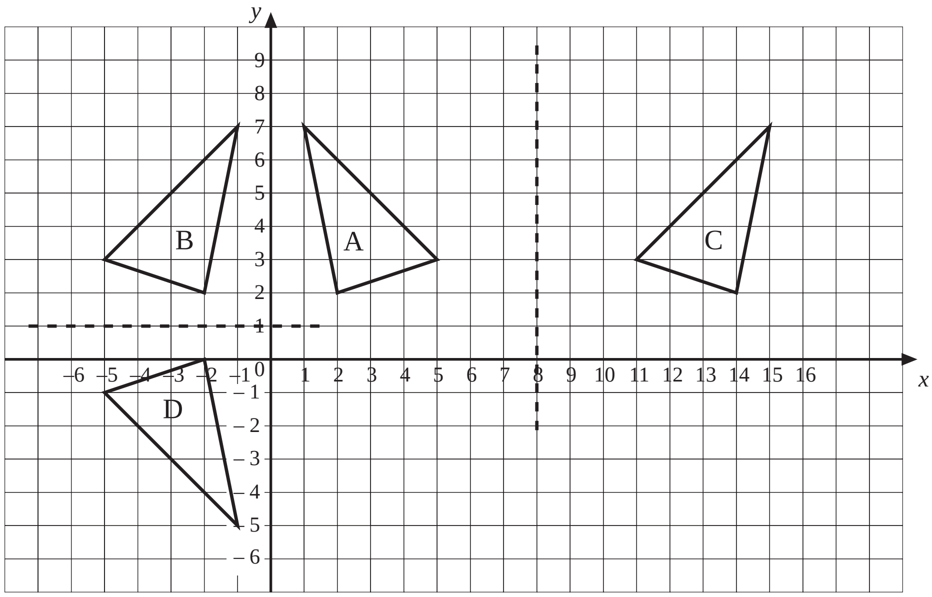
Revision Test 7.2 (Academic) Answers

1. (a) B1
   (b) B2
   (c) B2



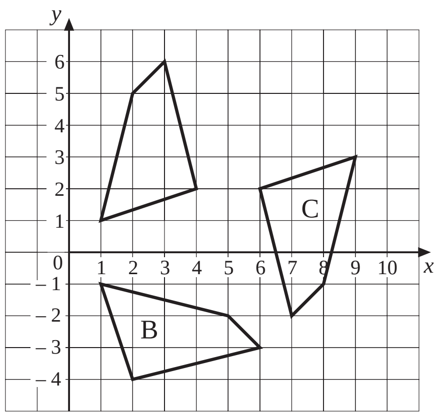
   (d) $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$ B1 B1 *(7 marks)*

2. (a) B1
   (b) B2
   (c) B2
   (d) B2



(7 marks)

3. (a) B1
   (b) B2
   (c) B2

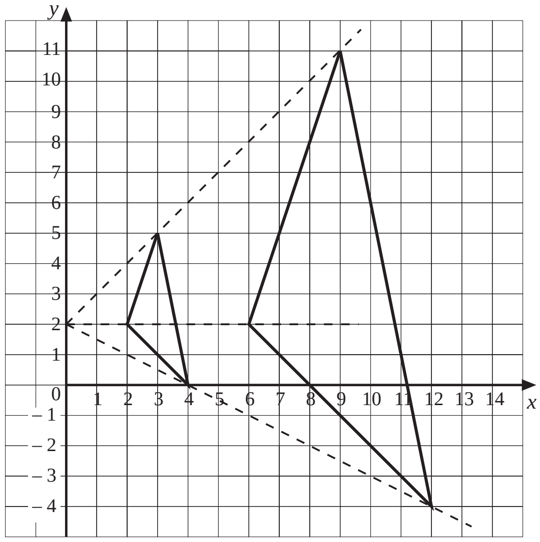


(5 marks)

Revision Test 7.2 (Academic) ANSWERS

4. (a) B1

(b) B2



(3 marks)

5. (a) Rotation, 90 ° clockwise around (5, 0) B1 B1 B1 B1

(b) Reflection in the line $y = -2$ B1 B1

(c) Translation by $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ B1 B1 *(8 marks)*

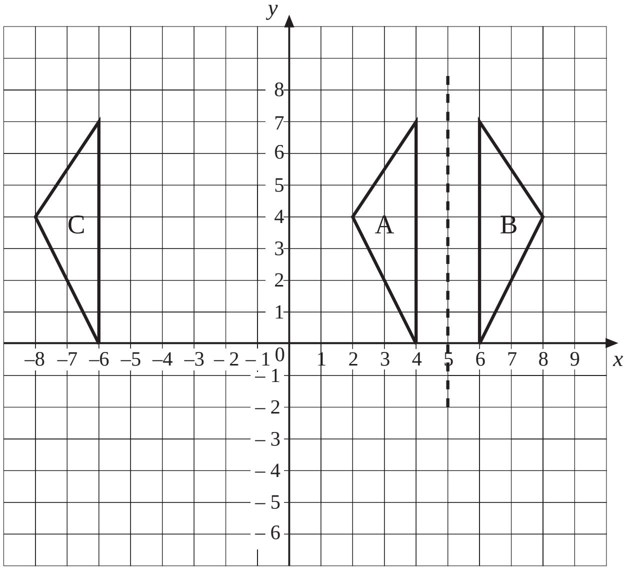
(TOTAL MARKS 30)

Revision Test 7.3 (Express) Answers

1. (a) B1

(b) B1

(c) B1

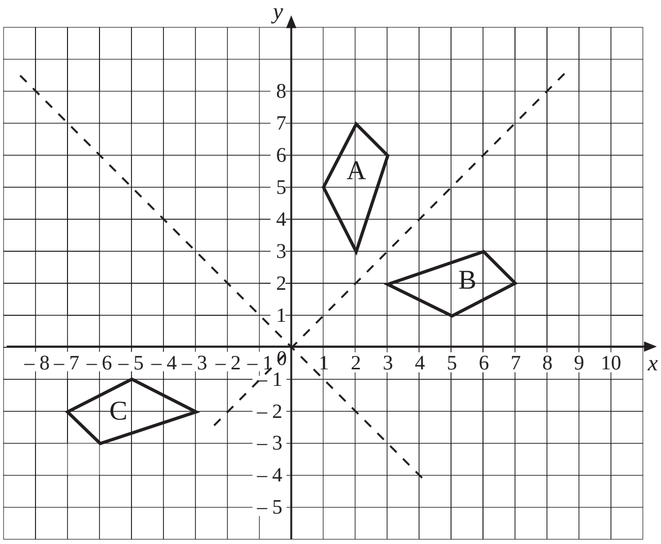


(d) Translation $\begin{pmatrix} -10 \\ 0 \end{pmatrix}$ B1 B1 *(5 marks)*

2. (a) B1

(b) B1

(c) B1

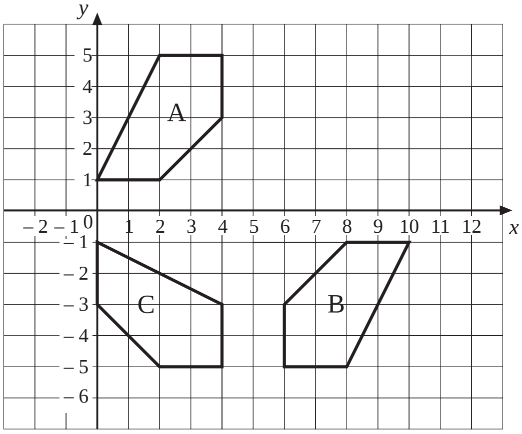


(d) Rotation 180 ° about (0, 0) B1 B1 *(5 marks)*

3. (a) B1

(b) B2

(c) B2



(d) Rotation 90 ° anticlockwise around (– 1, 0) B1 B1 B1 *(8 marks)*

Revision Test 7.3 (Express) ANSWERS

4. (a) A $\rightarrow$ B Reflection in the line $y = x + 1$ B1 B1

   (b) Rotation around (8, – 1) of $180°$ B1 B1 B1

   (c) Enlargement scale factor $\frac{1}{2}$, centre (– 3, 3) B1 B1 B1

   (d) Rotation $90°$ anticlockwise around (4, – 8) B1 B1 B1 B1 *(12 marks)*

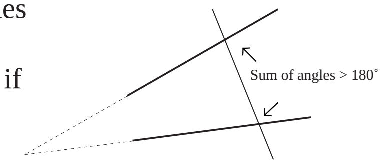
(TOTAL MARKS 30)

UNIT 7 *Transformations* Teaching Notes

Historical Background and Introduction

The 'traditional' approach to geometry, which predominated all school geometry up to the 1960s, was based essentially on *Euclid's 'Elements'*, written about 300 BC. Euclid's theorems were based on five assumptions or postulates:

1. For every point P, and for every point Q not equal to P, there exists a unique line, *l*, which passes through P and Q.
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
3. For every point O and every point A not equal to O, there exists a circle with Centre O and radius OA.
4. All right angles are equal to each other.
5. If a straight line falls on two other straight lines to make the interior angles on the same side less than two right angles, then the two lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.



(See: *Euclidean and Non-Euclidean Geometries* by M. J. Greenberg (Freeman) ISBN: 0 7167 1103 6)

The basic tools were congruent triangles and parallel lines and the subject matter mainly concerned

triangles, parallelograms and circles

and their properties.

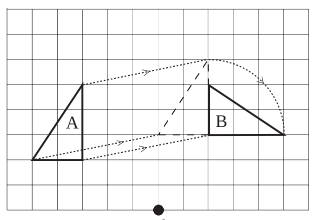
However, the German mathematician, *Felix Klein* (1849–1925), in his inaugural lecture as Professor of Mathematics at the University of Enlanger, gave a description of geometry as:

those properties of figures in space which remain unchanged under some fixed group of transformations.

This influential address led directly to the *Erlanger Programme*, which has changed radically the style of geometry taught in schools today. It led to a shift in emphasis away from congruence as the fundamental idea.

(See: *The Mathematics Curriculum: Geometry* by W.W. Willson (Blackie) ISBN: 0 216 90337 8)

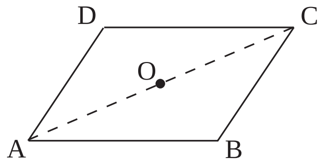
To say that two plane figures are *congruent* means that one can be moved to fit exactly onto the other. Klein's approach would be to view this as a translation and (possible) rotation. For example, A and B as shown opposite are congruent but one shape can be obtained from the other by a translation, followed by a rotation or, indeed, by a single rotation about the centre of rotation as shown.



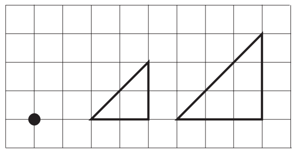
Centre of Rotation

UNIT 7 *Transformations* Teaching Notes

As another example, consider a parallelogram ABCD, as shown opposite. The triangles ABC and CDA are congruent. You can prove this either by *SAS* or *SSS* in the traditional way. Klein geometry, though, would consider rotating ADC through 180˚ about the midpoint, O, of the line AD and, in so doing, show that the triangle ADC fits exactly onto ABC. This is essentially the same mathematics but by a very different approach.



Also the traditional work on similarity can, in Klein geometry, be thought of as an enlargement with different scale factors (2 or $\frac{1}{2}$ shown opposite).



Some people may feel that geometry managed very well for more than 2000 years without transformations and that the introduction of transformation geometry is just a fad – but there are strong reasons for the use of transformations in school geometry.

One reason is that rotation, reflection, etc. can be introduced in a practical way and so should be more accessible to some pupils than the more theoretical traditional geometry. Another reason is that this geometry is, in fact, fundamental to future work, when the use of vectors becomes an integral part. It should also be noted that this approach does still provide logical and powerful analysis, although it is rather different in nature to that of traditional geometry.

This unit first revises shapes and then deals with the following transformations:

- *translations*
- *enlargements*
- *reflections*
- *rotation*

finally dealing with combinations of these transfromations.

Routes		**Standard**	**Academic**	**Express**
7.1	Shapes	✓	(✓)	×
7.2	Translations	✓	✓	(✓)
7.3	Enlargements	✓	✓	(✓)
7.4	Reflections	✓	✓	✓
7.5	Rotations	(✓)	✓	✓
7.6	Combining Transformations	×	(✓)	✓

UNIT 7 *Transformations* Teaching Notes

Language	**Standard**	**Academic**	**Express**
Translation	✓	✓	✓
Vector	✓	✓	✓
Enlargement	✓	✓	✓
Scale factor	✓	✓	✓
Centre of enlargement	✓	✓	✓
Reflection	✓	✓	✓
Mirror line	✓	✓	✓
Rotation	✓	✓	✓
Centre of rotation	(✓)	✓	✓

Misconceptions

- pupils need to realise that the mirror line for a reflection does not need to be vertical or horizontal
- rotations are not always about the origin - they can be about *any* point
- the direction, 'clockwise' or 'anticlockwise', for a rotation, *must* be stated (but note that $180°$ *clockwise* is, in fact, the same as $180°$ *anticlockwise*)